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THE
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SELECTIVE REFLECTION AND ABSORPTION IN THE ULTRA-VIOLET REGION OF THE SPECTRUM

By E. O. HULBURT

1. *Introductory.*—The phenomenon of selective reflection from the surface of certain liquids for monochromatic radiation of wave-length below 400 $\mu\mu$ has been observed by P. G. Nutting,[1] F. F. Martens,[2] and E. Flatow.[3]

Carbon disulphide, cassia oil, and monobromnaphthalene were found to exhibit traces of selective reflection for ultra-violet light. By photographic methods the positions of the maxima of reflection were determined, and the regions of absorption of thin films of the substances were mapped.

In the present investigation quantitative measurements of the reflecting powers of these and other substances have been made at various wave-lengths in the region from 210 $\mu\mu$ to 450 $\mu\mu$ by means of a grating spectrograph with a sodium photo-electric cell. At the same time the percentage transmission of thin films of the substances pressed between quartz plates has been determined. The liquids chosen for investigation were those possessing strong absorption bands and high values of the refractive index, for it is

[1] *Physical Review*, 13, 193, 1901.

[2] *Annalen der Physik*, 6, 603, 1901.

[3] *Ibid.*, 12, 85, 1903.

only in such cases that appreciable reflecting powers, i.e., values above 5 per cent, were to be expected.

The values of the reflecting power found in the present work, combined with the values of the refractive index, have been introduced into the formulae of the generalized dispersion theory to compute the optical constants of the substances.

2. *Theoretical.*—The three most important electron theories of dispersion—that of Helmholtz-Reiff-Drude, that of H. A. Lorentz, and that of M. Planck (first form)—differ only in respect to the interpretation of the meaning of the constants of the equations. The final formulae for n, the refractive index, and K, the extinction coefficient, obtained from these three theories are of the same form and are obtained by introducing a frictional term into the equation of motion of the dispersion electron.

Let ϵ be the base of natural logarithms, and let all independent variables contain the time only in the factor $\epsilon^{i\frac{2\pi c}{\lambda}t}$, where $\frac{2\pi c}{\lambda}$ is the frequency, λ the wave-length of the vibration, and c is the velocity of light *in vacuo*. Then the complete dispersion formula is

$$\frac{1}{\sigma+\frac{1}{[n(1-iK)]^2-1}}=\sum\frac{C_s\lambda_s^2\lambda^2/4\pi^2c^2}{\lambda^2-\lambda_s^2-ib_s\lambda_s^2\lambda/2\pi c}. \tag{1}$$

$\frac{2\pi c}{\lambda_s}$ is the frequency of the natural undamped vibration of the s'th electron. σ is a constant; two special types are obtained by making it zero, or $\frac{1}{3}$.

$$C_s=4\pi N_s\frac{e_s^2}{m_s},$$

where e_s is the charge, m_s the mass of the electron, and N_s is the number of such electrons per unit volume. b_s denotes absorption; if b_s is zero, K is zero, and (1) simplifies to the Sellmeier type of dispersion formula.

We assume that we are concerned with a region of absorption which is far removed from other similar regions, so that in the summation of (1) all the terms except one may be replaced by a

quantity independent of λ; hence in the vicinity of wave-length λ we write

$$\frac{1}{\sigma+\frac{1}{[n(1-iK)]^2-1}}=q_1+\frac{C_1\lambda_1^2\lambda^2/4\pi^2c^2}{\lambda^2-\lambda_1^2+ib_1\lambda_1^2\lambda/2\pi c}. \tag{2}$$

Separating real and imaginary parts, we find

$$\left.\begin{aligned}\frac{n^2(1-K^2)}{q_1'}&=1+\frac{g\lambda^2(\lambda^2-\lambda_1'^2)}{(\lambda^2-\lambda_1'^2)^2+g'^2\lambda^2}\\ \frac{2n^2K}{q_1'}&=\frac{gg'\lambda^3}{(\lambda^2-\lambda_1'^2)^2+g'^2\lambda^2}\end{aligned}\right\} \tag{3}$$

where q_1', g_2 λ_1', and g' are defined by the following equations,

$$\left.\begin{aligned}q_1'&=\frac{1+q_1(1-\sigma)}{1-q_1\sigma},\quad g=\frac{C_1}{(1-q_1\sigma)^2}\cdot\frac{\lambda_1'^2}{q_1'4\pi^2c^2}\\ \frac{4\pi^2c^2}{\lambda_1'^2}&=\frac{4\pi^2c^2}{\lambda_1^2}-\frac{C_1\sigma}{1-q_1\sigma},\quad g'=\frac{b_1\lambda_1'^2}{2\pi c}\end{aligned}\right\}. \tag{4}$$

We obtain the special case of no absorption by making b_1 zero. In that case (3) becomes

$$\frac{n^2}{q_1'}=1+\frac{g\lambda^2}{\lambda^2-\lambda_1'^2}. \tag{5}$$

The reflecting power at normal incidence, R, of the substance in contact with a medium of refractive index n' and extinction coefficient K′ is given by the formula

$$R=\frac{(n-n')^2+(nK-n'K')^2}{(n+n')^2+(nK+n'K')^2}.$$

In the present work the measurements of the reflecting powers were made on the surface of the substances in contact with quartz. We can take the extinction coefficient K' for quartz for the region from 210 to 450 $\mu\mu$ to be zero. Hence we write

$$R=\frac{(n-n')^2+n^2K^2}{(n+n')^2+n^2K^2}. \tag{6}$$

We shall be concerned with the reflecting power of the substance *in vacuo*, of the substance in contact with quartz, and of the substance in contact with fluorite. These reflecting powers are denoted by R_o, R_q, and R_f, respectively.

For many of the ideas used in the application of these formulae to the experimental results the writer is indebted to two papers by T. H. Havelock.[1]

3. *Apparatus and experimental details.*—A hydrogen end-on discharge tube, Fig. 1, served as the source of light. The tube

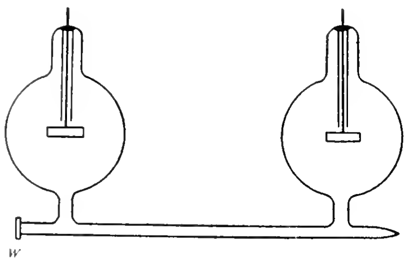

FIG. 1

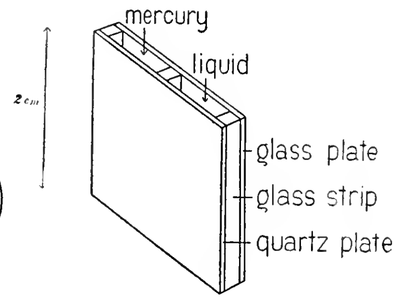

FIG. 3

was made by fusing two glass bulbs containing aluminum electrodes on to a small glass tube 14 cm in length and of 4 mm bore, and filling with hydrogen at a pressure of about 1 mm of mercury. This form of tube possessed the desirable qualities of being simple to construct and of being able to absorb considerable energy without becoming unduly heated around the quartz window, *W*, Fig. 1, which was fastened on with red sealing-wax. The tube was operated by a 250-to-1 transformer run on 60-cycle, 110-volt, alternating current, and taking 5–7 amperes in the primary circuit. The primary current was turned on and off with a contact key operated by a pendulum with a period of 5 seconds. By this means the hydrogen tube could be lighted for accurately known time-intervals of 5, 10, 15, etc., seconds.

The light from the hydrogen tube was focused on the slit *B* of the spectrograph by a quartz lens, *L*, Fig. 2, 2.8 cm in diameter

[1] *Proc. Roy. Soc.*, A, **84**, 492, 1911; A, **86**, 1, 1912.

and of focal length 15.8 cm. The spectrograph consisted of the Rowland mounting of a concave speculum metal grating, *C*, of 16 cm radius of curvature, ruled 15,000 lines to the inch, the area of ruling being 15 mm×24 mm. The spectrum was brought to a focus at slit *A*, back of which was placed the sodium photo-electric cell, *P*, with a quartz window. Slit *A* and the cell were on a movable arm so that various wave-lengths could be reached. The

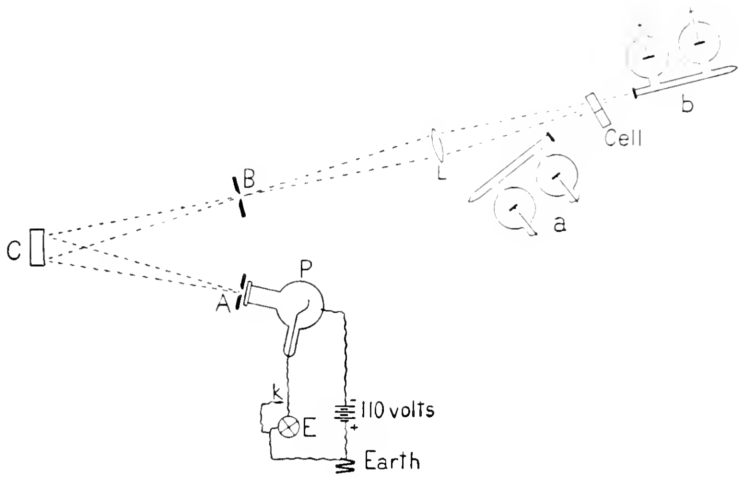

Fig. 2

dispersion was 106 A per mm. Each slit was 0.5 mm wide; the correction for finite width of slit was considered negligible. The photo-electric cell was connected to a Dolezalek quadrant electrometer, *E*, of sensibility 3600 mm per volt difference of potential between the quadrants, period 40 seconds. The deflections were observed by measuring the ballistic throw of the electrometer needle in a manner described by J. B. Nathanson.[1] The method consisted in first breaking the earth connection of those quadrants which were joined to the electrode of the photo-electric cell, then in turning on the light in the hydrogen tube for a convenient length of time, e.g., 10 seconds, and finally in observing the throw of the electrometer needle. In the present work this method had

[1] *Astrophysical Journal*, 44, 137, 1916.

especial advantage as a timesaver over the steady-deflection method because of the comparatively long period of the electrometer needle.

Single measurements of the reflecting powers, which were for the most part below 0.20, varied by as much as 25 per cent. Therefore the final recorded values were the means of many determinations and were considered accurate to perhaps 10 per cent.

The grating used in the present work was the same as the one described in a previous paper.[1] In that paper mention was made of the effect of scattered light produced by this grating and of how corrections were made for this effect. In the present case the correction for scattered light has been considered negligible, because a reflection measurement was the ratio of the intensity of a direct and a reflected beam, each of which contained probably nearly the same proportion of light of foreign wave-lengths. The spectrum produced by this grating was not particularly strong in the extreme ultra-violet, the deflections for an exposure of 15 seconds were less than 50 mm for wave-lengths below 210 $\mu\mu$. Therefore no trustworthy measurements were obtainable below 210 $\mu\mu$.

4. *Measurement of transmission.*—When the transmission (or absorption) of a liquid was to be studied, the liquid was pressed into a thin film between two plane plates of quartz. In the cases of the volatile substances the edges of the plates were surrounded by cotton wool which was kept saturated with the liquid. This insured the permanence of a film during the course of a measurement. The hydrogen tube was placed in position *b*, Fig. 2, and the absorption cell—i.e., the quartz plates and film—was placed close to the end of the tube. A measurement of transmission consisted in observing the deflection produced by the unscreened beam and that produced by the beam after passing through the cell. The ratio of the two deflections, increased by 20 per cent to correct for the transmission of the cell, gave the transmission, *t*, of the film; the values of the transmission at each wave-length plotted as ordinates against wave-lengths as abscissae formed the transmission curve.

5. *Measurement of reflection.*—For the purpose of the measurements of reflection a cell was constructed as shown in Fig. 3. This

[1] *Astrophysical Journal*, 45, 152, 1917.

cell had two compartments, one of which was filled with mercury and the other with the liquid under examination. The front face of the cell was a plane quartz plate 1 mm in thickness. The other parts were of glass; the cell was cemented with water-glass.

In the case of the reflection measurement the hydrogen tube was placed in position a, Fig. 2, and the light was reflected on the slit B of the spectrograph from the face of each compartment in turn at an angle of incidence of 7°. The cell was supported on a sliding carriage so arranged that the cell could be moved in its own plane, allowing either compartment to be placed in the path of the beam of light without displacement of the image of the source on the slit. The measurement consisted in observing d' and d, the deflections produced by light of a specified wave-length when reflected from the mercury and the liquid compartments, respectively. The ratio d/d' has been plotted in the figures on the same scale of ordinates as was used for the transmission t. Although the angle of incidence was 7°, all the reflection measurements have been treated as if they had been made for normal incidence. The error resulting from this was inappreciable in comparison with the error of experiment. The reflecting power, R_q, of the liquid surface in contact with quartz has been computed from the approximate relation

$$R_q = \frac{\frac{d}{d'} \times R' - r}{(1-r)^2}. \tag{7}$$

R' is the reflecting power of the face of the mercury compartment; r is the reflecting power of quartz, computed from the formula $r = \left(\frac{n-1}{n+1}\right)^2$, the values of n, the refractive index of quartz, for each wave-length being interpolated from the measurements of Martens (*loc. cit.*).

Before R_q could be computed, R' had first to be determined. This is described in the next section.

6. *Mercury*.—The refractive index n and the extinction coefficient K for mercury have been determined by W. Meier[1] for wave-lengths in the visible region of the spectrum and in the ultra-violet

[1] *Annalen der Physik*, 31, 1017, 1910.

as far as 325.5 $\mu\mu$, using the katoptric method of R. S. Minor.[1] P. Erochin[2] has continued the work, obtaining measurements at wave-lengths 295, 275, and 257 $\mu\mu$. In Fig. 4 the values of R_q computed by means of (6) from the values of n and K given by Meier and Erochin are shown by crosses and circles, respectively. The values of R_o are about 13 per cent greater than the corresponding values of R_q; these have not been plotted in Fig. 4.

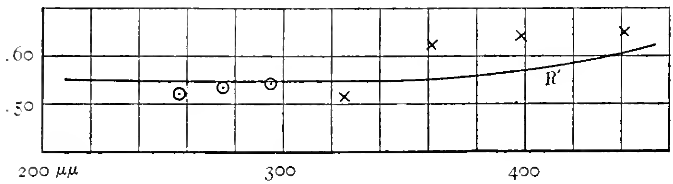

FIG. 4

The mercury used in the present work was freshly distilled according to the method of G. A. Hulett.[3] The reflecting power, R', of the face of the mercury compartment was equal to the deflection produced by the direct beam, with the hydrogen tube in position b, Fig. 2, divided into the deflection produced by the beam reflected from the face of the mercury compartment when the tube was swung around into position a, Fig. 2. The reflection cell was placed so that the optical path from the source to slit B was the same in each case.

The values of R' are shown by the smooth curve of Fig. 4; the curve represents the means of several measurements. The values of R' from this curve have been used in the subsequent calculations.

If we assume that there was no air film between the mercury and the quartz, then, approximately, for normal incidence

$$R_q = \frac{R'-r}{(1-r)^2},$$

where r is the reflecting power of the quartz. The values of R_q computed from this relation were found to be about 1 per cent greater than the values of R' shown in Fig. 4. It is seen that there

[1] *Annalen der Physik*, **10**, 581, 1903.

[2] *Ibid.*, **39**, 213, 1912.

[3] *Physical Review*, **21**, 388, 1905.

is fair agreement between these values of R_q and the values computed from the data of Meier and Erochin. This is evidence of the correctness of our assumption.

If, on the other hand, we assume that an air film existed between the mercury and the quartz, then, approximately, for normal incidence

$$R_o = \frac{R' - r - r(1-r)^2}{(1-r)^4}.$$

The values of R_o computed from this relation were found to be about 1.5 per cent greater than the values of R'. These values of R_o disagree unmistakably with those computed from the data of Meier and Erochin. Therefore it is concluded that the first assumption was correct—namely, that there was no appreciable air film between the mercury and the quartz.

7. *Carbon disulphide.*—The optical properties of this substance in the ultra-violet have been studied by Flatow,[1] Martens,[1] and W. Fricke.[2] The refractive index was found to increase with decrease in wave-length, showing a slight anomaly at 325 $\mu\mu$ (Fricke, *loc. cit.*), and then to increase rapidly for wave-lengths below 320 $\mu\mu$, the limit of the observations being at 266 $\mu\mu$. Absorption spectrograms of thin films of the pure substance, pressed between quartz plates, showed a region of minor absorption, with a maximum of absorption at about 320 $\mu\mu$ corresponding to the dispersion anomaly mentioned above, and a band of metallic absorption setting in below 250 $\mu\mu$. Martens found that a film of carbon disulphide diluted ten times with alcohol absorbed only the wave-lengths from 228 to 190 $\mu\mu$. He concluded that the pure substance probably became transparent again below 185 $\mu\mu$, and he pointed out that the dispersion-curve indicated another region of absorption at a still shorter wave-length. By a method of continued reflection from carbon disulphide in contact with fluorite, Flatow concluded that there was a band of selective reflection with its maximum at about 230 $\mu\mu$.

In the present work carbon disulphide, freshly purified by distillation, was used. The transmission-curve for a thin film of

[1] *Loc. cit.* [2] *Annalen der Physik*, **16**, 865, 1905.

the substance is given in Fig. 5. The values of the ratio d/d' (see sec. 5), determined from 210 $\mu\mu$ to 340 $\mu\mu$, are shown in Fig. 5; from these values R_q, Fig. 6, for each wave-length has been computed by means of (7). R_q has a maximum value of 0.092 at about 230 $\mu\mu$. We cannot as yet compare this result with Flatow's similar result upon the position of the maximum of reflection, because in the one case the reflection has occurred in quartz and in the other case in fluorite.

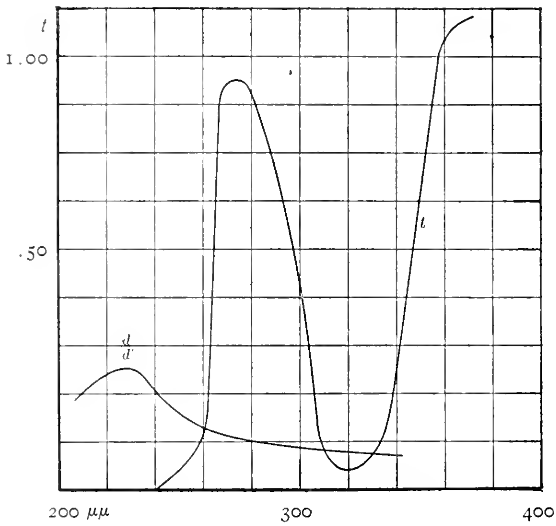

Fig. 5

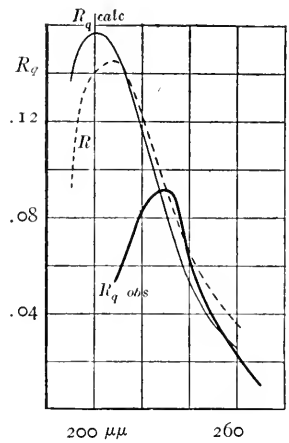

Fig. 6

In order to introduce these experimental results into the complete dispersion formula (3), only approximate methods are available. Following the method of Havelock, we ignore the region of minor absorption near 325 $\mu\mu$ and assume that we are dealing with a single principal frequency in the ultra-violet. We take the values of the refractive index as determined at 20° by Flatow—namely,

λ	441.6 $\mu\mu$	508.6	589.3
n	1.67180	1.64586	1.62806

We consider the absorption of carbon disulphide inappreciable at these wave-lengths, and hence use the simplified formula (5).

Substituting the above values of λ and n in this formula to determine the constants, we find

$$q'_1 = 1.7544 \qquad g = 0.434 \qquad \lambda'_1 = 228.4\,\mu\mu.$$

Formula (5) with these constants gives too high a value for n at 267 $\mu\mu$; e.g., $n = 2.39$, as compared with the index $n = 2.08884$ observed by Flatow. This is due to the fact that absorption has been neglected, i.e., g' has been taken to be zero. Havelock has proceeded by giving a value 0.2284×10^{-5} to g' such as to give $n = 2.0904$ at 267 $\mu\mu$ (using [3] for the calculation), which agreed sufficiently well with the observed value. In the present instance we make use of the reflection measurements to determine g'. From formula (3) at wave-length λ'_1, we have

$$n^2(1-K^2) = q'_1, \tag{8}$$

$$2n^2K = \frac{q'_1 g \lambda'_1}{g'}. \tag{9}$$

Eliminating n^2K^2 between (6) and (8), and solving for n, gives

$$n = \frac{n'}{2}\,\frac{1+R_q}{1-R_q} \pm \sqrt{\frac{n'^2}{4}\left(\frac{1+R_q}{1-R_q}\right)^2 - \frac{n'^2 - q'_1}{2}}. \tag{10}$$

At wave-length $\lambda'_1 = 228\,\mu\mu$ the observed value of R_q was 0.087. Taking the refractive index of quartz at this wave-length, n', to be 1.622 (the mean between $n_\omega = 1.616$ and $n_\epsilon = 1.628$, which have been found by interpolation from the data given by Martens (*loc. cit.*), and using the previously determined value of $q'_1 = 1.7544$, we find, from (10), $n = 1.66846$ at $228\mu\mu$. Substituting this value in (8) gives $K = 0.60810$. Using these values for n and K in (9) gives

$$g' = 0.5136 \times 10^{-5}.$$

We retain the above values of q'_1, g, and λ'_1, unaltered, for simplicity; this means that we regard K as inappreciable at the wave-lengths for which these constants were determined.

With the values of the constants thus determined n and nK have been computed by means of (3). With these values of n and nK, and with the values for the refractive index of quartz and fluorite from Martens' data, R_q and R_f have been computed for

each wave-length by means of (6); their graphs are shown in Fig. 6. The difference in R_q produced by using n_ω and n_ϵ, the indices for the ordinary and the extraordinary rays, respectively, is too slight to be of importance. The computed curve for R_q does not agree with the observed curve, giving values of R_q much too great. The maximum of the computed curve lies at about 200 $\mu\mu$, whereas the observed maximum is at about 230 $\mu\mu$. The computed curve for R_f has its maximum at about 207 $\mu\mu$, which does not agree with Flatow's observation.

The reason for the discrepancies between the observed and computed values is to be found in the incomplete nature of the dispersion formula (3). If we were dealing with the simple case of a single principal frequency, q'_1 would equal unity. The fact that q'_1 is much greater than unity shows that the carbon disulphide possesses another characteristic frequency farther in the ultra-violet, and that λ'_1 as computed above is too small. We then take $\lambda'_1 = 240\ \mu\mu$. At this wave-length we have the observed value 0.063 for R_q. Using these values for λ'_1 and R_q, and the former values of g and q'_1, in (8), (9), and (10), we find

$$g' = 0.7135 \times 10^{-5}.$$

With this new set of constants, namely,

$$q'_1 = 1.7544 \qquad g = 0.434 \qquad \lambda'_1 = 240\ \mu\mu \qquad g' = 0.7135 \times 10^{-5},$$

the values of n, nK, R_o, R_q, and R_f, have been computed from (3) and (6). The graphs are shown in Fig. 7.

The computed curve for R_q shows tolerable agreement with the observed curve. The computed values for n agree closely with the observed values for wave-lengths greater than 380 $\mu\mu$. For wave-lengths less than this the computed index falls considerably below the observed index. To lessen the discrepancy would probably necessitate a complete readjustment of all the constants. Such a procedure would be of little value at present, in view of the fact that the dispersion formula (3) is only an approximate one. The formula should contain several terms, one term for the characteristic frequency at 325 $\mu\mu$ (which is perhaps of minor importance), a second term for the frequency near 230 $\mu\mu$, a third term for a fre-

quency corresponding to some wave-length below 190 $\mu\mu$, and perhaps others. Further investigation in the extreme ultra-violet is necessary before one can attempt to evaluate the constants entering in the various terms.

Although neither of the two sets of constants used above yielded values of the optical constants which agreed satisfactorily with the observed values, yet in both cases the maxima of R_q and R_f were

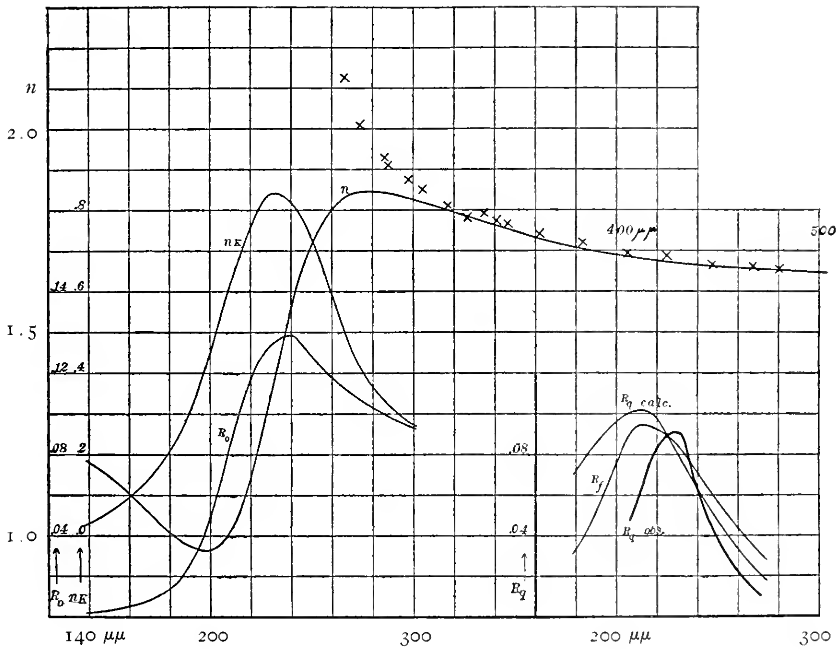

FIG. 7

found to occur at about the same wave-length (see Figs. 6 and 7). This indicates that if R_q and R_f had been computed from the true values of the optical constants their maxima would both have occurred at about the same wave-length. We conclude, therefore, that the observed maximum of R_q at about 230 $\mu\mu$ and Flatow's observed maximum of R_f also at 230 $\mu\mu$ are in accordance.

In the present instance we consider that the values of R_q and n, given in Fig. 7, which have been computed from equation (3) with the use of the second set of constants, agree sufficiently well with

the observed values to make it worth while to use the values of these constants to compute certain numerical facts concerning the dispersion electron which has its natural frequency near 230 $\mu\mu$. Putting the values for q'_1, λ'_1, g, and g', in equations (4), and using $\frac{1}{3}$ for σ, we obtain

$$q_1=0.603 \qquad b_1=2.33\times 10^{15}$$
$$C_1=30\times 10^{30} \qquad \lambda_1=219\ \mu\mu$$

We note that the natural frequency of the electron is at wave-length 219 $\mu\mu$, which is a shorter wave-length than λ'_1, the critical frequency for the medium as a whole. We next compute p the number of dispersion electrons per molecule, after the manner of Drude. From Sec. 2 we have

$$C_1=4\pi N\frac{e^2}{m_1}.$$

Let M be the molecular weight and d the density of carbon disulphide; let $\frac{e}{m_H}$ be the ratio of the charge to the mass of the hydrogen atom. Then

$$N=\frac{pd}{Mm_H} \text{ and } p\frac{e}{m_1}=\frac{m_H}{e}\frac{M}{d}\frac{C_1}{4\pi}.$$

With $M=76$, $d=1.26$, $\frac{e}{m_H}=9660$, we find

$$p\frac{e}{m_1}=1.66\times 10^7,$$

the charge being expressed in electro-magnetic units. This indicates one electron per molecule.

7. *α-monobromnaphthalene.*—The optical properties of this substance in the ultra-violet have been studied by Martens (*loc. cit.*). Absorption spectrograms showed that a thin film pressed between quartz plates transmitted no light of wave-length below 313 $\mu\mu$. Fourfold reflection from the substance in contact with fluorite showed only the lines 232 $\mu\mu$ and 228 $\mu\mu$.

The α-monobromnaphthalene used in the present work was obtained from Eimer and Amend, New York. The transmission-curve, given in Fig. 8, of a very thin film, between quartz plates,

shows complete absorption below 305 $\mu\mu$. The values of d/d' obtained from the reflection measurements are given in Fig. 8 for each wave-length in the region from 210 $\mu\mu$ to 290 $\mu\mu$. The values of R_q, computed from these values of d/d' by means of (7) are shown in Fig. 9; R_q has a maximum value of 0.092 at about 230 $\mu\mu$.

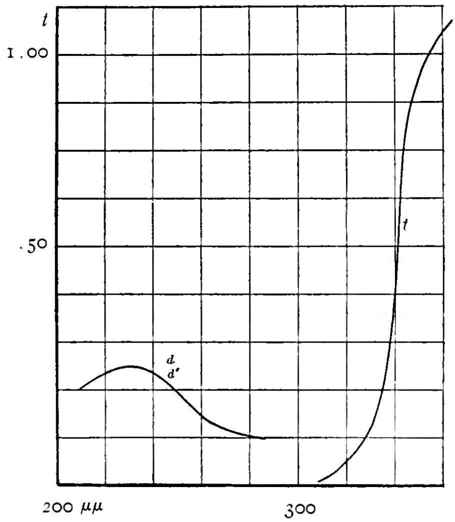

Fig. 8

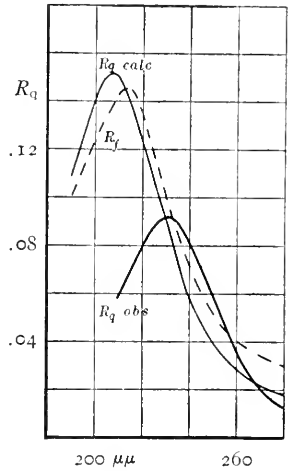

Fig. 9

We introduce these results into the dispersion formulae in a manner similar to the procedure used in the case of carbon disulphide. Using the values of the refractive index at 19°.4 C. determined by Brühl,[1] namely,

λ	434 $\mu\mu$	486	589
n	1.70433	1.68245	1.65876

to compute the constants of (5), we obtain

$$q_1'=1.811 \qquad g=0.439 \qquad \lambda_1'=229\ \mu\mu.$$

As before, we make use of the reflection measurements to determine g'. At wave-length λ_1', the observed value of R_q was 0.091. With

[1] *Ber. Chem. Ges.*, **22**, 388, 1897.

this value of R_q and with $n' = 1.622$, using (8), (9), and (10), we find

$$g' = 0.5096 \times 10^{-5}.$$

With the constants thus determined, n, nK, R_o, R_q, and R_f have been computed from (3) and (6). The graphs are shown in Figs. 9 and 10.

It is seen that the computed values of R_q are greater than the observed values and that the maximum of the computed curve for R_q lies at a shorter wave-length than does the maximum of the observed curve. These discrepancies are of the same character as were those in the case of carbon disulphide. R_q and R_f, although computed from values of the optical constants, which are probably only approximately correct, possess maxima at about the same wave-length. Following out the same reasoning as was given in the similar case of carbon disulphide, we conclude that the maximum of R_q observed at about 230 $\mu\mu$ is in accordance with the maximum of R_f also at 230 $\mu\mu$ observed by Martens.

Since data concerning the refractive index of α-monobromnaphthalene in the ultra-violet were not available, no adjustment of the constants to give better agreement between the observed and computed values of R_q was carried out. The discrepancy here, as in the case of the carbon disulphide, may be attributed to the incompleteness of the dispersion formula (3). This formula assumes that we are dealing with the region of a single principal frequency near 229 $\mu\mu$, which is remote from other similar regions. The large value of $q'_1 - 1$ and the difference between the observed and computed values of R_q indicate that such an assumption is erroneous, and that there exist other characteristic frequencies in the more remote ultra-violet.

If we use the constants determined above to compute the number of dispersion electrons per molecule, and take 1.49 for the density, we find

$$p\frac{e}{m_1} = 5.38 \times 10^7,$$

which indicates three electrons per molecule.

8. *Cinnamic aldehyde.*—Pure cinnamic aldehyde, obtained from Eimer and Amend, New York, pressed between quartz plates trans-

mitted no light of wave-length below 340 $\mu\mu$. The transmission-curve, Fig. 11, of a film of the substance diluted fifteen times with ethyl alcohol showed a strong absorption band with a maximum at about 285 $\mu\mu$ and indicated another region of absorption below 210 $\mu\mu$. The curve for R_q, Fig. 12, computed by means of (7) from the values of d/d', shown in Fig. 11, has a maximum value of 0.043 at about 285 $\mu\mu$.

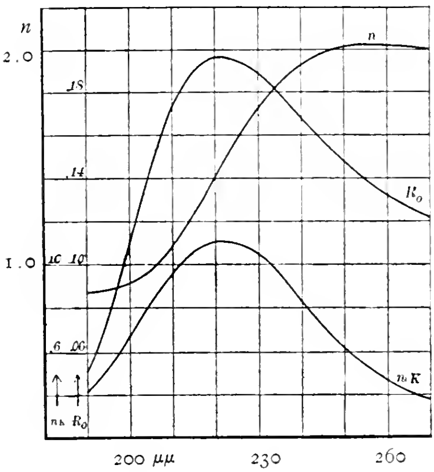

FIG. 10

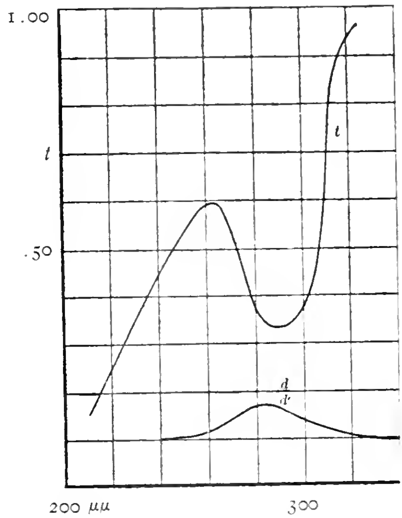

FIG. 11

Making use of the values of the refractive index determined at 20°C. by Brühl,[1] namely,

λ	434 $\mu\mu$	486	589
n	1.68295	1.65090	1.61949

to compute the constants of (5), we obtain

$$q_1' = 1.940 \qquad g = 0.275 \qquad \lambda_1' = 275\ \mu\mu.$$

Using the observed value of $R_q = 0.034$, and $n' = 1.592$ at wave-length λ_1', together with formulae (8), (9), and (10), we find

$$g' = 0.8475 \times 10^{-5}.$$

[1] *Liebigs Annalen*, **235**, 1, 1886.

With the constants thus determined, n, nK, R_o, and R_q, have been computed by means of (3) and (6). The graphs are shown in Figs. 11 and 12.

The discrepancies between the observed and computed values of R_q are of the same character as was noticed in the cases of carbon disulphide and α-monobromnaphthalene. This disagreement and the large value of $q_1' - 1$ lead to the conclusion that the cinnamic

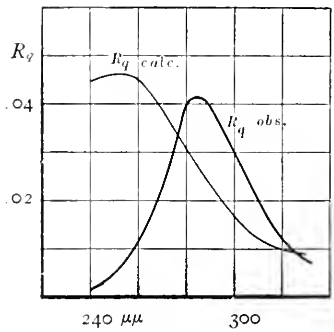

FIG. 12

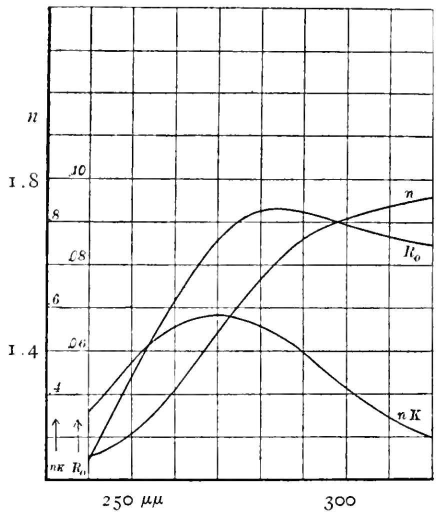

FIG. 13

aldehyde possesses other characteristic frequencies in the more remote ultra-violet.

Using the constants determined above, and with 1.05 for the density, we find

$$p\frac{e}{m_1} = 1.97 \times 10^7,$$

which indicates one electron per molecule.

9. *Cassia oil.*—This substance was given attention chiefly because it has been included by Martens in his investigations. Commercial cassia oil, such as was used in the present work, contained 75–80 per cent cinnamic aldehyde, the remainder being

cinnamic esters and terpenes. Whether this was the same as the substance used by the earlier investigators is unknown.

Martens (*loc. cit.*) found that pure cassia oil pressed between quartz plates was opaque below 336 $\mu\mu$. When diluted ten times with alcohol it was transparent as far as 305 $\mu\mu$ and at 256 $\mu\mu$ and 230 $\mu\mu$. Fourfold reflection from the substance in contact with fluorite showed only the line 274 $\mu\mu$, with no traces of the neighboring lines 257 $\mu\mu$ and 288 $\mu\mu$.

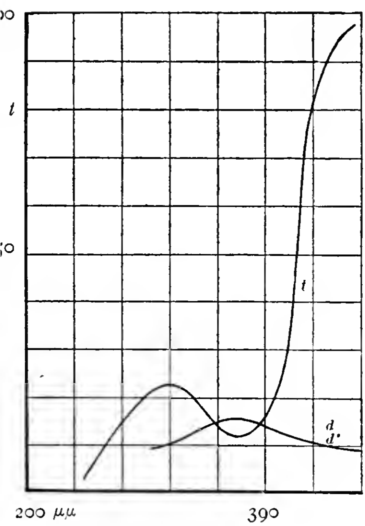

Fig. 14

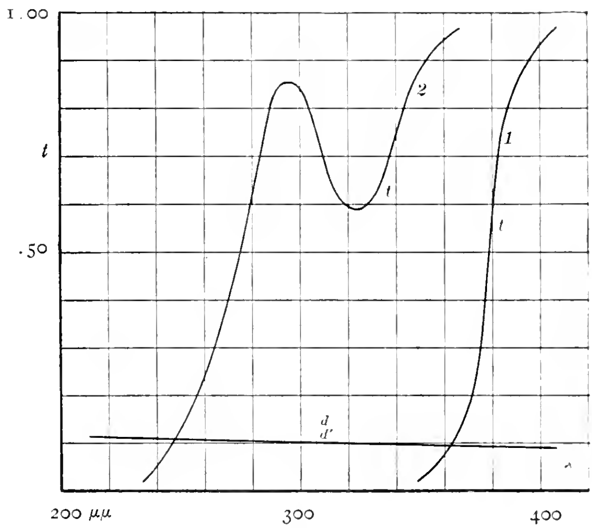

Fig. 15

The transmission-curve for a thin film of cassia oil diluted with ethyl alcohol, and the curve for d/d' (see Fig. 14), were very similar to the corresponding curves for the cinnamic aldehyde. This indicated that the cassia oil owed its ultra-violet optical properties to the presence of the cinnamic aldehyde.

10. *Barium mercury iodide.*—The transmission-curves for a thin film of a saturated aqueous solution of this salt, and for a film of a dilute solution, are given in Fig. 15, curves 1 and 2, respectively. There is a band of absorption with a maximum at about 325 $\mu\mu$; strong absorption begins again at 240 $\mu\mu$.

The refractive index of a saturated aqueous solution of this salt has been determined by C. Rohrbach.[1] Using his values—namely,

λ	486.15 $\mu\mu$	589.31	656.30
n	1.8488	1.7928	1.7752

to determine the constants of (5), we find

$$q_1' = 2.368 \qquad g = 0.2527 \qquad \lambda_1' = 319\ \mu\mu.$$

The value of λ_1' occurs near the observed maximum of the absorption band. The large value of $q_1' - 1$ indicates other critical frequencies farther in the ultra-violet.

Reflection measurements gave values of about 10 per cent for d/d', Fig. 15, throughout the region from 210 $\mu\mu$ to 350 $\mu\mu$. The values of R_q computed from (7) were less than 4 per cent, and therefore the substance showed no appreciable traces of selective reflection in this region. This was perhaps due to the fact, indicated by the absorption-curves, that the extinction coefficient did not attain a sufficiently great value in the regions of absorption at 325 $\mu\mu$ and below 240 $\mu\mu$.

11. *Phenol.*—The transmission-curve of a thin film of pure phenol, from Kahlbaum, is given in Fig. 16. There is a broad band of absorption in the region from 280 to 260 $\mu\mu$, with a maximum at about 270 $\mu\mu$. Strong absorption begins again at 230 $\mu\mu$.

Introducing the values of the refractive index determined at 40.6°C. by Eisenlohr,[2] namely,

λ	434 $\mu\mu$	486	589
n	1.56840	1.55581	1.54247

in formula (5) we obtain

$$q_1' = 1.9566 \qquad g = 0.1807 \qquad \lambda_1' = 237.6\ \mu\mu.$$

The wave-length λ_1' lies between the two regions of absorption. This fact and, also, the large value of $q_1' - 1$ indicate that we are concerned with at least two critical frequencies, one associated with the absorption band at 268 $\mu\mu$ and the other with the band beginning at 230 $\mu\mu$.

[1] *Wied. Ann.*, **20**, 172, 1883.

[2] *Ber. Chem. Ges.*, **44**, 3207, 1911.

The reflection measurements showed that phenol exhibited no marked selective reflection in the region from 210 μμ to 320 μμ. The values of R_q computed by means of (7) from the value of d/d', given in Fig. 16, were less than 4 per cent throughout this region. The low reflecting power for wave-lengths in the neighborhood of the absorption bands was due more probably to a low value of the refractive index than to a low value of the extinction coefficient.

12. *Bromine.*—F. F. Martens[1] found that bromine possessed an absorption band in the blue from 450 μμ to 390 μμ and a band in the ultra-violet which began at about 309 μμ. The refractive-index curve of bromine (Fig. 16), determined by W. Fricke,[2] showed an anomaly in the region of the blue absorption band, and then rose rapidly in the region of shorter wave-lengths, attaining a value of 1.851 at 309.19 μμ.

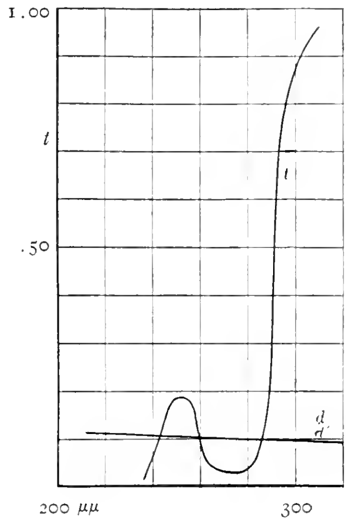

Fig. 16

In the present work the reflection measurements gave values of R_q computed from the values of d/d', Fig. 17, by means of (7), less than 4 per cent throughout the region from 210 μμ to 450 μμ. The transmission-curve, Fig. 17, of a thin film of bromine water pressed between quartz plates showed that the blue absorption band is comparatively weak and indicated that bromine is very strongly absorbing below 300 μμ. The absence of selective reflection in the region of the blue absorption band is due to the low values of the refractive index and the weak absorption. In the region of the ultra-violet band beginning at 300 μμ the data indicated high values of the refractive index and the extinction coefficient, which would imply values of the reflecting power greater than 4 per cent. There is no evident explanation for the low values actually observed.

[1] *Verhandl. d. Deutsch. Physik. Gesellsch.*, 4, 139, 1902.

[2] *Annalen der Physik*, **16**, 865, 1905.

13. *Discussion.*—It has been demonstrated that a dispersion formula such as (2), which is of the Ketteler-Helmholtz type, does not serve to express in an entirely satisfactory manner the optical properties in the ultra-violet of the three liquids, carbon disulphide, α-monobromnaphthalene, and cinnamic aldehyde. This is not surprising. The dispersion formula (2), a special case of the more general formula (1), describes the propagation of light in an ideally simple medium, one which contains a single type of dispersion

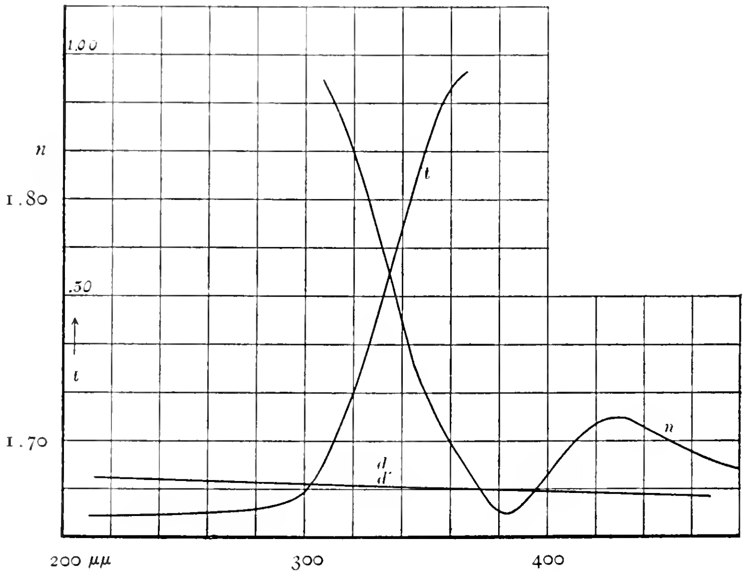

FIG. 17

electron actuated only by elastic and dissipative forces. The inapplicability of this formula in the case of the aforementioned three liquids merely goes to show that these substances cannot be considered ideal media of this kind. If we attempt to explain their optical properties by supposing them to contain additional types of electrons, each type controlled by a similar set of elastic and dissipative forces, but of varying magnitudes for each type, then (2) must be replaced by the more general formula (1), which contains a number of similar terms, one term for each type of electron. The

experimental data available at present are too meager to permit the applicability of (1) to be tested. Further, we know that in certain instances light of wave-length in the extreme ultra-violet causes the emission of electrons, etc., whereas light of longer wave-length does not. The absorption and dispersion in the substance of the short wave-lengths could not be represented by mathematical expressions of the same form as were used in the cases of the longer wave-lengths. It therefore seems probable that for the very short wave-lengths formula (1) is not a true, or at least not a complete, statement of the mechanism. At all events, further observations concerning the optical properties of these substances in the extreme ultra-violet are desirable.

The reflection coefficients of barium mercury iodide in aqueous solution, phenol, and bromine were found to be small for wave-lengths in the region of the strong ultra-violet absorption bands. The experimental data, although incomplete, have shown that, with one possible exception, the small reflectivity in these regions may be attributed to a small index of refraction, or to a small extinction coefficient, or to both.

14. *Summary.*—A grating spectrograph with a sodium photo-electric cell and electrometer has been used for quantitative measurements of monochromatic radiation in the region of the spectrum from 450 $\mu\mu$ to 210 $\mu\mu$.

In a preliminary experiment the reflecting power of mercury in contact with quartz has been determined.

The main portion of the work has consisted in a study of the selective reflection and absorption of seven liquids—carbon disulphide, α-monobromnaphthalene, cinnamic aldehyde, cassia oil, barium mercury iodide in aqueous solution, phenol, and bromine. The first four liquids have been found to exhibit traces of selective reflection; the last three have been found to show no such effect. The results have been discussed in connection with the electron theory of dispersion.

Western Reserve University
Cleveland, Ohio
May 1917

PHOTOGRAPHS OF NEBULAE WITH THE 60-INCH REFLECTOR 1911–1916[1]

By FRANCIS G. PEASE

The 60-inch reflector with its aperture ratio of F/5 is well adapted for the photography of faint nebulae and all such objects as require great light-gathering power together with considerable scale in order to show their details. For the program of observations, of which some account is given in this paper, the objects selected were in general those whose real nature was unknown or those which possessed curious or questionable characteristics. Many photographs were made under very unfavorable observing conditions, and, in consequence, for these little more than the type is discernible. At times of good seeing, well-known objects were photographed for purposes of measurement. Exposures on several of the bright planetaries were made with the 80- and 100-foot focus, Cassegrain arrangements, with a corresponding increase in scale.

The double-slide plate-holder and its manipulation have been previously described,[2] and but few changes in the instrument or in its usage have been made since that time. Since the actual photographic image is many times the theoretical diameter of the diffraction image, small changes of focus are unimportant in their effect on the images, and intervals of an hour and more were permitted between refocusing. When the image of the guiding star vibrates rapidly, it is found advisable to neglect its excursions to and fro and to follow only the slow drift of its mean center. Both eyes were used alternately for guiding; and on long exposures two guiding stars were used, and correction was made for variation in size and rotation of the field produced by refraction and imperfect adjustment of the instrument. Changes in character of the guiding image during the exposure are responsible for as many plates with elongated images as are irregularities in the driving of the telescope.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 132.

[2] *Mt. Wilson Contr.*, No. 47; *Astrophysical Journal*, **32**, 26, 1910.

Many objects are so situated as to place the observer in arduous positions for guiding, a difficulty now avoided by using a "gooseneck" microscope—a double-prism device which enables the guiding image to be viewed at a convenient angle.

All negatives were made on Seed 23 plates unless otherwise mentioned. For long exposures and for all those on fields containing bright stars the plates were backed. The usual development was with rodinal, of concentration 1/64 for 30 minutes at 20° C., though in some cases 1/32 for 15 minutes was used. All plates, except a few of early date and those otherwise noted, were taken with the full aperture of 60 inches, with a central stop of 23 inches diameter introduced to cut out the irregularly shaped cell of the Newtonian flat.

The appended list of the nebulae photographed gives (*a*) the approximate position for 1917 (the 1860 position of the *N.G.C.* corrected for precession); (*b*) the constellation; (*c*) the numbers assigned in various catalogues; (*d*) the type (El. = elliptical or elongated, Spl. = spiral, Plan. = planetary, Irr. = irregular, Spe. = spindle); (*e*) the 60-inch plate number, and (*f*), if illustrated, the number of the reproduction.

In the text following Table I is given the *N.G.C.* number of the nebula, its position for 1917, its galactic co-ordinates taken from a chart prepared by Professor Kapteyn, a list of the plates on which the object appears, and a brief description of its principal features. Much of the detail is faint and diffuse, unsuitable for comparator measures, so that the description is based on measures and estimates obtained with a magnifying glass of low power and a photographic reseau giving polar co-ordinates. The orientations have been derived from star trails, but as some of the plates are not provided with trails the results are approximate. Position angle (*p*) is measured from the north toward the east through 360°. The type of spectrum has been added wherever possible, a number of spectrograms, besides those by other observers, having been obtained with the focal-plane spectrograph of the 60-inch reflector.

A left-handed spiral has been defined as one in which an object traveling inward along one of the arms moves in a counter-clockwise direction. Many spirals which are inclined to the line of sight have

TABLE I

Negatives with the 60-inch Reflector

N.G.C.	α 1917	δ 1917	Constellation	G.C.	H and Others	h	Type	Plate Nos.	Illustrations
205	0^h 35^m 51^s	+41° 14.′2	Andromeda	105	V 18	44	El.	152	IV*a*
278	0 47 23	+47 6.0	Cassiopeia	158	I 159	71	Spl.	168	IV*b*
650-1	1 37 4	+51 9.2	Perseus	385-6	I 193, M 76	....	Irr.	57, 150, 151, 170	....
805	2 17 29	− 5 54.3	Cetus	529	II 438	219	Spl.	281	....
955	2 26 20	− 1 28.0	Cetus	551	II 278	229	Spe.	229	....
972	2 29 20	+28 56.9	Aries	560	II 211	232	El.	231	IV*e*
1023	2 35 12	+38 42.5	Perseus	575	I 156	242	El.	171	....
1068	2 38 26	− 0 21.9	Cetus	600	M 77	262	Spl.	49, 51, 53	IV*c*, *d*
1186	3 0 4	+42 30.2	Perseus	639	IV 43	281	Spl.	244, 245	....
1501	3 59 50	+60 41.9	Camelopardus	801	IV 53	....	Plan.	174, 178	I*a*
1535	4 10 23	−12 56.9	Eridanus	826	IV 26	2618	Plan.	163, 182	I*b*
1579	4 24 46	+35 6.0	Perseus	853	I 217	315	Irr.	227	III*a*
1931	5 25 55	+34 11.0	Auriga	1137	I 261	355	Irr.	277, 278	....
2022	5 37 33	+ 9 2.7	Orion	1225	IV 34	365	Plan.	183	I*c*
2366	7 20 7	+69 11.5	Camelopardus	1515	III 748	....	Irr.	279	....
2371-2	7 20 20	+29 39.1	Gemini	1519-20	II 316-17	444-5	Irr.	257	I*d*
2392	7 24 16	+21 5.0	Gemini	1532	IV 45	450	Plan.	187, 189, 190, 191, 255	I*e*, *f*
2403	7 28 49	+65 46.9	Camelopardus	1541	V 44	....	Spl.	169	V*c*
2681	8 47 36	+51 37.7	Ursa Major	1711	I 242	530	Spl.	179	IV*f*
2825	9 14 21	+34 5.1	Lynx	1806	....	581	Spe.	246, 247	....
2826	9 14 22	+33 58.0	Lynx	1807-1809	....	....	Spe.	246, 247	....
2827	9 14 23	+34 16.7	Lynx	1808	....	....	El.	246, 247	....
2828	9 14 33	+34 17.2	Lynx	1810	....	....	El.	246, 247	....
2829	9 14 39	+34 0.8	Lynx	1804	....	....	Neb. star	246, 247	....
2830	9 14 44	+34 6.0	Lynx	1811	I 113	582	Spe.	246, 247	....
2831	9 14 45	+34 6.6	Lynx	1812	....	....	Neb. star	246, 247	....
2832	9 14 46	+34 6.9	Lynx	1813	....	....	El.	246, 247	....
2834	9 15 3	+34 3.3	Lynx	1815	....	....	Neb. star	246, 247	....
2839	9 15 36	+33 59.2	Lynx	1821	....	....	Neb. star	246, 247	....
2841	9 16 18	+51 19.7	Ursa Major	1823	I 205	584	Spl.	58	V*a*
2976	9 40 24	+68 17.9	Ursa Major	1905	I 285	625	Spl.	175	V*b*
3115	10 1 6	− 7 18.9	Sextans	2008	I 163	668	Spe.	56	VI*a*
3593	11 10 17	+13 16.0	Leo	2347	I 29	840	Spl.	185	VI*b*
3666	11 20 8	+11 47.9	Leo	2405	I 20	882	Spl.	248	....
3938	11 48 31	+44 35.4	Ursa Major	2597	I 203	1002	Spl.	249	....
4206	12 11 6	+13 30.5	Virgo	2795	II 165	....	Spl.	192	....
4216	12 11 40	+13 36.4	Virgo	2806	I 35	1148	Spl.	192	VI*c*
4222	12 12 10	+13 48.0	Virgo	2841	II 109	....	Spe.	192	....
4236	12 12 48	+69 56.0	Draco	2825	V 51	1163	Irr.	201	....
4374	12 20 52	+13 20.8	Virgo	2930	M 84	1237	Neb. spot	234	....
4387	12 21 30	+13 16.0	Virgo	2955	II 167	1250	El.	234	....

4388	12 21 35	+13 7.2	Virgo	2949-2956	II 168	1244	El.	234	....
4402	12 21 55	+13 34.8	Virgo	2965	....	....	El.	234	....
4406	12 21 59	+13 24.3	Virgo	2961	M 86	1253	Neb. spot	234	....
4407	12 22 4	+13 6.0	Virgo	2968	....	1255	El.	234	....
4425	12 23 1	+13 11.7	Virgo	2983	II 170	1267	Spe.	234	....
4449	12 24 11	+44 33 0	Canes Venatici	3002	I 213	1281	Irr.	198	III*b*
4564	12 32 15	+11 53 6	Virgo	3105	II 68	1356	Spe.	235, 237	....
4567–8	12 32 20	+11 42.9	Virgo	3108–9	IV 8–9	1358–9	Spl.	235, 237	VI*d*
4594	12 35 40	−11 10.0	Virgo	3132	I 43	1376	Spl.	256, 262	VI*e*
4736	12 46 59	+41 34.3	Canes Venatici	3258	M 94	1456	Spl.	60, 3093P, 3094P	VII*a,b,c,d,e*
4900	12 56 28	+ 2 56.5	Virgo	3356	I 143	1509	Spl ?	188	....
5005	13 7 3	+37 30.2	Canes Venatici	3437	I 96	1547	Spl.	59	....
5308	13 44 15	+61 23.5	Ursa	3362	I 255	1674	Spe.	196	....
5383	13 53 41	+42 14.9	Canes Venatici	3723	I 181	1717	Spl.	195, 199	VIII*a*, *b*
5544–5	14 13 33	+36 57.3	Boötes	3833–4	II 410	1771	Spl.	127, 261	VIII*c*
5560	14 15 54	+ 4 22.3	Virgo	3845	II 579	1778	Spl.	250	....
5566	14 16 10	+ 4 19.8	Virgo	3846	I 144	1779	Spl.	250	....
5569	14 16 22	+ 4 20.1	Virgo	3848	....	....	Spl.	250	....
5740	14 40 21	+ 2 2.0	Virgo	3985	II 538	1872	Spl.	236	....
5746	14 40 43	+ 2 18.3	Virgo	3937	I 126	1874	Spl.	236	VIII*d*
5866	15 4 12	+56 5 0	Boötes	4058	I 215	1909	Spe.	129	VIII*e*
5907	15 13 44	+56 38 0	Draco	4087	II 759	1917	Spl.	197	....
6070	16 5 45	+ 0 55 7	Serpens	4164	III 553	1947	Spl.	205, 267	IX*a*
6210	16 41 1	+23 57.2	Hercules	4234	Σ 5	1970	Plan.	130	....
6217	16 36 32	+78 22.0	Ursa Minor	4236	I 280	....	Spl.	200	....
6309	17 9 24	−12 48 9	Ophiuchus	5851	....	....	Plan.	252, 263	II*a*
6412	17 32 4	+75 46.6	Draco	4321	VI 41	....	Spl.	128	....
6478	17 46 41	+51 11 6	Draco	....	SW III	....	Spl.	135	....
6543	17 58 35	+66 38 2	Draco	4373	IV 37	....	Plan.	9, 10, 11, 20, 34, 238	II*b*
6555	18 3 28	+17 35.3	Hercules	4383	II 902	....	Spl.	268	IX*b*
6702	18 44 33	+45 36.7	Lyra	5031	....	....	Neb. spot	269, 270	....
6703	18 44 55	+45 27.5	Lyra	5032	....	....	Plan. ?	269, 270	....
6804	19 27 37	+ 9 2 9	Aquila	4499	VI 38	2043	Plan.	120	....
6818	19 39 17	−14 21 1	Sagittarius	4510	VI 51	2047	Plan.	115, 116, 137	II*c*
6826	19 42 34	+50 19 4	Cygnus	4514	IV 73	2050	Plan.	130, 131, 132	....
6894	20 13 3	+30 18.6	Cygnus	4565	IV 13	2072	Plan.	4, 7	....
6905	20 18 42	+19 50 3	Delphinus	4572	IV 16	2075	Plan.	145, 147, 149	....
7008	20 58 8	+54 13 5	Cepheus	4627	I 192	2099	Irr.	243	II*d*
7009	20 59 39	−11 41 6	Aquarius	4628	IV 1	2098	Plan.	138, 140	II*e*
7023	21 0 35	+67 50 3	Draco	4634	IV 74	....	Irr.	12	....
7026	21 3 30	+47 30 8	Cygnus	....	Burnham	....	Plan.	156	....
7027	21 3 56	+41 54 0	Cygnus	....	Webb	....	Plan.	142	....
7129	21 41 4	+65 43 3	Cepheus	4702	IV 75	2131	Irr.	273, 274	....
7177	21 56 44	+17 20 4	Pegasus	4734	II 247	2139	Spl.	126	....
7217	22 4 10	+30 57.2	Pegasus	4760	II 207	2149	Spl.	216	IX*c*
7662	23 21 54	+42 4.8	Andromeda	4964	IV 18	2241	Plan.	29, 148, 157, 172, 173, 177	II*f*

NOTES

N.G.C. 2825–2829, 2831, 2832, 2834, 2839, see text description of N.G.C. 2830.
N.G.C. 4206, 4222, see text description of N.G.C. 4216.
N.G.C. 4374, 4387, 4388, 4402, 4407, 4425, see text description of N.G.C. 4406.
N.G.C. 4564, see text description of N.G.C. 4567–8.
N.G.C. 5566, 5569, see text description of N.G.C. 5560.
N.G.C. 5740, see text description of N.G.C. 5746.
N.G.C. 6702, see text description of N.G.C. 6703.

a different shading on one side of the major axis from that on the other, one side being soft while the other is more knotted and of stronger contrast, the latter resembling the effect produced by the sun shining through an irregular bank of clouds. All illustrations are placed with north (N) at the top and west (W, preceding) at the right.

All catalogued nebulae showing on any of the photographs here discussed have been inserted in the list; but those which appear only incidentally are described in the text under the *N.G.C.* number of the object for which the plate was primarily taken (see notes to Table I). A number of uncatalogued nebulae and nebulous stars are also shown. Mention of such is made in the text description of *N.G.C.* 1186, 2681, 2830, 2841, 4406, 5308, 5383, 6555, and 6703.

I am indebted to Mr. Ellerman for the preparation of the positives for the half-tones.

N.G.C. 205 Andromeda

$\alpha=0^h35^m51^s$, $\delta=+41°14'.2$ (1917); $\lambda=89°$, $\beta=-22°$
Plate No. 152, 1912, October 9, 230^m. Images large
Illustrated Plate IV*a*

This is the bright nebula lying on the N*p* side of the great nebula of Andromeda 36'.4 distant. On a diffuse ellipse $10'\times4'$, $p=172°$, gradually fading away at the edge, lies centrally a brighter patch about $1'.5\times1'$. Besides the sharp nucleus there is detail in the bright central part, and dark patches of irregular shape, similar in character to that surrounding the nucleus of the great nebula, lie one on the S end and the other on the N*f* end. The nebula is possibly spiral, but the exposure is not long enough to show detail in the outer nebulosity. Huggins[1] assigns it a continuous spectrum.

N.G.C. 278 Cassiopeia

$\alpha=0^h47^m23^s$, $\delta=+47°6'$ (1917); $\lambda=92°$, $\beta=-16°$
Plate No. 168, 1912, November 8, 240^m. Fine plate.
Illustrated Plate IV*b*

This fine left-handed spiral is seen in plan. Four or five whorls emanating from the nucleus make about a half-revolution before

[1] *Philosophical Transactions*, **156**, 388, 1866.

fading suddenly to a very much less intensity, thus forming a bright center irregularly round, 50″ in diameter. One arm, however, is interrupted, but again continues strong and forms a wing to the bright patch on the northern side. Along these arms lie many bright nebulous knots and condensations. Outside this the arms continue faint, and they fade out at 85″ diameter. The nucleus is irregular and appears to be a number of knots bunched together, each forming the starting-point of an arm.

N.G.C. 650–651 Persei

$\alpha = 1^h37^m4^s$, $\delta = +51°9'$ (1917); $\lambda = 99°$, $\beta = -11°$
Plate No. 57, 1912, February 19, 105^m. Poor plate
Plate No. 150, 1912, October 7, 180^m. Poor plate
Plate No. 151, 1912, October 8, 360^m. Poor plate
Plate No. 170, 1912, December 6, 15^m. Poor plate

The shape of this gaseous nebula reminds one of a moth; a strong, irregular, elliptical mass with diaphanous extension on either side. The body is about $1'.6 \times 1'$, $p = 41°$, with wings extending about 1′ either side. The whole forms the central zone of a circle of 3′ diameter, the wing periphery being tangent to the circle. On the E side a faint wisp extends beyond this; at the center of the circle lies a faint star. The brightest spot in the nebula, forming the S*p* portion, is N.G.C. 650; N.G.C. 651 is the N*f* portion. The spectrum of the nebula shows bright lines.[1]

N.G.C. 895 Cetus

$\alpha = 2^h17^m29^s$, $\delta = -5°54'.3$ (1917); $\lambda = 140°$, $\beta = -58°$
Plate No. 281, 1916, November 28, 75^m. Weak exposure

This is a right-handed spiral $3'.5 \times 2'$, $p = 120°$. Two prominent arms emerge from the faint nucleus, one of which, as is often the case, is interrupted only to fork out into two branches. This, however, may be merely an effect of contrast caused by an overlapping streamer. Quite away from the nucleus the arms are apparently strings of knotted nebulosity. N.G.C. 894 blends with the spiral.

[1] *Astrophysical Journal*, **33**, 59, 1911.

N.G.C. 955 Cetus

$\alpha = 2^h 26^m 20^s$, $\delta = -1°28'.6$ (1917); $\lambda = 138°$, $\beta = -54°$
Plate No. 229, 1913, November 25, 90^m. Images elongated

This is a spindle nebula, almost certainly spiral, about $80'' \times 7''$, $p = 20°$, with a stellar nucleus W of the center. At one time it was thought to be variable, but Dreyer has doubted this. The strong part of the nebula forms a rhombus $45'' \times 7''$.

N.G.C. 972 Aries

$\alpha = 2^h 29^m 20^s$, $\delta = +28°56'.9$ (1917); $\lambda = 116°$, $\beta = -28°$
Plate No. 231, 1913, November 26, 195^m. Good plate
Illustrated Plate IV*e*

This object has the shape of an irregular ellipse of greatly varying intensity, about $2' \times 1'.9$, $p = 152°$, almost certainly a left-handed spiral. The strong region E of the major axis is somewhat elliptical in shape, $40'' \times 15''$, with its axis parallel to the general figure of the nebula. Its detail is knotted and full of contrast, with a very strong knot at the N end. W of the major axis, except for the knots described below, the nebulosity consists of large softer wisps mingled with dark lanes. At 28″ N*p* the nucleus is a strong triple condensation and at 26″ S*f* the nucleus is a strong knot, both lying on the major axis. There are two more condensations beyond the N one, both surrounded by a dark ring except for a wisp on the rear side as met traveling along the arm from the center and a fine strong thread on the front side.

N.G.C. 1023 Perseus

$\alpha = 2^h 35^m 12^s$, $\delta = +38°42'.5$ (1917); $\lambda = 112°$, $\beta = -18°$
Plate No. 171, 1915, December 7, 240^m. Bad seeing

A bright nucleus 15″ in diameter lies on an ellipse $90'' \times 45''$, $p = 70°$. From this extend faint wings in either direction along the major axis, forming an ellipse $5' \times 1'$, $p = 85°$.

Fath[1] found for the nebula a spectrum of the solar type, and Slipher's[2] value of radial velocity is roughly +200 km.

[1] *Astrophysical Journal*, **37**, 199, 1913.

[2] *Popular Astronomy*, **23**, 36, 1915.

N.G.C. 1068 Cetus

$\alpha=2^h38^m26^s$, $\delta=-0°21'.9$ (1917); $\lambda=141°$, $\beta=-51°$
Plate No. 49, 1911, December 22, 120^m. Illustrated Plate IV*d*
Plate No. 53, 1911, December 25, 22^m. Illustrated Plate IV*c*
Good plates

This left-handed spiral is viewed in plan, with three striking gradations of intensity centrally superimposed. Surrounding the bright central nucleus, elongated in $p=40°$, is a strong, distinct spiral forming an ellipse $40''\times 25''$, $p=50°$, consisting mainly of two arms emanating from the ends of the nucleus, which extend a little more than a half-revolution. Distinct nebulous condensations lie along these arms. Outside this strong center is a fainter annulus $130''\times 110''$, $p=10°$, still holding the spiral form. Two of the arms are continuations of the central branches, but there are others besides, all softly mottled in appearance. Beyond this lie two immense and very faint arms, starting at about $p=30°$, and extending to an ellipse $6'\times 5'$, $p=60°$. These are so faint that it is difficult to say more than that they appear mottled. On the plate showing this external faint material the entire central spiral form is so overexposed that no detail shows. The spectrum[1] obtained with the focal-plane slit gives a value for the radial velocity of +765 km, while Slipher's[2] value is +1100 km. Fath[3] found an absorption spectrum with bright lines.

N.G.C. 1186 Perseus

$\alpha=3^h0^m4^s$, $\delta=+42°30'.2$ (1917); $\lambda=115°$, $\beta=-13°$
Plate No. 244, 1914, November 13, 75^m. Good plate
Plate No. 245, 1914, November 14, 130^m. Good plate, grain a little coarse

This supposedly variable nebula lies 2° north of Algol. It is a faint left-handed spiral $2'\times 0'.5$, $p=120°$, with faint stellar nucleus. A star several magnitudes brighter than the nucleus lies directly on one of the arms at $p=230°$, $12''$. There is a knot N of the nucleus at $p=330°$, $12''$. There are 15 faint nebulae lying within a radius of $15'$.

[1] *Publications of the Astronomical Society of the Pacific*, **27**, 134 and 192, 1915.
[2] *Popular Astronomy*, **23**, 36, 1915.
[3] *Astrophysical Journal*, **33**, 60, 1911.

N.G.C. 1501 Camelopardus

$\alpha = 3^h 50^m 50^s$, $\delta = +60°41'.9$ (1917); $\lambda = 112°$, $\beta = +7°$

Plate No. 174, 1912, December 10, 60^m

Plate No. 178, 1913, January 7, 120^m. Illustrated Plate I*a*

Good plates

This is a fine planetary of regular elliptical shape, $60'' \times 45''$, $p = 120°$, with protuberances at each end of the minor axis making the total breadth $53''$. The nebula is irregularly mottled, bearing a resemblance to the convolutions of the brain. Its periphery is in general denser than its center, and is twice as dense at the ends of the minor axis as at the extremities of the major axis. There is a bright central stellar nucleus. The spectrum shows bright lines.

N.G.C. 1535 Eridanus

$\alpha = 4^h 10^m 23^s$, $\delta = -12°56'.9$ (1917); $\lambda = 174°$, $\beta = -39°$

Plate No. 163, 1912, November 5, 60^m. 100-foot focus

Plate No. 182, 1913, February 4, 85^m. 25-foot focus. Illustrated Plate I*b*

This is a planetary of almost uniform intensity $46'' \times 40''$, $p = 23°$, on which lie a strong ring, a strong stellar nucleus, and a faint star. The ring is irregularly round, $18''$ in diameter, elongated in the same direction as the disk, quite sharp on the outer edge, but softer on the inner edge with branches running to the nucleus. The star is at $p = 320°$, $17''$. The spectrum shows bright lines; Keeler's[1] value of the radial velocity is -10.4 km.

N.G.C. 1579 Perseus

$\alpha = 4^h 24^m 46^s$, $\delta = +35°6'.0$ (1917); $\lambda = 133°$, $\beta = -8°$

Plate No. 227, 1913, October 29, 30, 31, 420^m. Illustrated Plate III*a*

This is an irregular nebula of the dispersive type; the dark lanes call to mind the Trifid nebula, and the outer faint nebulosity, that of the Orion nebula. The most prominent bit of nebulosity is a broad arrowhead adjoining the principal dark lane on the N and pointing due E; it is mottled in appearance and full of detail. Directly S of the arrowhead in the dark lane is a very faint star.

[1] W. W. Campbell, *Stellar Motions*, p. 210.

The principal part of the nebula lies within a circle of 5′ diameter, while faint patches extend Np to a distance of 10′ from the center. There are four stars which form nuclei for patches of nebulosity. Their positions with respect to the faint central star are as follows:

a.....	35° 12′.8	10	mag. (very rough)
b.....	213° 6.5	11	mag.
c.....	251° 10.0	11	mag.
B.D.+34°883.....	346° 3.1	9.5	mag.

N.G.C. 1931 Auriga

$\alpha=5^h25^m55^s$, $\delta=+34°11'.0$ (1917); $\lambda=143°$, $\beta=+1°$

Plate No. 277, 1916, October 27, 15^m
Plate No. 278, 1916, October 28, 60^m

This group of small stars, one of which is double, is here apparently superimposed upon an irregular, patchy, nebulous mass contained within a circle 1′.5 in diameter. Several loops appear in the nebulosity, and there are two prominent dark spots. At 3′.7 S of the nebula lies another star with a nebulous wing to the SW. The region is rich in stars, though the nebula lies in one of the so-called dark lanes, where the stars are much fewer than in the adjacent parts.

N.G.C. 2022 Orion

$\alpha=5^h37^m33^s$, $\delta=+9°2'.7$ (1917); $\lambda=164°$, $\beta=-9°$

Plate No. 183, 1913, February 4, 60^m. Good plate. Illustrated Plate Ic

This is a fairly bright planetary with a disk of almost uniform intensity, on which lies a ring and nucleus. The nebula is sharply outlined, and may best be described as the central zone of a circle 30″ in diameter; equator lies in $p=115°$, the distance between the sides being 27″; the S side is a little convex outward. The ring is very much stronger in intensity than the disk. It is elliptical, $20''\times15''$, $p=13°$ (median line), and on the average about 4″ wide. There is a condensation in the ring at each end of the axis in $p=30°$, making the axis 23″ long. There is an interruption in the N side of the ring and a wisp running toward the nucleus from the S knot. The spectrum shows bright lines.

N.G.C. 2366 Camelopardus

$\alpha = 7^h 20^m 7^s$, $\delta = +69°11'.5$ (1917); $\lambda = 113°$, $\beta = +29°$

Plate No. 279, 1916, October 30, 60^m. Very weak

This is the same general character as N.G.C. 4449, composed of nebulous knots and soft nebulosity. Very irregular in shape, $3' \times 1'$, $p = 30°$, with the brightest star or knot near the S end; 1′ E of this star lies a group irregularly 30″ in diameter.

N.G.C. 2371–2 Gemini

$\alpha = 7^h 20^m 20^s$, $\delta = +29°39'.1$ (1917); $\lambda = 157°$, $\beta = +21°$

Plate No. 257, 1916, March 6–7, 221^m. Bad seeing. Images large

Illustrated Plate I*d*

This gaseous nebula comprises an irregularly round, patchy mass of nebulosity lying within 33″ radius of a stellar nucleus; two wisps on opposite ends of an axis in $p = 120°$, each 1′ distant from the nucleus. The striking parts of the nebula are two strong condensations in the central part, diametrically opposite each other on an axis in $p = 60°$, each about 15″ in size and 15″ from the nucleus. In the *f* condensation is a faint star, and on the W edge of the *p* one a hazy nucleus. Faint wisps extend from the central mass to points halfway between the nucleus and the outer condensations, the W ones forming a loop, a feature seen also in N.G.C. 7026. Each of the outer wisps is $10'' \times 45''$ and lies perpendicular to the line joining them, being slightly convex outward with streamers turned inward.

N.G.C. 2392 Gemini

$\alpha = 7^h 24^m 16^s$, $\delta = +21°5'.0$ (1917); $\lambda = 165°$, $\beta = +19°$

Plate No. 187, 1913, March 6, 180^m. 25-foot focus
Plate No. 189, 1913, March 7, 10^m. 25-foot focus
Plate No. 190, 1913, March 7, 30^m. 25-foot focus
Plate No. 191, 1913, March 7, 60^m. 25-foot focus
Plate No. 255, 1915, December 8–9, 120^m. 80-foot focus
Illustrated Plate I*e*, and I*f*
Plate No. 258, 1916, April 1, 30^m. 25-foot focus

This bright planetary can best be described as the pupil of an eye with the surrounding iris. The iris varies only a few seconds from a mean outside diameter of 46″ and consists of a uniform

annulus with about 20 bright knots non-uniformly distributed along its median line. The knots are sharply bounded on the side toward the pupil and stringy toward the outside. There are several interruptions in the string of knots, notably at $p=144°$, where two lie diametrically opposite. The brighter knots are on the S side along $p=189°$, where four or five of them form an almost continuous mass. Opposite these is another continuous set, but not so bright. Between the ends of these two series the knots stand more apart. W of the "pupil," between it and the ring of knots, is an elongated condensation lying in a NS position. A 3^h exposure does not extend the disk any, but 3 threads spring from as many knots of the iris on the E side and curve out over the edge of the disk a distance of $5''$. The pupil is stirrup-shaped, $19''\times 17''$, with axis $p=5°$, and has a strong central nucleus. The N periphery is bright; the E has two bright knots, the W two, and the S one that reaches almost to the nucleus, and half-way between the nucleus and the N side is another; between this latter and the N side is a vacant space. The nucleus is listed as B.D. $+21°$ 1609, $9^m.1$. A focal-plane spectrum shows a continuous band crossed by projecting bright lines. Campbell and Moore[1] found internal motion in the nebula. Both illustrations are from the same negative, the one to show central detail, the other the external ring.

N.G.C. 2403 Camelopardus

$\alpha=7^h28^m49^s$, $\delta=+65°46'.9$ (1917); $\lambda=118°$, $\beta=+30°$

Plate No. 169, 1912, November 8, 210^m. Good plate. Illustrated Plate V*c*

This fine right-handed spiral nebula resembles M 33. The bright part is contained within an ellipse roughly $9'\times 5'$, $p=126°$. Faint knots and arms extend as far as $10'$ from the center. It does not possess the wealth of detail of M 33, but it has the same sharp stellar images, the nebulous stars, the bunching of these into knots, and the dark streaks where one can imagine one looks completely through the nebula to the sky beyond. While M 33 has two strong arms running out from the center, this nebula has but one, which starts from the central mass at the S*p* side, the opposite side being

[1] *Publications of the Astronomical Society of the Pacific*, **28**, 119, 1916.

a continuous sheet of nebulosity, not separable into arms. There is no distinct central nucleus.

N.G.C. 2681 Ursa Major

$\alpha=8^h47^m36^s$, $\delta=+51°37'.7$ (1917); $\lambda=134°$, $\beta=+41°$

Plate No. 179, 1913, January 7, 190^m. Good plate. Illustrated Plate IV*f*

This right-handed spiral is seen in plan. There is a strong central nucleus out of which springs at $p=35°$ a single arm which wraps itself concentrically around the nucleus at a mean radius of $20''$ and stops in the NE quadrant. It is possible that this arm is really continued inward in the ring of nebulosity which entirely surrounds the nucleus at a mean radius of $8''$. In the SW quadrant of this ring, however, are two strong condensations, which appear as if they might be very short arms, and a third fine wisp that starts at $p=185°$ and runs into the outer arm. Traces of the W half of a ring of $40''$ radius appear with ends forming elongated knots at $p=25°$ and $205°$, and at a mean radius of $1'.25$ is an almost continuous faint broad ring. A long exposure is necessary to determine whether the two outer rings really form part of the spiral arms. Five small faint nebulae appear on the plate.

N.G.C. 2830 Lynx

$\alpha=9^h14^m44^s$, $\delta=+34°6'.0$ (1917); $\lambda=158°$, $\beta=+46°$

Plate No. 246, 1914, November 15, 150^m
Plate No. 247, 1914, November 16, 60^m

This is a spindle nebula $50''\times6''$, $p=106°$, with faint stellar nucleus.

The ends of the nebula bend counter-clockwise, giving it a resemblance to the integral sign. The traces of detail show that it is almost certainly a spiral.

On this photograph $36'\times36'$ are 28 nebulous spots or stars, 6 spindles with nucleus, and 3 without; among them are the following:

N.G.C. 2825—A spindle about $40''\times4''$, $p=83°$, of about the same intensity as N.G.C. 2830. It has a stronger nucleus than N.G.C. 2830 and a knot N*f* the nucleus.

N.G.C. 2826—A rhombus about $1'\times6''$, $p=143°$, with nucleus and nebulosity much stronger than N.G.C. 2830.

N.G.C. 2827—Faint nucleus in elongated nebulosity in $p=8°$.
N.G.C. 2828—Nebulous star in very faint elongated nebulosity $p=45°$.
N.G.C. 2829—Nebulous star.
N.G.C. 2831—Nebulous star.
N.G.C. 2832—Strong nucleus surrounded by strong nebulosity that fades rapidly to about 14″ diameter, then gradually to 30″ diameter. The whole is elongated, $p=150°$.
N.G.C. 2834—Nebulous star.
N.G.C. 2839—Nebulous star.

N.G.C. 2841 Ursa Major

$\alpha=9^h16^m18^s$, $\delta=+51°19'.7$ (1917); $\lambda=135°$, $\beta=+45°$

Plate No. 58, 1912, February 19, 120^m. Good plate. Illustrated Plate V*a*

This is a fine left-handed spiral nebula $6'.5\times2'.2$, $p=150°$. The strong nucleus lies in an almost uniform glow of nebulosity, but the fine sweeping arms are streamers of nebulous knots. This nebula shows very nicely the difference in the nebulosity on the two sides of the major axis. The W side is softened and permeated with a glow, lacking in the E side, which is of marked contrast. In a field $36'\times36'$, 16 faint nebulae appear.

N.G.C. 2976 Ursa Major

$\alpha=9^h40^m24^s$, $\delta=+68°17'.9$ (1917); $\lambda=111°$, $\beta=+42°$

Plate No. 175, 1912, December 10, 180^m. Good negative
Illustrated Plate V*b*

This bright elliptical nebula $3'.2\times1'$, $p=142°$, is full of condensations and dark lanes, with faint extensions 25″–30″ at each end. The arrangement of some of the patches and dark lanes gives it somewhat the appearance of a spiral, but one cannot be certain. There is no nucleus, but at the center are three bright stellar knots and a fourth elongated knot. There are 11 more of these knots scattered about the nebula, together with many that range from small nebulous spots to areas barely distinguishable from the general nebulosity.

N.G.C. 3115 Sextans

$\alpha=10^h1^m6^s$, $\delta=-7°18'.9$ (1917); $\lambda=216°$, $\beta=+38°$

Plate No. 56, 1911, December 25, 100^m. Good plate. Illustrated Plate VI*a*

This is a bright spindle with an oblate center, which measures about $30''\times25''$. The disk lies in $p=45°$, is about 6″ wide, and

strong and continuous for about 45″ either side of the nucleus; then come several interruptions and knots. The whole lies within an elliptical halo of nebulosity 3′×1′. Slipher's[1] value for the radial velocity is roughly +400 km.

N.G.C. 3593 Leo

$\alpha = 11^h10^m17^s$, $\delta = +13°16'.0$ (1917); $\lambda = 210°$, $\beta = +64°$

Plate No. 185, 1913, March 5, 210^m. Good plate. Illustrated Plate VI*b*

This nebula has more or less of the spiral characteristics, but it cannot be said for certain that it is a spiral. On a faint patchy elliptical ground of nebulosity 4′.5×1′, $p = 89°$, lies centrally a bright patch 60″×13″. The E 15″ of this ellipse is not much brighter than the fainter nebulosity; the W end is crossed by two straight dark lanes, the one nearest the center running in $p = 39°$ to 219°, the other in $p = 178°$ to 358°, thus forming two isolated bright spots. At the E end of the bright central part are two stellar condensations. At the E end of the faint nebulosity there are a number of dark wisps or tongues curved left-handed. An irregular dark lane runs N of the bright center almost the length of the major axis.

N.G.C. 3666 Leo

$\alpha = 11^h20^m8^s$, $\delta = +11°47'.9$ (1917); $\lambda = 215°$, $\beta = +66°$

Plate No. 248, 1915, February 12, 150^m. Weak plate

This is a faint left-handed spiral 3′.5×0′.7, $p = 100°$, the central part 80″×20″ being brighter, and having a number of nebulous knots scattered through it, the brightest of which is the nucleus. At one time it was suspected of variability, but Dreyer's conclusions are opposed to this.

N.G.C. 3938 Ursa Major

$\alpha = 11^h48^m31^s$, $\delta = +44°35'.4$ (1917); $\lambda = 118°$, $\beta = +70°$

Plate No. 249, 1915, March 12, 39^m. Poor plate

The photograph is a very weak exposure, but sufficient to show a right-handed open spiral 4′ in diameter, having a small bright nucleus and resembling M 74 or M 101, with most of the material in the well-separated arms.

[1] *Popular Astronomy*, **23**, 36, 1915.

N.G.C. 4216 Virgo

$\alpha = 12^h 11^m 40^s$, $\delta = +13°36'.4$ (1917); $\lambda = 243°$, $\beta = +73°$
Plate No. 192, 1913, March 7, 90^m. Illustrated Plate VI*c*
Good plate, but needs longer exposure

This is a left-handed spiral $6' \times 1'$, $p = 21°$. The nucleus and the arm starting from it are bright; there is a condensation 1′ N of the nucleus. The *p* side is soft, while the *f* side shows the marked contrast usual in inclined nebulae. There are many nebulous spots scattered over the plate and 10′ *p*, 2′ N of the nucleus of 4216 lies a left-handed nebula about 30″ diameter. The two following nebulae also appear on the plate:

N.G.C. 4206—Faint spiral, $4' \times 30''$, $p = 1°$.
N.G.C. 4222—Faint spindle, $2'.5 \times 10''$, $p = 59°$.

N.G.C. 4236 Draco

$\alpha = 12^h 12^m 48^s$, $\delta = +69°56'.0$ (1917); $\lambda = 95°$, $\beta = +47°$
Plate No. 201, 1913, May 7, 180^m. Poor plate

This nebulosity lies S*p* the variable star SW Draconis. It is an irregular cloud of nebulous stars and nebulous haze roughly scattered about a region $15' \times 4'$, $p = 160°$. The strongest parts are a mixed group in the N*p* corner and a wisp extending from the N end a little to the E of southward for about 7′.

N.G.C. 4406 Virgo

$\alpha = 12^h 21^m 59^s$, $\delta = +13°24'.3$ (1917); $\lambda = 251°$, $\beta = +75°$
Plate No. 234, 1914, March 18, 90^m

The photograph shows a nebulous spot, having a bright nucleus, gradually decreasing in brightness until it fades away at a mean radius of 35″. It is slightly elongated in $p = 130°$. Besides a number of faint nebulae and spots the following nebulae appear:

N.G.C. 4374—Practically the same type and size as N.G.C. 4406. The nucleus is possibly a little larger, but the nebula lies near the edge of the plate.

N.G.C. 4387—Of the same type as N.G.C. 4406, $40'' \times 15''$. Elongated in $p = 140°$.

N.G.C. 4388—An elliptical nebula $3'.5 \times 30''$, $p = 90°$, lying near the S edge of the plate. A bright T-shaped knot, base to the N, cuts across

the center, with a dark spot on either side. A second bright elongated knot lies just W of the base of the T. Probably spiral.

N.G.C. 4402—A faint elongated nebula $3' \times 35''$, $p=89°$, dark elongated central space, bright periphery, soft outside and indented inside; probably spiral.

N.G.C. 4407—An elongated nebulous spot lying near the S edge of the plate, $20'' \times 8''$, $p=10°$.

N.G.C. 4425—A spindle consisting of nucleus 8″ diameter in uniform nebulosity about $90'' \times 22''$, $p=30°$.

N.G.C. 4449 Canes Venatici

$\alpha = 12^h 24^m 11^s$, $\delta = +44° 33'.0$ (1917); $\lambda = 96°$, $\beta = +72°$

Plate No. 198, 1913, April 7, 300^m. Illustrated Plate III*b*

This is an irregular nebulous mass in which many nebulous stars are distributed unevenly. The greater part of the nebula is roughly rectangular, about $4'.5 \times 2'.5$, $p=40°$, there being an assemblage of some dozen nebulous stars W of the SW corner. On the original negative there are 230 nebulous stars or patches, about 40 appearing bright and 190 faint, though the gradation is very uniform. Two-thirds of them are in the N half. The nebulosity proper is weak toward the edge, gradually increases inward, and culminates along a central ridge, where it is as strong as the stars themselves. A number of dark irregular rifts appear here and there in it. Wolf[1] found an absorption spectrum similar to that of the Andromeda nebula, with possible bright lines.

N.G.C. 4567–8 Virgo

$\alpha = 12^h 32^m 20^s$, $\delta = +11° 42'.9$ (1917); $\lambda = 264°$, $\beta = +73°$

Plate No. 235, 1914, March 19–20, 180^m

Plate No. 237, 1914, April 24, May 19, 360^m. Illustrated Plate VI*d*

The plate shows two fine overlapping spirals; one, N.G.C. 4567, seen more nearly in plan than N.G.C. 4568.

N.G.C. 4567 is a right-handed spiral $2' \times 1'.5$, $p=75°$, having a small bright nucleus and two arms consisting chiefly of nebulous stars. The nebulosity is weak and irregular in intensity with a number of dark lanes, notably one following the concave side of

[1] *Sitzungsberichte der Heidelberger Akad.*, August 26, 1912.

the *p* arm. There is evidence of much disturbance in the nebula, as the W arm is broken off and forked and the E arm offset.

N.G.C 4568 is a right-handed spiral $4' \times 1'.5$, $p=29°$, having a small nucleus which is fainter than that of N.G.C. 4567. It is hard to trace the arms distinctly about the nucleus, as the nebulous knots are irregularly placed. At their extremities the arms are much broken, especially on the N side, which is crossed with several great rifts; there is no question, however, as to the general trend. The dark rift at the apparent contact line of the two nebulae may really belong to N.G.C. 4568, which apparently lies beyond N.G.C. 4567.

N.G.C. 4564 appears on the plate as a spindle $90'' \times 20''$, $p=49°$, with strong nebulous nucleus, $15''$ diameter. There are 6 or more very small faint nebulous spots or spindle nebulae on the plate.

N.G.C. 4594 Virgo

$\alpha = 12^h 35^m 40^s$, $\delta = -11°10'$ (1917); $\lambda = 269°$, $\beta = +52°$

Plate No. 256, 1916, February 12, 55^m
Plate No. 259, 1916, April 6, 120^m
Plate No. 262, 1916, May 3, 132^m. Illustrated Plate VI*e*
Plate No. 264, 1916, May 26, 90^m. Seed 27 bathed Wallace 3 dye. Red screen

This fine spiral nebula, $7' \times 1'$, $p=89°$ is seen almost edge-on, the convolutions being so nearly concentric that it is not possible to state whether it is right- or left-handed. It is crossed by a dark streak which lies at the periphery, being possibly an outer ring of cooler material, or perhaps the unilluminated edge of the thin disk of nebulous matter surrounding the brilliant nucleus. The streak is $9''$ wide except near the ends, where it gradually broadens to twice this width. A trace of nebulosity runs almost centrally along the dark streak. A strong halo $2'$ in diameter surrounds the nucleus, and the first ring is much stronger than the surrounding ones. A test for possible differences in temperature, by exposures made on a Seed 23 plate and on a red-sensitive plate with a screen transmitting λ 5650–7600, showed no certain difference.

An 80-hour exposure with the focal-plane spectrograph, the slit parallel to the major axis and across the nucleus, showed that: (*a*) the spectrum is F5; (*b*) the velocity-curve is sensibly linear, $V = -278x + 1180$, the radial velocity accordingly being 1180 km

and the rotation, at a distance of $2'$, 330 km, the W side approaching and the E side receding from the observer.

N.G.C. 4736 Canes Venatici

$\alpha = 12^h 46^m 59^s$, $\delta = +41^\circ 34'.3$ (1917); $\lambda = 85^\circ$, $\beta = +76^\circ$

Plate No. 60, 1912,	February 20,	225^m.	Illustrated Plate VII*e*
Plate No. 3093P, 1916,	May 8,	5^m.	Illustrated Plate VII*b*
Plate No. 3093P, 1916,	May 8,	10^m.	Illustrated Plate VII*a*
Plate No. 3094P, 1916,	May 8,	20^m.	Illustrated Plate VII*d*
Plate No. 3094P, 1916,	May 8,	40^m.	Illustrated Plate VII*c*

This fine right-handed spiral has three regions of very marked difference in brightness. From a very bright, sharp stellar nucleus spring branches of smooth nebulosity, which after a turn about the nucleus break into a series of nebulous knots forming the periphery of an ellipse $2' \times 1'.5$, $p = 126^\circ$. Here the intensity suddenly diminishes while the arms continue in fine sweeping curves of faint smooth nebulosity, devoid of knots save for two or three small patches, to such an extent as to fill an ellipse $5' \times 4'$, $p = 105^\circ$. The nebulosity for a distance of $30''$ diameter around the nucleus is very strong. Plates 3093–4 P, taken by Mr. Seares, show the central parts well. The nebula has a solar-type spectrum,[1] with possible Wolf-Rayet bands. Its radial velocity[2] is roughly +200 km.

N.G.C. 4900 Virgo

$\alpha = 12^h 50^m 28^s$, $\delta = +2^\circ 56'.5$ (1917); $\lambda = 280^\circ$, $\beta = +66^\circ$

Plate No. 188, 1915, March 5, 210^m. Elongated images

The type of this interesting nebula cannot definitely be stated, though the lines are such as are followed by a left-handed spiral. It is irregularly round, $1'.5$ in diameter, patchy in appearance, with a number of nebulous stars, a single row around the N rim and a double row on the S side. There is a faint central stellar nucleus crossed by a bright patch of nebulosity $15'' \times 6''$, $p = 145^\circ$. A star much brighter than the nucleus lies on the S*f* point of the rim.

[1] *Astrophysical Journal*, **37**, 199–200, 1913.

[2] *Popular Astronomy*, **23**, 36, 1915.

N.G.C. 5005 Canes Venatici

$\alpha = 13^h 7^m 3^s$, $\delta = +37°30'.2$ (1917); $\lambda = 64°$, $\beta = +78°$

Plate No. 59, 1912, February 19, 50^m

This left-handed spiral $3'.5 \times 1'.3$, $p = 69°$, has a stellar nucleus surrounded by strong nebulosity which gradually fades toward the edge. The N half shows contrast, and the S half is smooth.

N.G.C. 5308 Ursa Major

$\alpha = 13^h 44^m 15^s$, $\delta = +61°23'.5$ (1917); $\lambda = 76°$, $\beta = +55°$

Plate No. 196, 1913, April 4, 180^m. Good plate

This is a spindle with oblate center $2' \times 0'.25$, $p = 60°$, having almost no detail. On either side of the bright nucleus the major axis appears as a bright line, gradually fading toward the rim and apparently interrupted at the nucleus on the *p* side. Several nebulous spots appear on the plate.

N.G.C. 5383 Canes Venatici

$\alpha = 13^h 53^m 41^s$, $\delta = +42°14'.9$ (1917); $\lambda = 48°$, $\beta = +69°$

Plate No. 195, 1913, April 3, 180^m. Images elongated. Illustrated Plate VIII*b*
Plate No. 199, 1913, May 5, 6, 360^m. Good plate. Illustrated Plate VIII*a*

This right-handed spiral resembles a pinwheel lying about a very bright multiple nucleus within a circle $2'.25$ diameter. Some disturbance has altered the regularity of distribution of the typical form. On the *f* side much of the nebulosity has been swept into a broad band running in a SE direction to a row of bright nebulous knots lying along the rim at right angles to it; enough nebulosity remains, however, to show the spiral form. In the *p* arm there are no traces of the spiral form, everything being swept into a broad band running in a NW direction from the nucleus for some distance, when it suddenly turns counter-clockwise in a bright ridge diametrically opposite and similar to that of the E arm. A dark streak runs from between the central and S parts of the nucleus and separates the *p* band into two arms; opposite this another dark interrupted streak emanating from between the central and the N members of the nucleus, runs E and S following the line of the spiral. The nucleus consists of 3 almost parallel bright condensations $15''$ to $20''$

long, elongated in $p=100°$, their central lines separated about $6''.5$, the northernmost slightly f, and the southernmost p the central one. A dark streak cuts across the three members of the nucleus, running almost due NS.

S of N.G.C. 5383, $3'.25$, lies a faint S-shaped left-handed spiral about $1'$ in diameter.

N.G.C. 5544–5 Boötes

$\alpha=14^h13^m33^s$, $\delta=+36°57'.3$ (1917); $\lambda=32°$, $\beta=+68°$

Plate No. 127, 1912, June 13, 180^m

Plate No. 261, 1916, May 1, 2, 3, 4, 5, 360^m. Illustrated Plate VIII*c*

Good negatives

These are two overlapping spirals, the E one in plan, the W one very much inclined to the line of sight.

N.G.C. 5544 is a left-handed spiral $70''\times15''$, $p=60°$, its f end just tangent to the NW point of the nucleus of N.G.C. 5545. The nucleus is faint and stellar. The arms are about equal in intensity where they start from the nucleus, but that on the E side continues bright for a much greater distance, being interrupted, however, at several points.

N.G.C. 5545 consists of a bright stellar nucleus, a nebulous ring $28''$ outside diameter, a fainter diametral streak crossing the nucleus in $p=130°$, and another ring about the same intensity as the inner one, irregularly round, $45''$ outside diameter, both being slightly elongated $p=120°$.

In N.G.C. 5545 the nebulosity is entirely soft; in N.G.C. 5544 several knots and condensations appear.

N.G.C. 5560 Virgo

$\alpha=14^h15^m54^s$, $\delta=+4°22'.3$ (1917); $\lambda=318°$, $\beta=+57°$

Plate No. 250, 1915, April 11, 200^m

This right-handed spiral is very much inclined to the line of sight. It has a weak nucleus, several knots in nebulosity near the nucleus, and two arms that make a half-revolution, then sweep outward very rapidly and fade away. It is $3'\times0'.3$, $p=105°$.

N.G.C. 5566 is a right-handed spiral, with a bright nucleus, $18''\times8''$, $p=30°$. Surrounding the nucleus is faint nebulosity from

which the arms emerge. More exposure is needed to bring out the arms well, but they sweep outward so as practically to fill an ellipse $6' \times 1'.5$, $p=32°$. An elliptical ring of nebulosity $90'' \times 45''$, $p=20°$, gives the appearance of overlapping arms.

N.G.C. 5569 is a faint right-handed spiral in plan.

N.G.C. 5746 Virgo

$\alpha=14^h40^m43^s$, $\delta=+2°18'.3$ (1917); $\lambda=323°$, $\beta=+52°$

Plate No. 236, 1914, March 20, 21, 22, 360^m. Good plate

Illustrated Plate VIII*d*

This fine right-handed spiral is seen almost edge-on, having the characteristic oblate center surrounding the bright nucleus. It is crossed by a dark streak parallel to the major axis; nebulosity full of contrast occurs on the W side, the E side being smooth. The spiral form measures $7'.5 \times 0'.75$, $p=170°$, and the oblate halo projects to a semi-minor axis of $30''$.

N.G.C. 5740 lies near the S edge of the plate. It is a left-handed spiral $3' \times 1'.25$, $p=160°$, with bright nucleus, and arms gradually weakening toward the edge. An asteroid trail appears (March 20) $p=234°$, $19'$, S*p* the nucleus, the trail lying in $p=158°$. Through the kindness of Dr. Leuschner, Miss Levy identified this as (304) Olga.

N.G.C. 5866 Boötes

$\alpha=15^h4^m12^s$, $\delta=+56°5'.0$ (1917); $\lambda=58°$, $\beta=+52°$

Plate No. 129, 1912, June 14, 165^m. Illustrated Plate VIII*e*

This nebula is lenticular, $2'.5 \times 0'.75$, $p=126°$, with no apparent nucleus, but with a bright center gradually decreasing in intensity toward the edge. Lying across the center and making an angle of $3°$ with the major axis is a narrow dark streak, $p=123°$, about $1'$ long. Overlapping this at either end, and extending along the major axis to a distance of $45''$ in either direction from the center, lies a streak as bright as the central nebulosity, which gradually fades out toward the end. Slipher's[1] value for the radial velocity is +600 km.

[1] *Popular Astronomy*, **23**, 36, 1915.

N.G.C. 5907 Draco

$\alpha=15^h13^m44^s$, $\delta=+56°38'$ (1917); $\lambda=57°$, $\beta=+51°$

Plate No. 197, 1913, April 4, 90^m. Weak plate

This nebula is a spiral seen edge-on, similar to N.G.C. 5746 in having a longitudinal absorption streak lying just to one side of the nucleus. It measures $11'\times0'.7$, $p=156°$.

N.G.C. 6070 Serpens

$\alpha=16^h5^m45^s$, $\delta=+0°55'.7$ (1917); $\lambda=340°$, $\beta=+35°$

Plate No. 265, 1916, May 26, 75^m
Plate No. 267, 1916, May 27, 150^m. Illustrated Plate IX*a*

This is a right-handed spiral $3'\times1'.3$, $p=59°$. Several arms dotted with nebulous stars make a full turn or more; those ending on the *f* side are well defined, those on the S*p* side diffused and free of knots. Directly *f* the nucleus two arms of the same curvature appear to overlap. There is a small stellar nucleus in an elongated nebulous knot. In a line a little to E of N of the nucleus the arms are much reduced in intensity.

N.G.C. 6210 Hercules

$\alpha=16^h41^m1^s$, $\delta=+23°57'.2$ (1917); $\lambda=11°$, $\beta=+36°$

Plate No. 139, 1912, July 13, 60^m. 100-foot focus

This is the well-known bright planetary nebula, Struve No. 5. The negative is overexposed, but one can see a bright central nucleus, bright nebulous streaks, fainter short curves bowed outward, some of them fading before returning, and several faint extensions, all of which give the nebula an angular appearance. The focal-plane slit spectrum is continuous for the star and crossed by bright lines projecting on either side. Keeler's[1] value of the radial velocity is -34.3 km and Campbell and Moore[2] have found internal motion.

[1] W. W. Campbell, *Stellar Motions*, p. 210.

[2] *Publications of the Astronomical Society of the Pacific*, 28, 120, 1916.

N.G.C. 6217 Ursa Minor

$\alpha = 16^h 36^m 32^s$, $\delta = +78°22'$ (1917); $\lambda = 79°$, $\beta = +33°$

Plate No. 200, 1913, May 6, 60^m. Weak plate

This right-handed spiral has a stellar nucleus. Nebulosity is noticeably absent near nucleus. There are nebulous knots along the arms. It measures $1'.5 \times 1'$, $p = 160°$.

N.G.C. 6309 Ophiuchus

$\alpha = 17^h 9^m 24^s$, $\delta = -12°48'.9$ (1917); $\lambda = 337°$, $\beta = +13°$

Plate No. 252, 1915, May 10, 90^m. Illustrated Plate II*a*

Plate No. 263, 1916, May 5, 10^m

The plate shows a nebulous spot with a well-defined arrowhead at the S end, a blunt arrowhead at the N end, both pointing outward, the two connected by faint nebulosity. A faint stellar nucleus lies at the base of the N head a little N of the center of the nebula. It measures $22'' \times 12''$, $p = 162°$. Keeler's[1] value of the radial velocity is -51.5 km.

N.G.C. 6412 Draco

$\alpha = 17^h 32^m 4^s$, $\delta = +75°6'.6$ (1917); $\lambda = 74°$, $\beta = +31°$

Plate No. 128, 1912, June 13, 80^m. Weak plate

This is a faint left-handed spiral $90'' \times 70''$, $p = 160°$, having a faint stellar nucleus, with respect to which there is a knot of about the same brightness, $p = 350°$, $36''$.

N.G.C. 6478 Draco

$\alpha = 17^h 32^m 4^s$, $\delta = +51°11'.6$ (1917); $\lambda = 46°$, $\beta = +29°$

Plate No. 135, 1912, July 11, 270^m

This is a right-handed spiral, $90'' \times 30''$, $p = 32°$, having a stellar nucleus. A star a little brighter than the nucleus lies at $p = 10°$, $14''$. It is surrounded by a dark space $5''$ to $6''$ in diameter, the arms being cut off sharply. Does the star have an absorbing atmosphere? The detail is stronger in contrast on the *p* side. There are over a dozen very small and faint nebulae on this plate.

[1] W. W. Campbell, *Stellar Motions*, p. 210.

N.G.C. 6543 Draco

$\alpha = 17^h 58^m 35^s$, $\delta = +66°38'.3$ (1917); $\lambda = 63°$, $\beta = +9°$

Plate No. 9,	1911, July	23, 10^m.	25-foot focus
Plate No. 10,	1911, July	23, 20^m.	25-foot focus
Plate No. 11,	1911, July	23, 6^m.	25-foot focus
Plate No. 20,	1911, July	25, 90^s.	25-foot focus
Plate No. 34,	1911, October	18, 50^m.	100-foot focus
	Illustrated Plate II*b*		
Plate No. 238,	1914, May 17,	90^m.	80-foot focus

This is the well-known planetary nebula in Draco. The long exposures show a sharply outlined ellipse $24'' \times 18''$, $p = 40°$, without detail, beyond which project ansae $2''$ to $3''$ on an axis in $p = 20°$. On the shorter exposures the brighter nebulosity has the appearance of a curved thread which crosses itself several times, the ansae forming the ends. Its continuity is more or less disturbed in several places. Overlapping points are of greater brightness. At two points the thread approaches the bright stellar nucleus, giving almost the appearance of arms starting therefrom, but in neither case do they connect directly with the nucleus. The focal-plane slit spectrum appears as a continuous band corresponding to the nucleus, crossed by bright lines which project on either side. Keeler's[1] value for the radial velocity is -64.7 km, and Campbell and Moore[2] have found internal motion.

N.G.C. 6555 Hercules

$\alpha = 18^h 3^m 28^s$, $\delta = +17°35'.3$ (1917); $\lambda = 12°$, $\beta = +16°$

Plate No. 268, 1916, May 28–29, 360^m. Illustrated Plate IX*b*

This left-handed spiral $1'.5 \times 1'$, $p = 111°$, is in a rich field of stars. There is a star which might be taken for the nucleus, but the real nucleus is a slightly elongated knot a few seconds N. The brightest part of the nebula is a double knot at $p = 240°$, $13''$, with respect to star. A number of small faint nebulae appear on the plate.

[1] W. W. Campbell, *Stellar Motions*, p. 210.

[2] *Lick Observatory Bulletin*, No. 278, 1916.

N.G.C. 6703 Lyra

$\alpha = 18^h 44^m 53^s$, $\delta = +45°27'.5$ (1917); $\lambda = 42°$, $\beta = +18°$

Plate No. 269, 1916, July 1, 60^m
Plate No. 270, 1916, July 1, 180^m

A long exposure is necessary to show whether this is a ring nebula of a type very different from that usually met with, or whether a spiral, though indications point toward the former. The central nucleus lies in a bright nebulous spot which gradually fades out at 30″ diameter. The surrounding ring is very faint and narrow and about 80″ diameter.

N.G.C. 6702 appears as a nebulous spot very similar to, but fainter and smaller than, the central part of 6703. It is slightly elongated in $p = 60°$.

There are six additional small faint nebulae on the plate, three being nebulous spots and three spindles, the brightest two of the latter with respect to N.G.C. 6703 being at $p = 177°$, $8'.9$, elongated in $p = 120°$, and $p = 132°$, $9'.8$, elongated in 129°.

N.G.C. 6804 Aquila

$\alpha = 19^h 27^m 37^s$, $\delta = +9°2'.9$ (1917); $\lambda = 13°$, $\beta = -6°$

Plate No. 120, 1912, June 11, 60^m. Weak

The plate shows the nebula as a faint annulus with an irregularly shaped ring 5″ to 10″ in width and about $32'' \times 25''$, $p = 60°$. It has the typical central star and another of about the same magnitude lying directly upon the ring at the W end of the major axis. As in many planetaries, the nebulosity is stronger near the ends of the minor axis than near the ends of the major axis. Huggins[1] found that it had a bright-line spectrum.

Plate 294, 1917, June 24–25, $5^h 25^m$ exp., shows a planetary nebula with uniform disk $65'' \times 55''$, $p = 165°$, on which lies the ring described above. The N*p* and S points of the rim are strengthened while the *f* side is missing.

[1] *Philosophical Transactions*, **166**, 386, 1866.

N.G.C. 6818 Sagittarius

$\alpha=19^h39^m17^s$, $\delta=-14°21'.1$ (1917); $\lambda=353°$, $\beta=-19°$

Plate No. 115, 1912, June 10, 5^m. 25-foot focus
Plate No. 116, 1912, June 10, 2^m. 25-foot focus
Plate No. 137, 1912, July 12, 75^m. 100-foot focus. Illustrated Plate II*c*

This bright planetary nebula is about 25″ in diameter and contains a small faint nucleus and much detail. Upon the faint uniform disk lies a moderately bright elliptical ring $25''\times16''$, $p=9°$, and varying in width and intensity, the N end almost fading into the disk and the other end crossed by a dark streak. From each of two strong knots, about 8″ apart and equidistant from the nucleus in the W limb of the ring, a wisp runs inward to the major axis, the two wisps being parallel. A thread runs from the nucleus NE to the E limb. The spectrum has bright lines; Keeler's[1] value for the radial velocity is -16.7 km.

N.G.C. 6826 Cygnus

$\alpha=19^h42^m34^s$, $\delta=+50°19'.4$ (1917); $\lambda=52°$, $\beta=+12°$

Plate No. 130, 1912, June 14, 10^m
Plate No. 131, 1912, June 14, 5^m
Plate No. 132, 1912, June 14, 25^m
Bathed-process plate and red screen; Pan-Iso developer

This is a bright planetary nebula $30''\times27''$, $p=127°$. The central nucleus is bright, sharply defined, and about 10″ in diameter. A knot appears at each end of the major axis, otherwise the disk is uniform. On the red-sensitive plate the nucleus is about 5″ in diameter and lies in a faint nebulous haze that fades out at about 18″ diameter. The Cassegrain short-focus camera gives a spectrum strongly continuous for the star, crossed by bright lines, some projecting but a slight distance beyond the nucleus, others clear across the disk. Values obtained for the radial velocity are -5.3[2] and -8[3] km.

[1] W. W. Campbell, *Stellar Motions*, p. 210.

[2] *Ibid.*

[3] *Publications of the Astronomical Society of the Pacific*, **27**, 239, 1915.

N.G.C. 6894, Cygnus

$\alpha=20^h13^m3^s$, $\delta=+30°18'.6$ (1917); $\lambda=37°$, $\beta=-3°$

Plate No. 4, 1911, July 2, 60^m
Plate No. 7, 1911, July 3, 14^m

This is a well-defined ring; on the outside there is a haze, particularly on E and W sides; and on the inside, many protrusions toward the center giving an internal-toothed appearance. The strongest thread of the ring measures $42''\times32''$, $p=45°$. The central nucleus is stellar and small. In the N*p* section of the ring is a star with a very faint companion partly surrounded by a dark ring. A. Searle[1] found the spectrum to show bright lines.

N.G.C. 6905 Delphinus

$\alpha=20^h18^m42^s$, $\delta=+19°15'.3$ (1917); $\lambda=29°$, $\beta=-11°$

Plate No. 145, 1912, August 16, 180^m. 100-foot focus
Plate No. 147, 1912, September 5, 175^m. 100-foot focus
Plate No. 149, 1912, September 6, 205^m. 100-foot focus
All plates weak

This is a planetary nebula $47''\times34''$, $p=175°$, containing much detail, strongest on E and W sides and weak along the NS line. Huggins[2] found it to have a bright-line spectrum.

N.G.C. 7008 Cepheus

$\alpha=20^h58^m8^s$, $\delta=+54°13'.5$ (1917); $\lambda=61°$, $\beta=+5°$

Plate No. 243, 1914, July 22, 180^m. Illustrated Plate II*d*

This planetary nebula is elliptical in shape, $95''\times75''$, $p=5°$, containing much detail. The strongest bits of nebulosity are two condensations just E of the N end of the major axis. On the S*f* side the elliptical form seems eaten away, but traces of nebulosity may be seen connecting with a star which lies $p=156°$, $53''$. A number of stars a magnitude or two fainter than the nucleus appear in the nebula. Except for one, they are surrounded by a dark ring which in turn opens directly on a dark region. As the nucleus

[1] *Harvard Annals*, **33**, 145.

[2] *Philosophical Transactions*, **156**, 385, 1866.

itself presents this appearance, it is suggested that these stars lie within the nebula. One of these stars, $p=240^\circ$, $23''$, appears elongated; it may be a double star or a very bright bit of nebulosity. One star at $p=65^\circ$, $29''$, with respect to the nucleus, is but partly surrounded by the dark ring. Huggins[1] found the spectrum to be gaseous.

N.G.C. 7009 Aquarius

$\alpha=20^h59^m39^s$, $\delta=-11^\circ41'.6$ (1917); $\lambda=5^\circ$, $\beta=-36^\circ$

Plate No. 138, 1912, July 12, 90^m. 100-foot focus

Plate No. 140, 1912, July 13, 210^m. 100-foot focus. Illustrated Plate II*e*

This is a most striking planetary, owing to its resemblance to Saturn. The brightest nebulosity is in the form of an elliptical ring, outside diameter $30''\times 13''$, $p=78^\circ$. A second ring, more or less complete in outline, lying across the nucleus and in $p=160^\circ$, almost at right angles to the first, shows as a condensed knot on the N side, broadening out as it approaches the S side. Another feature is the rhomboid-shaped uniform mass with the E and W edges, approximately $18''$ long lying parallel to a NS line, and with the N and S sides, $23''$ long, slightly convex outward and running parallel to the major axis of the bright ellipse. The faint ansae are about $51''$ apart and connect with the central parts of the nebula through a bar lying in $p=78^\circ$. Other knots and threads besides those described also show. The reproduction has been prepared with a view to detail rather than to relative intensities. The focal-plane slit spectrograph shows a bright-line spectrum. The radial velocity is $+10.1$ km,[2] and Campbell and Moore have found internal motion.[3]

N.G.C. 7023 Draco

$\alpha=21^h0^m35^s$, $\delta=+67^\circ50'.3$ (1917); $\lambda=72^\circ$, $\beta=+14^\circ$

Plate No. 12, 1911, July 23, 149^m

This large nebula contains a great wealth of detail and is intimately connected with a star of the seventh magnitude lying in it.

[1] *Philosophical Transactions*, **156**, 387, 1866.

[2] W. W. Campbell, *Stellar Motions*, p. 210.

[3] *Lick Observatory Bulletin*, No. 278.

It occupies one of those dark regions which appear devoid of stars and which have been explained in several ways: the stellar material is at present in the nebula, or the nebula, containing both dark and bright material, lies this side of the stars and cuts off their light. Traces of the nebula extend as far as 9′ from it, but only in those regions that are starless. The nebulosity surrounding the star is strongest and projects on its S*p* side. Several bright filaments run S*p* from the star, and there are two more S of the star running in an EW direction. The spectra of both the star and the nebulosity 13″.6 N of the nucleus photographed with the focal-plane spectrograph are continuous with absorption lines. The velocity of the nucleus is +0.7 km.[1]

N.G.C. 7026 Cygnus

$\alpha = 21^h 3^m 30^s$, $\delta = +47°30'.8$ (1917); $\lambda = 57°$, $\beta = 0°$
Plate No. 156, 1912, October 10, 120^m. 100-foot focus

This bi-nuclear planetary is formed by two nebulous knots of about the same intensity, elongated NS, one lying 6″ almost due E of the other. They are connected by a bar but there is no central star. There are traces of two loops, one on the N and another on the S, so connecting the knots that the whole forms an ellipse, 27″×11″, $p = 10°$. The spectrum shows bright lines and Campbell and Moore[2] have found internal motion.

N.G.C. 7027 Cygnus

$\alpha = 21^h 3^m 56^s$, $\delta = +41°54'.0$ (1917); $\lambda = 53°$, $\beta = -5°$
Plate No. 142, 1912, August 15, 30^m. 100-foot focus

This is Webb's bright planetary, a bright starlike patch with faint wings extending 3″ to 4″ to the N and E. A second condensation of about the same intensity as the wing, elongated in $p = 80°$, and of about the same size as the star lies at $p = 130°$, 5″. The focal-plane spectrum shows very strong bright lines with practically no continuous background. Internal motion has been detected[3] by Campbell and Moore.

[1] *Publications of the Astronomical Society of the Pacific*, **27**, 239, 1915.
[2] *Ibid.*, **29**, 55, 1917. [3] *Ibid.*, **28**, 119, 1916.

N.G.C. 7129 Cepheus

$\alpha = 21^h 41^m 4^s$, $\delta = +65° 43'.3$ (1917); $\lambda = 73°$, $\beta = +9°$

Plate No. 273, 1916, August 31, 150^m
Plate No. 274, 1916, September 3, 25^m

This nebula lies in one of the vacant Milky Way regions in which one or more bright stars are involved in nebulosity, faint traces of which may be followed over a large part of the dark area. Seven stars are involved in nebulosity, their positions with respect to the brightest, *a*, being as follows:

ab	$p = 298°$, 58″	*ac*	$p = 50°$, 62″	*ad*	$p = 15°$, 60″
ae	$p = 355°$, 4′.5	*af*	$p = 346°$, 4′.9	*ag*	$p = 63°$, 5′.9

The nebulosity about *a* for a distance of 12″ is very strong, completely masking the star; beyond it is fainter and more or less interrupted by dark patches up to a distance of 30″, where the curve begins. From *a* the nebulosity sweeps N*f*, just passes *c* and *d* on the N, and sweeps backward to the W underneath *b*. Fainter clouds appear S*p* as far as 4′.5 from *a*. Not including *e*, *f*, and *g* the nebula lies within a circle 6′ in diameter, the center of which is about 1′.5 S*p* *a*. Two stars lie near this center, but do not seem involved in the nebula; *e*, *f*, and *g* are each centers of nebulosity of medium intensity and irregular form about 1′.5 to 2′ in diameter. A long exposure will probably show all these stars connected by nebulosity.

N.G.C. 7177 Pegasus

$\alpha = 21^h 56^m 44^s$, $\delta = +17° 20'.4$ (1917); $\lambda = 43°$, $\beta = -30°$

Plate No. 126, 1912, June 12, 75^m

This nebula is a right-handed spiral with moderately bright nucleus, arms fairly strong over roughly 30″ diameter. Outside of this are traces of arms filling an ellipse $2'.5 \times 1'.5$, $p = 75°$. One arm starts from the nucleus and branches in two, the arm opposite being deformed or missing altogether.

N.G.C. 7217 Pegasus

$\alpha = 22^h 4^m 10^s$, $\delta = +30°57'.2$ (1917); $\lambda = 54°$, $\beta = -21°$

Plate No. 216, 1913, September 2, 330^m. Illustrated Plate IX*c*

This nebula is a fine right-handed spiral seen in plan. It meaures $3' \times 2'.5$, $p = 80°$. For one turn about the nucleus the nebulosity is quite bright; then it drops in intensity over a space $75'' \times 60''$ where it fades away, to increase again in brightness so as to form a ring, at the periphery 20″ to 30″ wide, almost as bright as the second stage. Instead of two separate arms there seem to be many small ones composed of fine, knotted, and stringy nebulosity lying closely parallel. It lies in a region rich in stars. Huggins[1] found the spectrum to be continuous.

N.G.C. 7662 Andromeda

$\alpha = 23^h 21^m 54^s$, $\delta = +42°4'.8$ (1917); $\lambda = 74°$, $\beta = -18°$

Plate No. 29, 1911, October 17, 90^m. 100-foot focus, 44-inch aperture
Illustrated Plate II*f*

Plate No. 148, 1912, September 5, 60^m. 100-foot focus
Plate No. 157, 1912, October 10, 60^m. 100-foot focus. Seed 27 plate
Plate No. 172, 1912, December 10, 1^m, 3^m, 6^m, 10^m
Plate No. 173, 1912, December 10, 7^m, 30^m
Plate No. 177, 1913, January 7, 4^m, 7^m
Plate No. 275, 1916, September 3, 1^m, 4^m, 10^m

This is the well-known planetary in Andromeda with sharp bright nucleus and a mottled disk $31'' \times 27''$, $p = 40°$, on which lies a very bright elliptical ring, 2″ to 3″ wide, whose median line is $15'' \times 9''$, $p = 45°$, weakened on the minor axis. The E edge is tipped with a bright line of nebulosity. The longer exposures show a considerable number of knots on the rim, and from two of these at opposite ends of a diagonal in $p = 200°$ threads appear which project over the edges and form ansae. The spectrum with focal-plane slit has bright lines, with practically no continuous background. Keeler's[2] value of the radial velocity is -11.4 km.

Mount Wilson Solar Observatory
April 1917

[1] *Philosophical Transactions*, **156**, 391, 1866.

[2] W. W. Campbell, *Stellar Motions*, p. 210.

LIGHT-CURVES AND ORBITAL ELEMENTS OF TT LYRAE AND Y CAMELOPARDALIS

By MARTHA BETZ SHAPLEY

Photographic light-curves of the eclipsing variables TT Lyrae and Y Camelopardalis have been obtained recently by Miss Harwood, and the observations, in advance of their publication, have been kindly sent to me for a discussion of the orbits. Miss Harwood's paper, which will appear shortly in *Harvard Annals*, **84**, contains a detailed treatment of the observations and reductions. The present note is devoted to a brief statement of the orbital computations, and gives the elements and the related data derived from them.

Provisional orbits for both stars have been published,[1] but because of additional and more accurate observations the new orbits entirely replace the ones computed on the basis of the visual estimates by Nijland. The observations upon which the computations in this paper are based are of considerable accuracy and can be satisfactorily represented by the theoretical curves.

The light-elements derived from the new observations are:

TT Lyrae Min. = J.D. $2410000.757 + 5^{d}.243708 \cdot E$
Y Camelopardalis Min. = J.D. $2410002.642 + 3.305568 \cdot E$

To the period of Y Camelopardalis was applied a small correction read from a curve to be published in Miss Harwood's paper. The positions of the stars for 1900 are as follows:

	R.A.	Dec.	Gal. Lat.	Gal. Long.
TT Lyrae................	$19^{h}24^{m}.3$	$+41°30'$	$+11°$	$42°$
Y Camelopardalis.........	7 27.6	+76 71	+30	105

The solution for orbital elements follows the methods outlined in *Astrophysical Journal*, **35** and **36**, and in *Princeton Contributions*, No. 3. The light-curves were assumed symmetrical. The observa-

[1] *Contributions from the Princeton University Observatory*, No. 3, p. 90, 1915.

tions with positive and negative phases, counted from minimum, were plotted as a single branch, and theoretical curves were computed which would best fit the observations so plotted.

TT LYRAE

The variation at the principal minimum of TT Lyrae is two and a half magnitudes—one of the dozen largest ranges known among eclipsing variables. The maximum light-observations give positive evidence (1) of a shallow secondary minimum, (2) of a light-variation due to the ellipsoidal form of the components, and (3) of an unusually large "reflection" effect that may be wholly a conspicuous differential interradiation phenomenon, or a normal condition of reflection enhanced by periastron brightening. The latter alternative demands an eccentric orbit with periastron passage occurring at or near the time of secondary minimum. The observations are not sufficient to show orbital eccentricity through differences in the widths of the two minima. Accordingly, the orbit has been assumed circular, and the increase of maximum light around secondary minimum is attributed to interradiation. On this interpretation one hemisphere of the faint star (the one that always faces the bright companion) is eleven times as bright as the other. Since only a small part of the light of the bright star remains visible at the deepest phase of principal minimum, a large fraction of the total loss of light at that time, when measured in magnitudes, is due merely to the rotation of this unequally illuminated faint companion. The reflection-constant computed from the photographic curve is more than twice that derived for any eclipsing star heretofore. A further observational study of the maximum light would be of much interest.

The observations of TT Lyrae are from photographic plates made at Harvard. For the purpose of the orbital computations it was found convenient to bring zero phase to the mid-point of the minimum by adding $+0^{d}.004$ to the phases derived by Miss Harwood. Table I gives the deviations of the normal points from the theoretical curve, together with other data concerning the observations. The data referring to the principal minimum, from the fourth and sixth columns of the table, are plotted in Fig. 1.

The first orbit computed for this star was based upon the assumption that the stellar disks are uniformly luminous, except for the reflection, the effect of which is eliminated from the light-curve

TABLE I

NORMAL PLACES FOR TT LYRAE

Harwood Number	Number of Observations	Phase	sin θ	Observed Magnitude	Observed Magnitude Rectified	O−C Uniform
1– 4.......	34	0^d810	+0.825	9.24	9.14	$+0^M.03$
5– 8.......	39	1.200	+ .991	9.205	9.13	+ .02
9–12.......	26	1.601	+ .939	9.19	9.11	.00
13–16.......	47	2.016	+ .667	9.20	9.13	+ .02
17–19.......	28	2.353	+ .317	9.16	9.11	− .02
20–22.......	27	2.663	− .049	9.23	9.20	.00
23–26.......	27	3.004	− .441	9.185	9.12	+ .01
27–30.......	25	3.396	− .800	9.16	9.09	− .02
31–34.......	32	3.808	− .989	9.20	9.12	+ .01
35–38.......	35	4.213	− .945	9.18	9.09	− .02
39–42.......	51	4.592	− .705	9.23	9.11	.00
43–45.......	9	4.878	− .428	9.28	9.13	+ .02
46..........	4	4.978	− .314	9.40	9.23	+ .02
47..........	2	5.032	− .252	9.66	9.44	+ .03
48..........	7	5.079	− .198	9.97	9.69	+ .01
49..........	5	5.141	− .125	10.54	10.10	.00
50..........	6	5.174	− .086	10.98	10.37	.00
51..........	6	5.222	− .028	11.59	10.67	− .01
52..........	7	0.039	+ .045	11.46	10.62	.00
53..........	7	0.083	+ .098	10.85	10.30	.00
54..........	5	0.133	+ .157	10.23	9.88	.00
55..........	7	0.176	+ .208	9.82	9.57	− .03
56..........	2	0.237	+ .278	9.59	9.39	+ .08
57..........	4	0.290	+ .339	9.34	9.17	.00
58..........	2	0.345	+ .401	9.27	9.12	+ .01
59–63.......	16	0.488	+0.552	9.22	9.09	−0.02

by rectification. Another orbit was computed for disks completely darkened at the edge. The principal elements of these solutions are as follows:

Solution	Semi-Duration of Eclipse	k	α_0	L_b	cos i
U............	0^d331	0.62	0.97	0.799	0.109
D............	0.352	0.73	0.90	0.850	0.122

A solution intermediate between U and D is the one adopted.[1] It is given in detail in Table III. For this the darkening coefficient

[1] For an application of the method of computing orbits for intermediate degrees of darkening, see *Mt. Wilson Contr.*, No. 86; *Astrophysical Journal*, **40**, 219, 1914.

chosen is $x=0.75$, a value comparable with that observed photographically on the sun.[1] The orbital inclination has been corrected for the probable polar flattening of the stars; and the densities have been corrected for polar flattening as well as for unequal mass distribution by means of the approximate relation

$$\rho=\frac{b\bar{\rho}}{c}(0.4+1.2L),$$

where b and c are the middle and shortest axes of the ellipsoidal stars, $\bar{\rho}$ is the "equal mass" density, and L is the luminosity in terms of the combined light of both components.

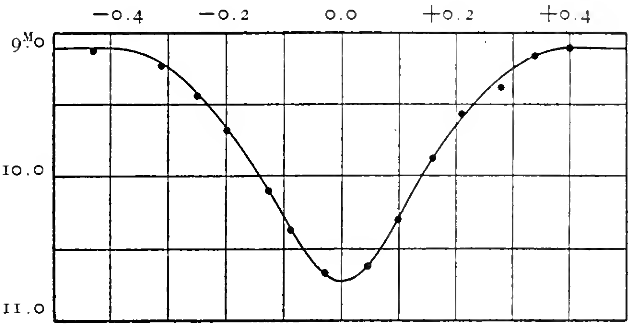

FIG. 1.—Observations and theoretical light-curve of TT Lyrae at principal minimum. Abscissae are sines of phase-angles; ordinates are magnitudes rectified for ellipticity and reflection.

Y CAMELOPARDALIS

The other eclipsing star treated in this paper, Y Camelopardalis, also has a large range, losing 78 per cent of its light at principal eclipse. Though the surface intensities of the components in this system are very unequal, there is an indication that the spectra are quite similar, for the variation is practically the same in visual and photographic light. Observations of maximum light give some evidence of a secondary minimum of a tenth of a magnitude.

Two series of observations are available—measures of photographic plates from the Harvard College Observatory and visual

[1] *Mt. Wilson Contr.*, No. 99; *Astrophysical Journal*, 41, 292, 1915.

photometric observations at Harvard by Wendell.[1] The latter are used for the orbital computations because the photographic observations at primary eclipse are relatively few in number and are derived generally from single-exposure plates taken at widely separated intervals; the phases, therefore, may be effected to some extent by the recognized irregularities of the period. Wendell's observations extend over a much shorter period of time and are exceptionally accurate and homogeneous. To the phases published in *Harvard Annals*, **69**, in addition to the correction (mentioned above) depending on the epoch, it was found necessary to apply a constant correction of $+0^{d}.011$ in order to bring the mid-point of minimum to zero phase.

TABLE II

Y CAMELOPARDALIS

No.	Phase	Obs. Mag.	O−C "Uniform"	No.	Phase	Obs. Mag.	O−C "Uniform"
1......	$3^{d}.087$	10.70	$-0^{M}.04$	26......	$3^{d}.357$	11.86	$-0^{M}.04$
2......	3.140	10.86	− 1	27......	3.358	11.92	+ 3
3......	3.160	10.96	− 4	28......	3.363	11.86	+ 1
4......	3.184	11.20	+ 3	29......	3.370	11.77	+ 1
5......	3.192	11.23	0	30......	3.374	11.66	− 5
6......	3.205	11.42	+ 6	31......	3.383	11.39	− 22
7......	3.208	11.44	+ 5	32......	3.384	11.63	+ 2
8......	3.220	11.47	− 4	33......	3.397	11.42	− 4
9......	3.240	11.72	0	34......	3.417	11.38	+ 11
10......	3.245	11.69	− 10	35......	3.434	11.20	+ 8
11......	3.254	11.92	+ 3	36......	3.442	11.04	− 2
12......	3.266	12.14	+ 11	37......	3.450	11.05	+ 4
13......	3.269	12.03	− 3	38......	3.451	10.98	− 3
14......	3.271	12.08	0	39......	3.452	11.05	+ 5
15......	3.274	11.98	− 12	40......	3.463	10.98	+ 5
16......	3.290	12.30	+ 6	41......	3.464	10.98	+ 6
17......	3.300	12.27	0	42......	3.476	10.83	− 3
18......	3.308	12.33	+ 5	43......	3.479	10.81	− 3
19......	3.311	12.22	− 5	44......	3.480	10.87	+ 4
20......	3.323	12.16	− 6	45......	3.491	10.73	− 5
21......	3.325	12.21	0	46......	3.495	10.72	− 4
22......	3.347	12.06	+ 4	47......	3.499	10.78	+ 4
23......	3.348	11.92	− 3	48......	3.506	10.69	− 3
24......	3.349	12.03	+ 4	49......	3.527	10.63	−0.03
25......	3.352	11.98	+0.02	....	....	....	....

For Y Camelopardalis the observations were not combined into normals but are plotted singly in Fig. 2, together with the "uniform" computed curve. The adopted range at secondary minimum

[1] *Harvard Annals*, **69**, 151, 1914.

is $0^{M}.10$. There is one "uniform" and one "darkened" solution, with elements as shown in the following table:

Solution	Semi-Duration of Eclipse	k	a_0	L_b	cos i
U...........	$0^{d}.245$	0.694	0.968	0.812	0.103
D...........	0.258	0.78	0.92	0.854	0.116

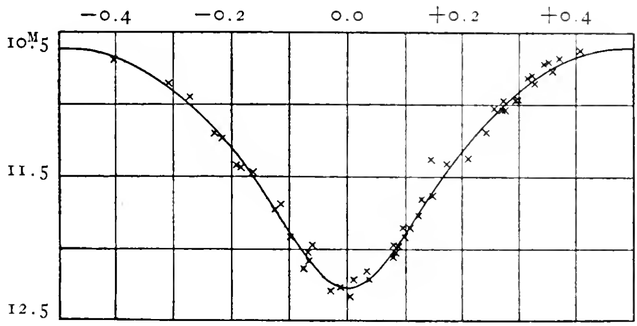

FIG. 2.—Observations and computed uniform light-curve of Y Camelopardalis. Abscissae are sines of phase-angles; ordinates are visual magnitudes.

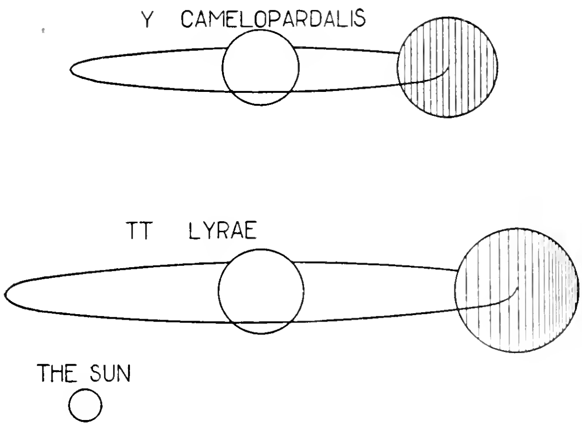

FIG. 3

The orbit adopted and given in Table III is intermediate for $x = 0.6$, the value of the darkening coefficient which has been found to be

in best agreement with solar observations at the wave-length corresponding to mean visual light. Table II gives Wendell's observations during minimum, together with the phase (corrected as explained above) and the residuals from the computed curve.

TABLE III

ORBITAL ELEMENTS

Elements and Other Data	TT Lyrae	Y Camelopardalis
Period . . . . . P	$5^d 2437$	$3^d 3056$
Spectrum	A	A–F
Maximum . . . . . m_0	$9^M 17$	$10^M 61$
Depth of primary minimum . . . . . m_1-m_0	$2^M 48$	$1^M 67$
Depth of secondary minimum . . . . . m_2-m_0	$0^M 08$	$0^M 10$
Adopted coefficient of darkening x	0.75	0.6
Semi-duration of eclipse . . . . . t'	$0^d 347$	$0^d 253$
Nature of eclipse	Partial, large star in front at primary	Partial, large star in front at primary
Ratio of radii . . . . . k	0.703	0.745
Fraction of light lost at primary eclipse . . . . . a_0	0.92	0.944
Light of bright star . . . . . L_b	0.835	0.832
Light of faint star, brighter end . . . . . L_f	0.165	0.168
Light of faint star, fainter end . . . . . L_f-2b	0.015	. . . . .
Major semi-axis of bright star . . . . . a_b	0.175	0.202
Minor semi-axis of bright star . . . . . b_b	0.172	. . . . .
Major semi-axis of faint star . . . . . a_f	0.249	0.271
Minor semi-axis of faint star . . . . . b_f	0.244	. . . . .
Probable ratio of polar axis to major axis . . . . . c/a	0.970	. . . . .
Cosine of inclination . . . . . $\cos i$	0.118	0.111
Ratio of surface brightness of the bright ends of the two stars . . . . . J_b/J_f	10.2	8.9
Ratio of surface brightness of the ends of the faint star $L_f/(L_f-2b)$	11.	. . . . .
Most probable density of bright star . . . . . ρ_b	0.067	0.11
Most probable density of faint star . . . . . ρ_f	0.010	0.02
Maximum radius of bright star in terms of sun . . . . . $\bar{a}_b$	2.80	2.36
Maximum radius of faint star in terms of sun . . . . . $\bar{a}_f$	3.98	3.18
Hypothetical absolute magnitude of bright star . . . . . M_b	−0.4	+0.7
Hypothetical parallax . . . . . π	+0."0011	+0."001

Fig. 3 contains a graphical representation of the two systems at greatest elongation, and below to the left the sun is drawn to scale.

SUMMARY

1. Orbits of two eclipsing binaries, TT Lyrae and Y Camelopardalis, have been computed on the basis of curves derived by Miss Harwood from Harvard observations.

2. The theoretical curves represent the observations well within their probable errors. For both variables shallow secondary minima have been observed.

3. In the system of TT Lyrae the stars are slightly ellipsoidal, and the "reflection" effect is peculiarly large.

4. The two systems are remarkably alike in many respects (see Table III and Fig. 3). Both are estimated to be more than 3000 light-years distant from the earth.

5. The densities of the brighter components are normal for the spectral types to which they belong.

PASADENA, CAL.
April 1917

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

By HARLOW SHAPLEY

FIFTH PAPER: COLOR-INDICES OF STARS IN THE GALACTIC CLOUDS[1]

The interpretation of the color-results[2] for Messier 11 has involved a more thorough investigation of the general stellar fields of that part of the sky than can be made on the basis of the stars shown on the photographs of the cluster. In this system the presence of faint stars that are conspicuously bluer than those found in Messier 67 and other open clusters of high galactic latitude opens the question of the relation of color to galactic latitude and necessitates an inquiry into the frequency of color-classes in the dense galactic clouds, one of the richest of which surrounds Messier 11. The present paper is concerned with the results for color and magnitude in four fields in the immediate neighborhood of the cluster.

The position of the center of each field is given in Table I. Fields I and II are respectively southwest and northwest of

TABLE I

GALACTIC CLOUD FIELDS NEAR MESSIER 11

Field	Central Star	R.A. 1900	Decl. 1900
I.........	B.D. $-7^\circ 4736$	$18^h 45^m 6^s$	$-7^\circ\ 8'$
II........	B.D. $-6^\circ 4913$	18 43 18	−6 7
III.......	....	18 46 10	−6 28
IV.......	....	18 45 42	−6 17

Messier 11, each nearly a degree distant, and in the denser parts of the galactic clouds, as may be seen by referring to Barnard's photographs of the Milky Way.[3] The photographic and photo-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 133.

[2] *Mt. Wilson Contr.*, No. 126; *Astrophysical Journal*, **45**, 164, 1917.

[3] *Publications of the Lick Observatory*, **11**, Plates 62–65, 1915.

visual magnitudes in these two fields were determined by direct comparison with the stars in Messier 11, and those for Field I were also derived through Polar comparisons. Table II contains a list and description of the plates used for the first two fields.

TABLE II

OBSERVATIONS ON FIELDS I AND II

Plate Number	Date	G.M.T.	Kind of Plate	No. Exp.	Exp. Time	Hour-Angle	Remarks
3201....	1916 July 6	18^h56^m	Iso	3	15^m, 2^m	10° E	I and M 11
3202....	6	19 28	Seed 27	3	3	2° E	I and M 11
3219....	7	18 52	Seed 27	3	3	11° E	I and M 11
3220....	7	19 2	Iso	3	15	8° E	I and M 11
3221....	7	20 10	Seed 27	1	7	9° W	I, for chart
3402....	Aug. 25	16 4	Iso	6	1–10	4° E	II, exp.-ratio
3403....	25	16 34	Iso	5	0.4–4	4° W	II, exp.-ratio
3404....	25	16 52	Seed 27	3	2	8° W	II and M 11
3407....	25	17 59	Iso	6	1–10	25° W	I, exp.-ratio
3431....	Sept. 6	16 4	Iso	3	5	8° W	II and M 11
3463....	24	15 23	Seed 27	3	2	16° W	II and M 11
3464....	24	15 52	Seed 27	3	2	22° W	II and M 11
3465....	24	16 0	Iso	3	10	24° W	II and M 11
3563....	Nov. 22	13 57	Iso	5	3	51° W	I and N.P.
3564....	22	14 3	Seed 27	5	1	53° W	I and N.P.

Field III is about 9′ southeast, and Field IV 7′ north of the cluster. The magnitudes were determined directly from the plates used for the study of Messier 11, which are described in the first table of the preceding paper of this series. A further partial check of the colors in all fields was obtained by the method of exposure-ratios.[1]

The fifty stars in Field I for which final results appear in Table III were selected at random from the several hundred within the measurable limits of the plates. Details concerning the measures and reductions can be omitted. It will suffice to note that the average deviation from the mean of one determination of a photographic magnitude is ±0.10; for a photo-visual magnitude the average deviation is ±0.06. The average probable error of a color-index is about ±0.08. In this field the exposure-ratios for a hundred stars give an independent determination of the colors, and, though the reduction-curve now available is provisional, the

[1] F. H. Seares, *Mt. Wilson Communications*, No. 33; *Proceedings of the National Academy of Sciences*, **2**, 521, 1916.

TABLE III

MAGNITUDES AND COLORS IN FIELD I

Star	Δα cos δ	Δδ	Photographic Magnitude	Photo-visual Magnitude	Color-Index
1	−188″	−190″	15.46	13.88	+1.ᴹ58
2	−177	+448	15.54	14.28	+1.26
3	−154	− 33	14.78	13.65	+1.13
4	−154	+418	14.84	14.59	+0.25
5	−153	− 46	15.42	15.00	+0.42
6	−129	− 50	14.94	13.64	+1.30
7	−128	−149	13.26	12.93	+0.33
8	−119	− 1	15.12	14.62	+0.50
9	−118	+428	15.57	14.61	+0.96
10	−114	+372	14.95	14.53	+0.42
11	−103	−221	14.40	13.77	+0.63
12	− 81	− 20	14.32	14.46	−0.14
13	− 69	+174	15.41	14.43	+0.98
14	− 68	+ 95	15.38	14.86	+0.52
15	− 63	+145	16.24	15.04	+1.20
16	− 55	+330	14.42	13.81	+0.61
17	− 47	− 60	15.49	14.30	+1.19
18	− 44	+319	13.70	12.64	+1.06
19	− 38	+ 72	13.98	13.72	+0.26
20	− 37	+ 12	15.37	13.83	+1.54
21	− 35	+ 18	14.54	13.84	+0.70
22	− 34	− 67	15.31	14.43	+0.88
23	0	+189	15.07	14.64	−0.43
24	+ 25	+146	15.19	14.16	+1.03
25	+ 35	+417	14.45	13.17	+1.28
26	+ 43	+123	15.25	14.58	+0.67
27	+ 44	+196	11.45	11.34	+0.11
28	+ 49	+205	14.99	14.53	+0.46
29	+ 53	+401	14.40	12.64	+1.76
30	+ 59	+231	14.85	14.12	+0.73
31	+ 70	+223	14.77	14.37	+0.40
32	+ 80	+297	15.01	13.05	+1.96
33	+ 83	+ 14	14.96	14.05	+0.91
34	+ 87	+ 66	15.58	14.22	+1.36
35	+ 88	+ 72	13.73	12.57	+1.16
36	+ 93	+234	12.78	12.30	+0.48
37	+106	+119	14.46	14.14	+0.32
38	+116	+ 89	14.36	13.98	+0.38
39	+145	+ 22	14.36	13.83	+0.53
40	+174	+197	13.89	12.93	+0.96
41	+200	+ 79	15.39	14.56	+0.83
42	+220	+138	13.17	13.11	+0.06
43	+229	+109	13.04	12.71	+0.33
44	+244	+188	14.61	13.98	+0.63
45	+351	+216	13.29	11.48	+1.81
46	+253	+154	15.32	14.02	+1.30
47	+358	+133	14.89	13.80	+1.09
48	+361	+126	15.17	14.10	+1.07
49	+363	+169	14.89	14.57	+0.32
50	+367	+113	13.20	12.12	+1.08

results from the different methods are in satisfactory agreement. The positions given in Tables III–VI, which are intended to serve only for identification, were determined, with the aid of a reseau, from enlargements of the original negative, the orientation and scale being derived from catalogue stars.

Table IV contains results for Field II. The average deviation of a magnitude from one plate is $\pm 0^{M}.07$, giving an average probable

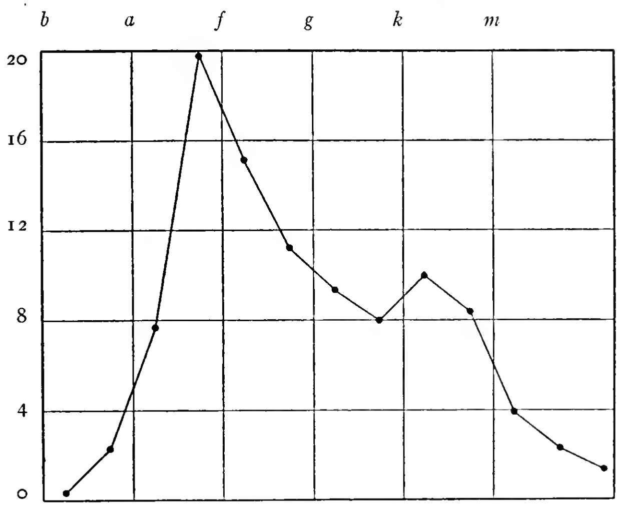

FIG. 1.—Frequency of color-classes in galactic clouds
Abscissae are color-classes; ordinates are percentages

error for a color-index of less than $\pm 0^{M}.06$. This value is independent of the accidental errors in the magnitudes in Messier 11, as thirty or forty of the cluster stars were used in the intercomparison, but it is not free from possible systematic errors in those magnitudes. From the results of the Polar comparisons for Field I and from the colors derived by exposure-ratios, this latter source of error is found to be unimportant.

The errors in the magnitudes for Fields III and IV are essentially the same as those given previously for the stars in Messier 11. Stratonoff's catalogue of stars in and near the cluster[1] includes a number of these outlying objects, and, when available, his numbers are given in the second column of Tables V and VI.

[1] *Publications de l'Observatoire astronomique et physique de Tachkent*, No. 1, 1899.

TABLE IV

MAGNITUDES AND COLORS IN FIELD II

Star	$\Delta\alpha \cos\delta$	$\Delta\delta$	Photographic Magnitude	Photo-visual Magnitude	Color-Index
1	−484″	+ 76″	12.89	12.48	+0.41 M
2	−481	+144	15.68	14.50	+1.18
3	−474	+ 1	14.01	13.81	+0.20
4	−468	+ 19	13.99	13.61	+0.38
5	−467	+155	14.82	13.84	+0.98
6	−466	+ 10	16.58	14.35	+2.23
7	−463	−436	12.92	12.75	+0.17
8	−450	+ 42	14.72	14.28	+0.44
9	−449	−431	13.86	13.83	+0.03
10	−445	+ 15	15.12	14.52	+0.60
11	−440	−477	12.64	12.34	+0.30
12	−427	−500	15.84	15.01	+0.83
13	−422	− 25	15.16	13.61	+1.55
14	−411	+147	14.72	14.55	+0.17
15	−389	− 45	14.77	13.48	+1.29
16	−388	− 68	14.24	14.02	+0.22
17	−377	−521	14.47	13.45	+1.02
18	−376	+ 33	15.19	13.28	+1.91
19	−367	−353	15.30	14.97	+0.33
20	−367	−371	16.29	14.83	+1.46
21	−365	−445	14.88	13.28	+1.60
22	−364	+ 90	14.18	12.45	+1.73
23	−356	−316	15.17	14.85	+0.32
24	−354	−327	15.64	15.00	+0.64
25	−353	−526	14.50	14.50	0.00
26	−352	+107	16.69	14.69	+2.00
27	−336	−379	13.99	13.85	+0.14
28	−330	−307	15.40	13.96	+1.44
29	−325	−447	15.05	13.83	+1.22
30	−325	+ 90	16.01	14.61	+1.40
31	−322	+ 60	16.61	14.98	+1.63
32	−318	+256	14.34	14.25	+0.09
33	−315	+198	14.03	12.82	+1.21
34	−315	−373	12.66	11.82	+0.84
35	−314	−366	14.79	14.55	+0.24
36	−314	−323	14.45	14.23	+0.22
37	−303	+239	15.12	14.79	+0.33
38	−302	−208	15.20	14.66	+0.54
39	−299	−317	15.17	14.08	+1.09
40	−294	+127	14.69	14.40	+0.29
41	−292	−350	15.88	15.02	+0.86
42	−292	−199	14.99	13.98	+1.01
43	−289	+280	15.68	14.27	+1.41
44	−288	−330	14.82	14.55	+0.27
45	−283	+ 69	14.41	13.94	+0.47
46	−282	−254	14.21	13.94	+0.27
47	−281	−301	14.79	14.33	+0.46
48	−279	+120	16.64	14.56	+2.08
49	−278	+470	15.63	15.03	+0.60
50	−265	−217	15.13	14.81	+0.32
51	−260	+298	15.57	14.98	+0.59
52	−259	−249	14.74	14.30	+0.44

TABLE IV—*Continued*

Star	$\Delta\alpha \cos \delta$	$\Delta\delta$	Photographic Magnitude	Photo-visual Magnitude	Color-Index
53	−258″	+330″	15.16	14.63	+0.M53
54	−255	+107	14.69	14.35	+0.34
55	−254	+469	13.91	13.83	+0.08
56	−248	+178	14.82	13.28	+1.54
57	−245	+240	15.32	14.81	+0.51
58	−244	+302	16.03	14.56	+1.47
59	−243	+348	16.01	14.87	+1.14
60	−243	+372	13.78	12.03	+1.75
61	−238	+128	13.75	13.55	+0.20
62	−237	+236	14.52	14.18	+0.34
63	−233	−290	14.38	14.16	+0.22
64	−232	+241	14.15	13.85	+0.30
65	−228	−246	15.48	14.30	+1.18
66	−225	+295	14.40	13.34	+1.06
67	−224	−336	13.49	12.83	+0.66
68	−224	+146	14.79	13.94	+0.85
69	−221	+270	14.27	13.85	+0.42
70	−218	+469	15.00	14.74	+0.26
71	−211	+252	14.90	14.43	+0.47
72	−211	+470	13.63	12.39	+1.24
73	−210	+319	14.32	12.50	+1.82
74	−210	+386	14.77	14.55	+0.22
75	−210	+139	16.57	14.52	+2.05
76	−207	−259	16.50	14.57	+1.93
77	−200	−314	14.80	13.51	+1.29
78	−197	−280	15.40	14.23	+1.17
79	−195	+242	16.13	14.66	+1.47
80	−195	−445	12.27	11.43	+0.84
81	−186	+405	15.48	14.52	+0.96
82	−171	−475	15.78	14.48	+1.30
83	−166	−476	15.78	15.04	+0.74
84	−163	−482	16.08	15.09	+0.99
85	−155	−189	15.65	14.51	+1.14
86	−146	+376	16.04	14.79	+1.25
87	−141	−215	15.59	14.92	+0.67
88	−134	+411	14.77	14.90	−0.13
89	−125	+354	15.53	14.23	+1.30
90	−122	−480	14.31	13.96	+0.35
91	−119	−249	15.80	14.55	+1.25
92	−116	+429	15.40	14.80	+0.60
93	− 92	−268	14.87	14.42	+0.45
94	− 86	−514	14.69	13.58	+1.11
95	− 85	+401	15.56	14.15	+1.41
96	− 81	−492	16.08	14.72	+1.36
97	− 79	−479	14.72	14.52	+0.20
98	− 77	+282	14.85	13.96	+0.89
99	− 75	−534	15.49	14.95	+0.54
100	− 67	−282	14.90	14.86	+0.04
101	− 67	−493	15.12	14.85	+0.27
102	− 52	+299	16.18	14.77	+1.41
103	− 49	−482	13.46	13.39	+0.07
104	− 45	+283	15.56	14.35	+1.21
105	− 24	−265	16.26	14.88	+1.38
106	− 10	+315	14.72	14.23	+0.49

TABLE IV—*Continued*

Star	Δα cos δ	Δδ	Photographic Magnitude	Photo-visual Magnitude	Color-Index
107	− 7″	−237″	12.93	12.18	+0.M75
108	+ 2	+301	14.54	14.48	+0.06
109	+ 30	+332	14.45	13.05	+1.40
110	+ 31	+291	14.50	14.13	+0.37
111	+ 73	+289	15.20	14.66	+0.54
112	+ 89	−354	15.12	14.61	+0.51
113	+ 94	−349	16.61	14.74	+1.87
114	+100	+339	13.26	12.33	+0.93
115	+100	+150	15.20	14.86	+0.34
116	+100	−165	15.35	14.53	+0.82
117	+101	− 31	13.13	12.36	+0.77
118	+101	+247	15.68	14.37	+1.31
119	+101	− 59	12.36	11.35	+1.01
120	+105	−194	16.11	14.58	+1.53
121	+107	−320	13.91	12.68	+1.23
122	+108	−142	14.80	14.50	+0.30
123	+110	− 69	14.93	13.46	+1.47
124	+118	−424	14.77	14.50	+0.27
125	+121	−218	15.33	14.83	+0.50
126	+127	−350	14.03	13.94	+0.09
127	+130	−353	13.93	13.61	+0.32
128	+144	−177	16.22	14.86	+1.36
129	+150	−347	14.65	14.20	+0.45
130	+154	−424	15.99	14.55	+1.44
131	+155	+281	16.10	14.66	+1.44
132	+163	+ 79	15.94	14.27	+1.67
133	+163	−401	13.56	12.82	+0.74
134	+167	−120	16.07	14.76	+1.31
135	+174	−108	14.99	14.39	+0.60
136	+179	−328	14.90	14.46	+0.44
137	+180	+293	14.86	13.40	+1.46
138	+183	+145	15.17	13.22	+1.95
139	+187	−322	15.83	14.80	+1.03
140	+193	−321	15.50	14.80	+0.70
141	+195	+ 93	15.91	14.48	+1.43
142	+210	−453	15.88	14.50	+1.38
143	+214	−432	12.41	11.93	+0.48
144	+216	−155	15.75	14.27	+1.48
145	+219	−443	14.91	14.97	−0.06
146	+229	−481	13.27	13.32	−0.05
147	+241	−435	14.91	14.25	+0.66
148	+248	−180	14.92	14.02	+0.90
149	+249	−186	13.19	12.19	+1.00
150	+278	−187	16.54	14.86	+1.68
151	+297	−278	15.54	15.01	+0.53
152	+306	−497	12.46	11.50	+0.96
153	+310	−291	15.58	14.20	+1.38
154	+311	−195	16.87	15.25	+1.62
155	+313	−517	12.89	13.06	−0.17
156	+333	−491	13.78	14.20	−0.42
157	+334	−224	13.38	11.99	+1.39

TABLE V

MAGNITUDES AND COLORS IN FIELD III

Star	Stratonoff Number	R.A. 1900	Decl. 1900	Photographic Magnitude	Photo-visual Magnitude	Color-Index
1.......	697	$18^h45^m55^s8$	$-6°28'26''$	14.45	14.14	$+0^M31$
2.......	698	56.3	27 37	13.14	11.98	+1.16
3.......	....	56.7	27 42	16.03	14.83	+1.20
4.......	710	57.6	28 27	13.23	12.84	+0.39
5.......	713	58.1	28 43	14.50	14.17	+0.33
6.......	730	59.9	28 46	14.46	14.21	+0.25
7.......	....	46 0.2	27 32	15.07	14.74	+0.33
8.......	....	0.5	28 20	15.38	15.06	+0.32
9.......	738	0.7	29 2	13.52	11.84	+1.68
10.......	....	0.9	27 44	15.04	14.58	+0.46
11.......	740	1.1	28 25	12.02	11.96	+0.06
12.......	....	2.0	27 56	15.22	14.85	+0.37
13.......	....	2.1	28 54	15.12	15.18	−0.06
14.......	....	2.5	28 5	15.04	14.71	+0.33
15.......	748	2.5	29 20	14.30	14.17	+0.13
16.......	....	2.7	28 55	14.72	14.24	+0.48
17.......	....	5.0	28 28	15.02	14.02	+1.00
18.......	....	5.4	30 35	14.88	13.80	+1.08
19.......	....	5.4	29 34	14.80	14.74	+0.06
20.......	769	6.9	27 57	14.21	14.06	+0.15
21.......	770	6.9	30 18	13.23	12.91	+0.32
22.......	....	9.1	30 29	15.07	14.95	+0.12
23.......	....	9.3	30 24	15.27	14.40	+0.87
24.......	780	9.4	29 10	11.60:	10.67	+0.93
25.......	....	9.6	27 22	14.84	14.55	+0.29
26.......		9.7	27 35	15.55	14.55	+1.00
27.......	....	9.8	27 16	14.97	14.55	+0.42
28.......	782	10.1	30 32	13.76	12.47	+1.29
29.......	783	10.1	26 28	13.15	11.61	+1.54
30.......	786	10.3	27 12	13.76	13.64	+0.12
31.......	....	10.8	25 37	14.76	13.89	+0.87
32.......	788	10.9	25 54	14.59	13.64	+0.95
33.......	789	11.2	28 57	14.11	13.84	+0.27
34.......	....	13.2	28 52	14.93	14.68	+0.25
35.......	....	13.3	28 42	14.93	14.48	+0.45
36.......	....	13.6	25 39	15.98	14.55	+1.43
37.......	795	14.0	25 50	13.93	13.68	+0.25
38.......	....	14.1	29 43	14.34	12.73	+1.61
39.......	796	14.2	29 57	13.89	13.49	+0.40
40.......	....	15.3	27 18	15.55	14.93	+0.62
41.......	804	15.9	27 16	13.64	13.56	+0.08
42.......	809	16.6	27 34	13.52	13.36	+0.16
43.......	811	17.3	28 22	13.80	13.70	+0.10
44.......	814	17.9	26 13	14.42	14.17	+0.25
45.......	....	19.0	27 38	14.80	13.84	+0.96
46.......	823	19.3	27 18	13.19	11.64	+1.55
47.......	....	20.5	27 47	14.93	14.21	+0.72
48.......	....	21.5	28 15	15.46	13.98	−1.48
49.......	....	21.9	28 2	14.88	14.10	+0.78
50.......	....	22.3	28 49	14.80	14.48	+0.32
51.......	....	23.1	28 41	14.84	14.21	+0.63
52.......	831	23.2	28 29	13.56	12.82	+0.74

TABLE VI

MAGNITUDES AND COLORS IN FIELD IV

Star	Stratonoff Number	R.A. 1900	Decl. 1900	Photographic Magnitude	Photo-visual Magnitude	Color-Index
1	177	$18^h45^m26^s.7$	−6° 16′ 38″	13.98	13.68	+0.^{M}30
2	....	27.1	16 24	15.32	15.00	+0.32
3	....	28.4	17 30	15.84	15.16	+0.68
4	....	28.5	17 1	16.56:	14.97	+1.59:
5	195	28.7	18 0	14.74	13.24	+1.50
6	196	28.7	16 29	14.86	14.65	+0.21
7	....	30.6	16 30	16.94:	14.58	+2.36:
8	....	30.8	17 4	15.26	14.83	+0.43
9	....	31.2	17 42	15.50	14.17	+1.33
10	220	31.5	17 37	13.85	12.99	+0.86
11	....	32.3	18 32	14.93	14.52	+0.41
12	241	33.2	15 7	13.37	12.82	+0.55
13	....	33.4	16 29	15.80	14.06	+1.74
14	246	33.5	15 13	13.85	13.33	+0.52
15	255	33.9	18 34	14.89	14.58	+0.31
16	....	34.8	15 22	14.78	14.14	+0.64
17	....	35.0	17 5	16.38	14.71	+1.67
18	....	35.2	17 2	14.86	14.17	+0.69
19	....	35.3	15 12	15.46	14.85	+0.61
20	....	35.6	17 35	15.92	14.58	+1.34
21	304	36.8	15 56	12.67	11.63	+1.04
22	312	37.0	16 30	14.86	14.24	+0.62
23	326	37.7	14 59	14.51	14.17	+0.34
24	339	38.1	18 31	13.30	12.77	+0.53
25*	340	38.1	16 2	12.74	12.04	+0.70
26	367	39.2	16 26	14.03	13.49	+0.54
27	371	39.3	16 38	14.10	13.88	+0.22
28	382	39.8	16 49	13.36	12.64	+0.72
29	....	40.0	16 28	15.35	14.91	+0.44
30	....	40.0	16 35	15.42	14.21	+1.21
31	388	40.1	17 50	14.51	14.21	+0.30
32	....	42.4	16 46	15.92	15.20	+0.72
33	....	42.6	16 16	14.86	13.42	+1.44
34	463	42.8	18 25	14.07	13.76	+0.31
35	....	43.2	16 52	15.01	13.72	+1.29
36	473	43.2	15 34	14.24	13.72	+0.52
37	479	43.3	15 58	13.72	13.42	+0.30
38	....	44.0	15 30	15.39	15.00	+0.39
39	....	45.0	18 41	14.97	14.62	+0.35
40	530	45.4	18 36	13.81	13.08	+0.73
41	549	46.2	18 30	13.02	12.15	+0.87
42	575	47.5	18 26	14.48	14.06	+0.42
43	578	47.5	17 43	14.26	14.02	+0.24
44	587	48.1	17 50	12.45	11.95	+0.50
45	595	48.6	18 18	14.55	14.44	+0.11
46	605	49.1	18 30	14.93	14.62	+0.31
47	626	50.5	18 30	13.18	12.39	+0.79
48	627	50.7	18 3	13.74	13.24	+0.50
49	632	51.0	17 44	14.32	13.49	+0.83
50	641	51.8	16 53	13.02	12.15	+0.87
51	646	52.2	17 26	14.97	14.48	+0.49
52	665	53.3	18 14	13.06	12.39	+0.67
53	673	53.8	18 37	14.44	14.06	+0.38
54	683	54.4	17 26	14.07:	14.10	−0.03:

*This star apparently varies through six-tenths of a magnitude.

The results for the four fields are collected in Tables VII and VIII, the former showing the distribution of the stars among the various color-classes,[1] the latter giving statistical data bearing on the relation of average color-index to luminosity. A wide range in color is immediately apparent; and the plot of the relative frequency of color-classes in Fig. 1 indicates that the distribution of spectral types among the fourteenth-magnitude stars is much the same in this distant galactic region as in the immediate vicinity of the sun.

TABLE VII

PERCENTAGE FREQUENCY OF COLORS IN GALACTIC CLOUDS

Field	Color-Class												Number of Stars
	*b*0	*b*5	*a*0	*a*5	*f*0	*f*5	*g*0	*g*5	*k*0	*k*5	*m*0	*m*5	
I*	0	2	6	12	18	12	12	16	12	4	2	4	50
II	1	3	7	17	13	8	8	8	12	12	5	6†	157
III	0	2	17	29	10	10	10	8	4	8	4	0	52
IV	0	0	2	27	24	22	10	0	8	4	4	0	51
All fields	0.3	2	8	20	15	11	9	8	10	8	4	4‡	310

* The corresponding numbers for Field I from the 100 provisional exposure-ratio color-determinations are 0, 6, 22, 17, 11, 6, 8, 10, 7, 9, 3, 1.

† 2.5 per cent have color-indices greater than +2.00.

‡ 1.3 per cent have color-indices greater than +2.00.

A small but probably real increase of mean color with brightness is suggested by Table VIII. In Messier 11 the corresponding change is very striking,[2] the average color-index decreasing a whole magnitude as the brightness decreases three magnitudes. An examination of the individual values for the fields in the galactic clouds shows that in Table VIII, for all cases, the average deviation of the color-index from the means is large. In other words, stars of all colors are included in each interval of magnitude; and, so far as color is an index of intrinsic luminosity, this may be accepted as an indication of considerable difference in the distances of such stars. Herein the galactic clouds apparently differ from the stellar

[1] Under *b*0 are included stars with color-indices between $-0^{M}.40$ and $-0^{M}.20$; under *b*5 those between $-0^{M}.20$ and $0^{M}.00$, etc.

[2] *Mt. Wilson Contr.*, No. 126, Table VIII and Fig. 3.

group comprising the nucleus of Messier 11. There the color-phenomenon is reasonably explained, in the light of results for globular clusters as well as for moving groups of stars, as the characteristic of a relatively condensed physical group whose distances from the sun are sensibly the same.[1] These cluster-stars are probably giants in luminosity, and accordingly the distance of the group must be of the order of 15,000 light-years.

TABLE VIII

RELATION OF COLOR INDEX IN GALACTIC CLOUDS TO PHOTO-VISUAL MAGNITUDE

Field I	Mean mag.	12.08	13.02	13.81	14.14	14.50	14.80
	Mean color	+ 1.07	+ 0.91	+ 0.87	+ 0.78	+ 0.59	+ 0.67
	No. stars	6	8	10	10	10	6
Field II	Mean mag.	12.05	13.13	13.82	14.27	14.67	15.01
	Mean color	+ 0.96	+ 1.07	+ 0.63	+ 0.75	+ 0.93	+ 0.69
	No. stars	15	19	21	36	51	15
Field III	Mean mag.	11.62	13.02	13.78	14.15	14.54	14.89
	Mean color	+ 1.15	+ 0.62	+ 0.68	+ 0.46	+ 0.61	+ 0.37
	No. stars	6	8	9	11	9	9
Field IV	Mean mag.	12.10	13.01	13.62	14.14	14.56	14.96
	Mean color	+ 0.78	+ 0.74	+ 0.64	+ 0.72	+ 0.44	+ 0.66
	No. stars	7	8	9	11	8	8
All fields	Mean mag.	11.99	13.07	13.77	14.21	14.62	14.94
	Mean color	+ 0.98	+ 0.90	+ 0.69	+ 0.70	+ 0.80	+ 0.60
	No. stars	34	43	49	68	78	38

The wide dispersion in magnitude of both blue and red stars indicates a similarly great distance for the neighboring galactic clouds. It suggests that the extent of the stellar clouds in the line of sight is relatively very great—in fact, the depth may be as great as, or greater than, the distance to the nearer boundary.

Fainter than the fourteenth photo-visual magnitude the average color-index in Messier 11 begins to increase (see Fig. 3 of the preceding paper). These faint stars might perhaps be considered as at the head of the series of dwarf stars in a stellar system that includes all the grades of luminosity found among the stars at large;

[1] The correction of the cluster-results (if it were rigorously possible) for the stars not belonging to the physical group would not affect these conclusions.

but a preferable interpretation is that the physical group in Messier 11, containing chiefly stars of high luminosity, has but a few stars fainter than the fourteenth apparent magnitude. The increasing redness, then, is due to the increasing predominance of the members of the star-clouds, which we now find are of all color-classes with an average index, between the fourteenth and fifteenth magnitudes, of +0.70.

SUMMARY

1. Magnitudes and colors have been determined for three hundred stars in four fields in the vicinity of the open galactic cluster Messier 11. Twenty-three plates have been used in the discussion, and the various errors are satisfactorily small.

2. As in Messier 11, a wide range of color is present; but the change of color with brightness is hardly perceptible. The magnitudes of the blue stars seem to indicate the remoteness of the star-clouds and also their great dimensions. Similar results have been obtained in a number of other galactic fields,[1] all pointing to a diameter of the galactic system considerably greater than generally supposed.

3. The cluster Messier 11 proves to be a physical group in the midst of the star-clouds, which on their own part have the general appearance and some of the properties of an enormous but definitely outlined physical system. There is no certain evidence as yet of the existence of dwarf stars, either in Messier 11 or in the galactic clouds.

Mount Wilson Solar Observatory
April 1917

[1] *Mt. Wilson Communications*, No. 44; *Proceedings of the National Academy of Sciences*, 3, 269, 1917.

EFFECT OF A LONGITUDINAL MAGNETIC FIELD UPON GLOW-DISCHARGE

By R. F. EARHART and C. B. JOLLIFFE

A relation probably exists between variation in solar activity and magnetic phenomena. This is the general conclusion reached from observations on sun-spots and recorded variation in terrestrial magnetism. The application of the Zeeman effect to magnetic variations in the sun-spots made by Professor Hale and colaborers at Mount Wilson bears out this connection. The authors' information, while exceedingly incomplete, may be briefly summarized.

Accompanying variations in the solar activity which are observed visually, there are magnetic disturbances on the earth. These magnetic disturbances are probably due to change in the magnetization of the sun itself, the magnetic field in the surrounding region being disturbed thereby. Our terrestrial observations are records of this general change. It has never been made clear to the authors which is the cause and which the effect—that is, whether the luminous solar variations are due to magnetic changes or whether variations in the streams of luminous and possibly electrified matter cause the magnetic variations. Possibly both are effects of some more fundamental cause. Any relation between variation of the magnetic field and variation of the luminous glow-discharge will have at least a bearing in the general subject.

The senior author of this paper has previously made some experiments on the effect of a magnetic field on glow-discharge in a vacuum tube.[1]

In the former experiment the discharge took place between flat plates, and the current-density was small. It was found that for a pressure less than the critical pressure (low pressure) the longitudinal field tended to suppress the discharge, while for higher pressure the current-density was increased with increasing field-strengths, but that a maximum value of the field was attained beyond which increase in field no longer increased the discharge.

[1] *Physical Review*, 3, 103, 1914.

The intent of the present experiment is to extend the measurements to larger currents. The method of controlling, measuring the current passing through the discharge-chamber, and determining the potential-difference between the terminals is precisely as in the former experiment, save that all electrical measurements

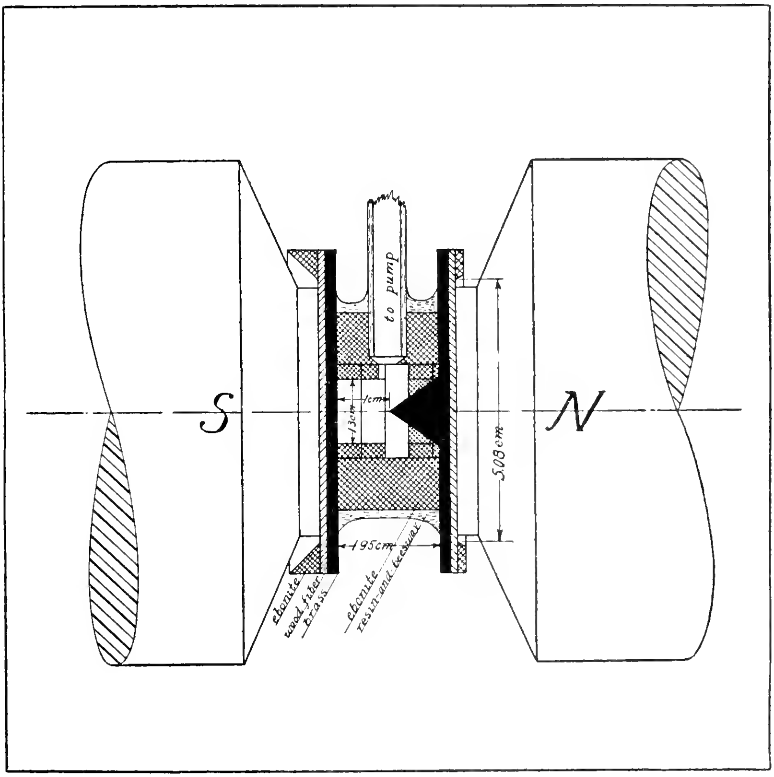

FIG. 1

were made with direct-reading voltmeters and millimeters. The discharge-chamber indicated in Fig. 1 was a cylindrical tube closed at one end by a flat plate and at the other by a conical electrode. The tube was coaxial with the poles of a large electromagnet. The pole-pieces of the magnet measured 5.08 cm across the face. Exploration with a flip-coil showed a uniform magnetic field to exist in the central region. The intent was to produce a

discharge with the electric and magnetic lines of force parallel. This was accomplished, barring the distortion of the electric field due to the shape of the conical electrode. It will be noted later that this distortion greatly modifies the result where the cone was the cathode.

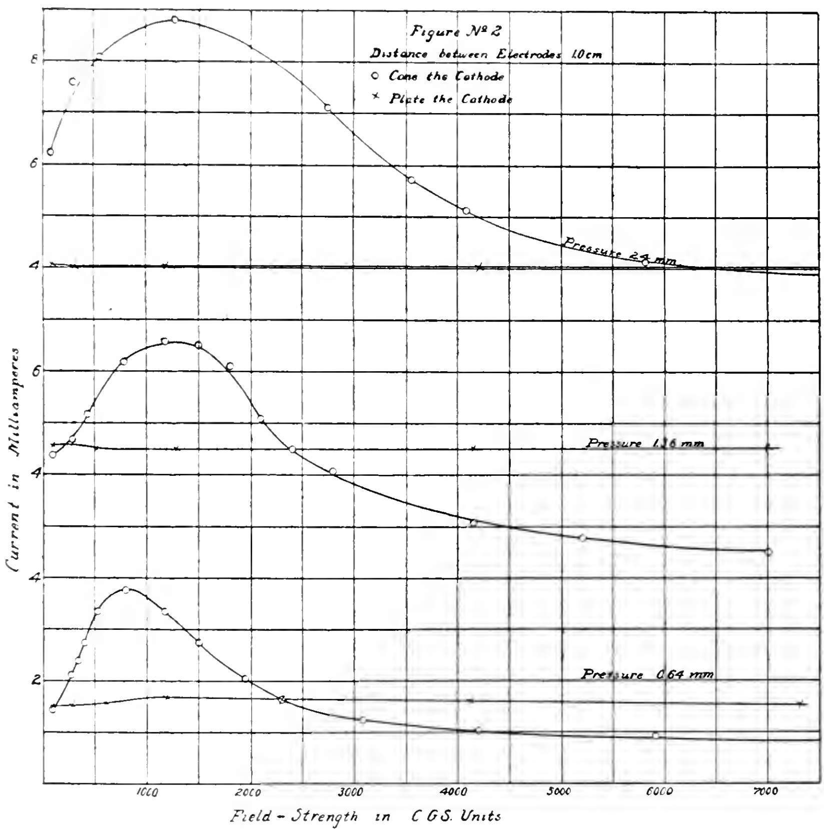

FIG. 2

The plan adopted was to produce a current through a circuit consisting of a set of storage cells, a controlling device, an ammeter, and the discharge-chamber; then, without altering any other control factor, to make readings of the current simultaneously with variation in the magnetic field. Measurements of pressure were made with a McLeod gauge. Dry air was the only dialectric used.

A large number of curves showing relation between current and field-strength were obtained. The various factors altered in obtaining these results were pressure, length of gap, area of electrodes, polarity, and potential-difference. A few typical results are reproduced. Fig. 2 shows three members of a family of curves obtained

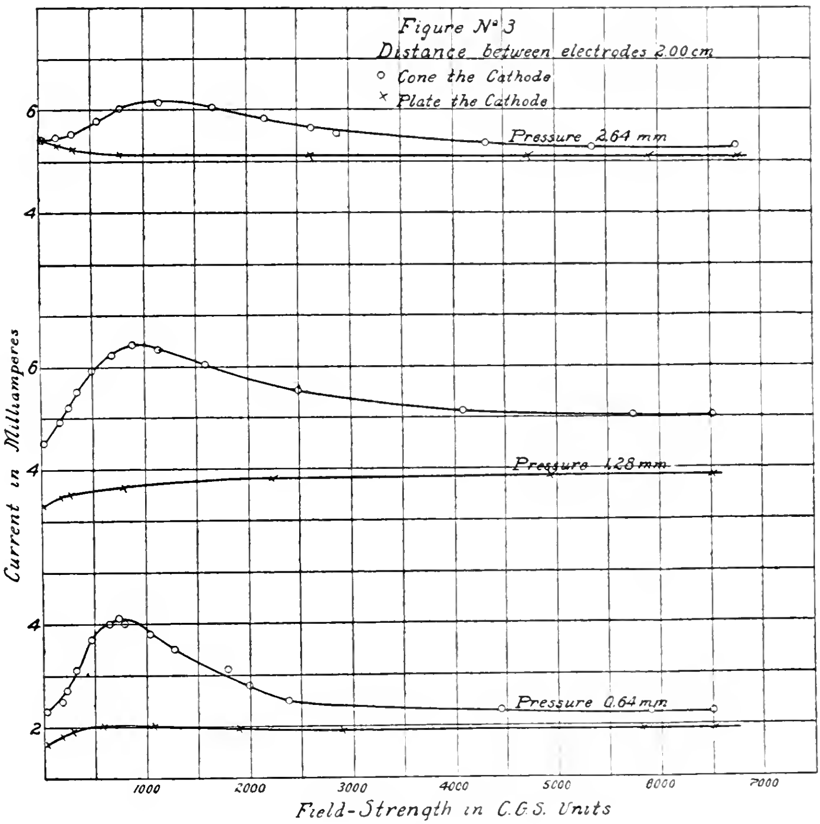

FIG. 3

when the gap was 1 cm. Circles (○) represent values when cone is cathode; crosses (X) give values when plate is cathode. Similar families were obtained for distances of 0.5 cm, 1.5 cm, and 2 cm. The results obtained in the last case are shown in Fig. 3. Finally, the effect of concentrating the discharge from one electrode, leaving the other undiminished, was studied by covering one-half of the

cone by a ring of insulating material, leaving the point projecting. Similarly the plate electrode was reduced one-half, the original area of the cone being restored at the same time. The results are indicated in Fig. 4. It may be noted that when the cone is made

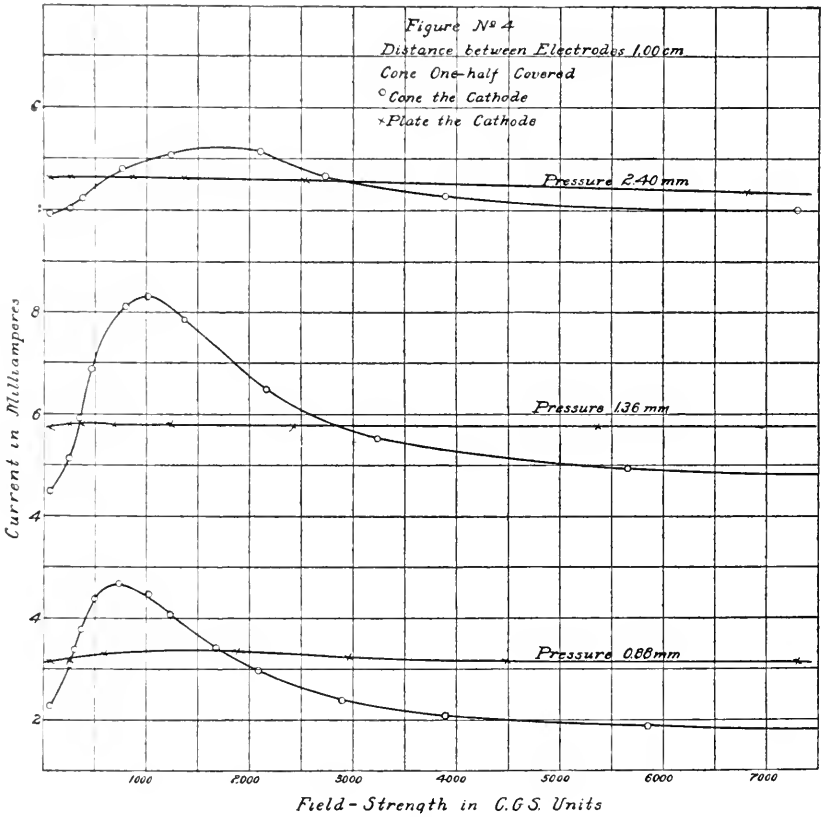

FIG. 4

the cathode an increase in field causes a rapid increase in current up to a well-defined maximum, which is best defined at low pressures. A further increase causes a rapid decrease in current, followed by a more gradual decrease. The final values in many cases are less than the original value. Increase in pressure causes a shift in maximum toward higher fields. The omitted curves bear out this statement completely. Lengthening the discharge-gap, as well

as decreasing the area of the electrodes, serves to flatten the peaks, which means that the variations due to magnetic fields are less abrupt, thus making the wider area more sensitive to small changes in the critical region. When the flat plate is made cathode, the

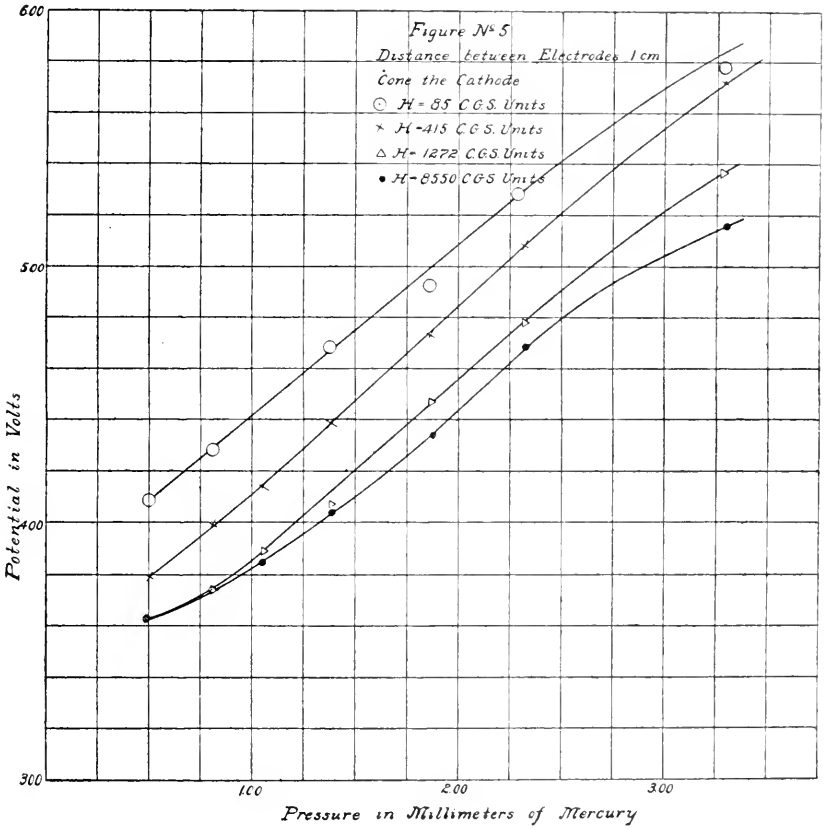

FIG. 5

shape of the curves is different. Changes due to field-strength are not great, and no well-defined maximum appears.

SPARK-POTENTIALS

Measurements were made on the potentials required to start a discharge, using the apparatus shown in Fig. 1. The spark-length chosen was 1 cm. Pressures were varied from 0.5 mm to

3 mm of mercury. The cone was made cathode and the minimum potential required to produce a discharge was determined. The pressure remaining constant, the field was varied from 85 to 8550 c.g.s. units in eleven steps. The value 85 represents the

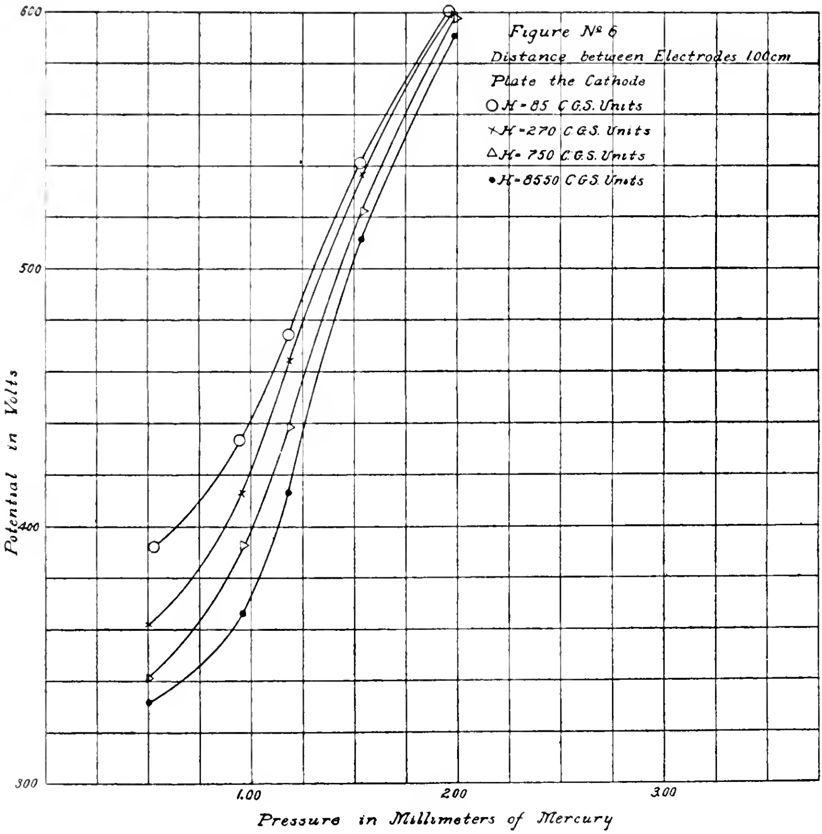

FIG. 6

residual magnetism, i e., the field-strength with zero current in the coils of the magnet.

Fig. 5 indicates the results for several pressures with the cone as cathode. A reduction of spark-potential with increase in fields up to 1300 c.g.s. units is quite marked. Above such values changes are small. Fig. 6 shows values secured with the polarity reversed.

PHYSICAL LABORATORY
OHIO STATE UNIVERSITY
COLUMBUS, OHIO
June 2, 1917

Minor Contributions and Notes

NOTE ON THE SOUTHERN TAIL OF HALLEY'S COMET SEEN MAY 17 AND 18, 1910

In a paper printed in the *Astrophysical Journal*, **39**, 373, 1914, I gave two drawings of the tail or tails of Halley's comet on the nights of May 17 and 18, 1910. Below the long bright tail a luminous region is shown in each drawing, extending to the southeastern horizon, with a definite dark rift between it and the bright tail. On the original drawing this dark rift is more strongly marked, and the lower illumination is quite definitely bounded at its upper part, where it makes the lower edge of the dark rift. In the paper referred to, this lower illumination is attributed to a second tail (which I now think was probably the main tail) of the comet, which perhaps blended into the zodiacal light, since this tail extended along the ecliptic.

This second tail is an interesting factor in the study of the close approach of Halley's comet to the earth at that time and the probability of the earth having passed through a part of the tail. It is therefore important to know just how much of this object belonged to the comet and how great a part the zodiacal light played in the phenomenon.

The southern observers (among them Perrine at Cordoba, who has given a good account of the two tails in the *Astronomical Journal*, **26**, 145, 1910, and Innes at the Union Observatory, South Africa), who saw the comet under better conditions, unhesitatingly describe it as having two tails—a northern tail and a southern one much broader and only one-half as bright. Their location of these tails among the stars agrees closely with the drawings given in the *Astrophysical Journal* referred to (Plate X). The question with me was not whether the comet had two tails, for this fact was evident from the definite northern boundary of the southern branch, which is not a characteristic of the edges of the zodiacal light, but how much of the phenomenon belonged to the comet and how much to the zodiacal light. Fortunately this is a question that can be

decided by an actual observation of the region of the comet under similar atmospheric conditions on or about May 18 of any year.

In the present year the smoky condition of the sky prevented any observations of this kind here until the night of May 22, at which time the conditions were good and the sky was certainly as clear as on the nights of May 17 and 18, 1910. At frequent intervals on the night above mentioned, while working with the Bruce telescope, I carefully examined the region of the drawings of 1910 from $12^h 30^m$ to $15^h 0^m$, Central Standard Time, or later, for traces of the zodiacal light. The effect of this light was not visible at any time. The sky was free from any illumination where the supposed southern tail was seen. The dawn came up as usual with a uniform illumination rising from the horizon, and was not specially noticeable until after $14^h 45^m$. As late as $14^h 50^m$ the arch of dawn was only beginning to whiten the horizon, though the sky was beginning to whiten generally.

From these observations it is clear to me that as seen from this latitude the zodiacal light could not have entered at all into the phenomenon of 1910. It is also clear that the brightness along the southeastern horizon in 1910 before $14^h 45^m$ C.S.T. was due entirely to the second branch of the tail of Halley's comet. This is a fact that can be substantiated at the Yerkes Observatory any year in the latter half of May under proper conditions of the sky. This fact may be of interest in connection with the observations in 1910 made in the Southern Hemisphere, where the large angle made by the ecliptic with the horizon may have introduced an effect from the zodiacal light.

In the *Astronomical Journal*, **26,** 140, 1910, there is a report by Professor Mary Whitney, of Vassar College Observatory, where the second tail was seen but was attributed to the zodiacal light. Doubtless similar observations of this tail were made elsewhere in the Northern Hemisphere with the same inference.

So far as the observations at the Yerkes Observatory are concerned, the phenomenon of 1910 was in no way due to the zodiacal light, but belonged entirely to the comet.

E. E. Barnard

Yerkes Observatory
May 24, 1917

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THE MATHEMATICAL STRUCTURE OF BAND SERIES[1]

By RAYMOND T. BIRGE

INTRODUCTION

It is generally agreed[2] at the present time that the mathematical relationships found in band series are conditioned primarily by molecular structure, while those of line series are due mainly to atomic structure. An exact knowledge of the structure of spectral series is therefore considered of the utmost importance in the study of the ultimate constitution of matter.

This paper is concerned exclusively with the relationship between the individual lines of a single band series, i.e., with that relationship for which the first Deslandres' law is a first approximation. An admirable review of the entire subject of regularities in band spectra is given by Konen,[3] and the "cause for action" in this field is there stated very clearly (p. 272): "Keine der bisher aufgestellten Formeln stellt die längeren, bisher untersuchten Serien genau dar." Ritz[4] has called attention to the apparently anomalous structure of very long series and to the great difficulty

[1] Presented to the American Physical Society, April, 1917.

[2] For full discussion and references see Konen, *Das Leuchten der Gase und Dampfe*, pp. 339–366; also, in particular, Königsberger, *Astrophysical Journal*, **35**, 139, 1912, and Whittaker, *Proc. Roy. Soc.* (A), **85**, 262, 1911.

[3] *Ibid.*, pp. 199–278. [4] Ritz (Weiss), *Astrophysical Journal*, **35**, 75, 1912.

of finding for them any formula, empirical or otherwise. It might be added also that those formulae which are least in error are of a purely empirical type, and not at all adapted for formulating any theory of molecular structure. The situation in the case of line series is very different, for, while the exact *functional form* of the equation is still unknown, the very fact that the frequency can be expressed as a difference of two functions ("combination principle"), taken in connection with considerations as to energy leads directly to a possible theory of atomic structure.[1] In the present paper there is presented a formula for band series which not only holds with the greatest possible accuracy for the longest known series, but which, it is hoped, is of a type suitable for application to a theory of molecular structure.

In 1886 Deslandres[2] stated his now famous law for band series, as follows: "Les intervalles d'une raie à la suivante, calculés en nombres de vibrations ou inverse de longueurs d'onde sont à peu prés en progression arithmétique." If Δv expresses this first frequency-difference, we then have as the algebraic form of the foregoing law

$$\Delta\nu_m = s + mr, \qquad m = 0,\ 1,\ 2,\ 3,\ \text{etc.} \tag{1}$$

Also, if the difference of any two successive $\Delta\nu$'s is taken, we have

$$\Delta\nu_{m+1} - \Delta\nu_m = r = \text{constant.} \tag{2}$$

The fact that the second frequency-difference is constant is the most general and concise statement of Deslandres' law, and any series for which the second difference *is* constant therefore obeys the law. This fact has not always been recognized,[3] and some confusion over a very simple matter has thereby resulted.

As shown by (1) and (2), the first frequency-difference is given by the general equation of the first degree in m, and the second difference by that of zero degree. Conversely, the frequencies themselves are given by the general equation of the second degree:

$$\nu_m = \nu_0 + am + bm^2 \tag{3}$$

or

$$\nu_m = A + B(m + c)^2. \tag{3'}$$

[1] See, for instance, Millikan, *Science*, **45**, 321–330, 1917.

[2] *Comptes rendus*, **103**, 375, 1886.

[3] I.e., Lester, *Astrophysical Journal*, **20**, 81, 1904.

(3′) is the most convenient form for reference, since the constant c (the "phase") plays an important rôle in Thiele's theory of band series.

The exact relationship between the constants in (3) or (3′) and those in (1) depends on the assumptions made regarding the values of m to be used to express each ν and each $\Delta\nu$. The most convenient assumptions in this regard are the following: that the frequencies of successive lines shall be given in (3) or (3′) by the succession of integral values $m=0, 1, 2, 3$, etc., while the corresponding first frequency-differences shall be given in (1) by $m=0.5, 1.5, 2.5$, etc., i.e.,

$$\Delta\nu_m = s+mr, \qquad m=0.5, 1.5, 2.5, \text{ etc.} \tag{1′}$$

Thus

$$\nu_4-\nu_3=\Delta\nu_{3.5}.$$

By these assumptions the $\Sigma\Delta\nu_m$ becomes identical with the $\int\Delta\nu_m$. For, if the frequency of the first line $[m=0]$ is called ν_0, we have for the frequency of the $(n+1)$th line $[m=n]$

$$\nu_n=\nu_0+[\Sigma\Delta\nu_m]_{m=0.5}^{m=n-0.5}=\nu_0+sn+r/2\ n^2. \tag{4}$$

This is identical with the integral of (1′). A comparison of (4) with (3) and (3′) gives the relations between the various constants

$$[r/2=B=b;\ c=a/2b=s/r;\ \nu_0=A+Bc^2].$$

Equation (1′) is that of a straight line with the intercept $m=-c$, while (4), (3), and (3′) represent a parabola with the vertex at $m=-c$, $\Delta\nu=A$ (see Fig. 1). I call the equation (1′) the "slope" form, since it is identical with the equation in terms of the slope of the parabola $d\nu/dm$ and m, the integral of which gives the original parabola.[1] In general, however, the $\Sigma\Delta\nu$ will not be identical with the $\int\Delta\nu$, for, while the integral gives the true area under the $\Delta\nu$-m curve (from $m=0$ to $m=n$), the $\Sigma\Delta\nu$ gives the

[1] In respect to the values of m to be applied to ν and $\Delta\nu$, Deslandres made the same assumptions as are expressed by (1′), but instead of having the corresponding frequencies given by $m=0, 1, 2, 3$, etc., he used $m=1, 2, 3, 4$, etc. Therefore in (4) the "n" should be replaced by $(n-1)$, in order to obtain the exact equation used by him (*ibid.*). By Deslandres' conventions the parabola, instead of having its vertex directly below the intercept of the straight line (Fig. 1) would be shifted one unit to the right, and (1′) is no longer the derivative of the frequency equation, thus complicating the computations.

area under a series of chords drawn along the $\Delta\nu$-m curve, joining the points corresponding to $m=0.5, 1.5, 2.5 \ldots . (n-0.5)$, *plus* the two rectangles (one at each end) $\frac{1}{2}\Delta\nu_{0.5}$ and $\frac{1}{2}\Delta\nu_{n-0.5}$. It is of course only because the arc and chord coincide for a straight line that we can write

$$\nu=\nu_0+\Sigma\Delta\nu=\int\Delta\nu.$$

The importance of this point will appear later.

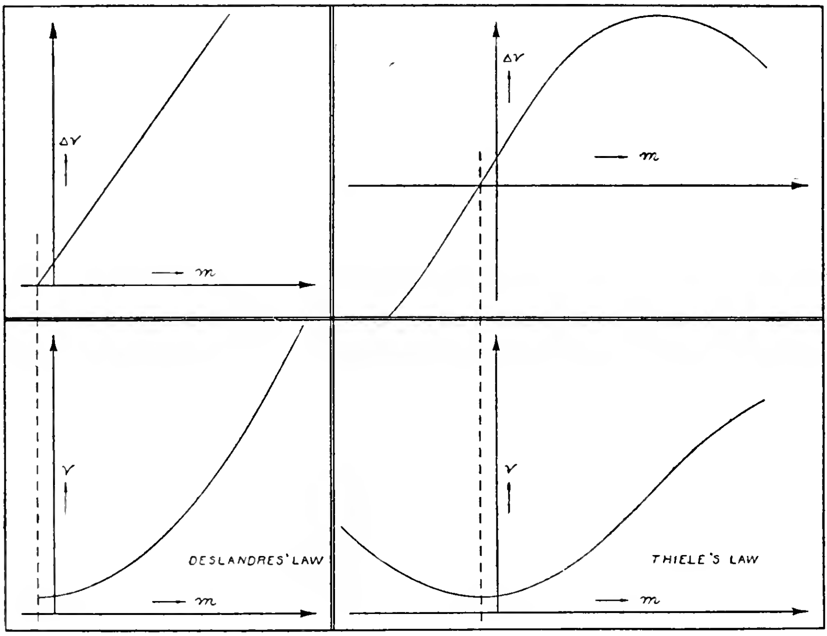

FIG. 1

Now there is nothing in Deslandres' law to specify what particular integral value of m shall be assigned to any particular line of a series. In other words, the straight line and parabola of Fig. 1 can be shifted at will parallel to the m-axis, the vertex of the parabola, however, always remaining under the intercept of the straight line. But it is customary to make this intercept (c) as small as possible, i.e., equal to 0.5 or less. Under these conditions it is found that substituting $n=0$ in (4) usually, *but not always*, gives the frequency of a definite, observable "head." The only

reason, in the author's opinion, why $n=0$ does not always give the observed head is that many series begin with zero intensity, and the observed head belongs to some other series which happens to begin with great intensity. It seems best to *define* the head of a series as that frequency obtained by putting $m=0$ in any suitable frequency-equation (shifted, as explained above), regardless of whether or not there is any observable radiation at the predicted location. This matter will be discussed in more detail in another paper.

Deslandres found that when he had thus shifted the origin so as to make the intercept in (1′) as small as possible, he might equally well *assume* this intercept to be zero. In fact, even for that small portion of a band series for which Deslandres' law is practically true, the wave-lengths must ordinarily be known with certainty to at least 0.005 A in order to get any determinate value of c. With the data at Deslandres' disposal, the appropriate values of m were usually indeterminate to one or more whole units. Deslandres therefore *actually used* his law in the *approximate* form

$$\nu_m = A + Bm^2. \tag{3''}$$

This, however, is *not* Deslandres' law, for it assumes in (1′) the special relationship $s=0$, which is not assumed in any way in the original statement of the law.[1] Thiele found that for some series the phase $c=0$. In general it did not. In other words, the shifted $\Delta\nu$-m curve sometimes seems to head straight for the origin, but in general it does not.

As was recognized by Deslandres himself, the foregoing law (3) is only a first approximation, applying at best to a short portion, near the head, of a band series. For some series it will hold apparently for nearly 60 members, in others, for a much smaller number. The actual form of the $\Delta\nu$-m and of the ν-m curve for the longest known band series (those of the CN 3883 band) is given roughly by the portion of the curves to the right of the vertical axis in the right half of Fig. 1. The first frequency-difference, instead of increasing linearly, actually attains a maximum value, corresponding to a point of inflexion on the ν-m curve.

[1] Deslandres, *op. cit.;* see also *Comptes rendus*, 138, 317, 1904.

Thiele[1] attempted to explain these facts, and many others, by assuming that the general law for band series is

$$\nu = f[(m+c)^2], \tag{5}$$

where c = constant, and was called by Thiele the "phase" of the series. The Thiele theory has been definitely disproved in every essential point in two articles, one by H. S. Uhler[2] and the other by Uhler and Patterson.[3] The writer could give additional evidence in this matter, but it seems superfluous.

The essential points in the Thiele theory are shown by the two right-hand curves of Fig. 1. In equation (5) the series obtained by using *negative* integral values of m (negative branch) is assumed to exist, as well as that corresponding to positive values (positive branch). Thus the ν-m curve is symmetrical with respect to the line $m = -c$, and both branches of the curve represent series. If $c = 0$ or $\frac{1}{2}$, the lines of the two series coincide. If $c = \frac{1}{4}$, the lines of one series fall approximately (*not exactly*) halfway between those of the other series. The simple fact that in many cases the two series which leave the same "head" of a band actually cross over one another is sufficient proof against the validity of Thiele's hypothesis.

The other essential point in Thiele's theory is that, while $m = 0$ gives the "head" of a series, $m = \infty$ gives the "tail." This indicates that the ν-m curve becomes asymptotic to the horizontal line ν_∞, while the $\Delta\nu$-m curve becomes asymptotic to the m axis. The tail of a band series is thus identical in structure with the head of a line series. *No tails of the structure assumed by Thiele have ever been found.* The so-called tails of the CN bands, first measured by King,[4] either have no structure at all or else have the structure of an ordinary band head, and are thus composed of a finite number of lines instead of an infinite number. Uhler and Patterson[5] and others[6] make this point very clear, and their conclusions have been verified by the author, using his own negatives. Thus there is at the present time no proven theory concerning the structure of band series.

[1] *Astrophysical Journal*, **6**, 65, 1897; **8**, 1, 1898.

[2] *Ibid*, **42**, 72, 1915.

[3] *Ibid.*, **42**, 434, 1915.

[4] *Ibid*, **14**, 323, 1901.

[5] *Ibid.*

[6] Ritz (Weiss), *ibid.;* Strutt and Fowler, *Proc. Roy. Soc.* (A), **86**, 105, 1912.

It might be well to state that, in searching for new spectral formulae, it is very convenient to work with the $\Delta\nu$-m ("slope") curve, since its equation must necessarily be of degree one lower than the final equation in frequency, and must have one less arbitrary coefficient.[1]

EXPERIMENTAL MATERIAL

When this investigation was started, it appeared, from the literature, that the A_1 series of the 3883 CN band was not only the longest band series known, but also that series which showed the most radical deviations from Deslandres' law. It was therefore deemed to furnish the most promising material for testing any new formula. This series is the so-called "singlet" series running from the first head of the band. Its structure has been studied by Kayser and Runge,[2] Jungbluth,[3] Uhler and Patterson,[4] Kilchling,[5] and others. The lines of the series have been observed from $m=4$ to $m=168$ inclusive.[6] In Fig. 2 the solid line marked A_1 represents a *smooth* curve drawn through the experimental values, in terms of $\Delta\nu$ and m,[7] the mean position of all resolved doublets being used. The actual data for the A_1 series used in this article are taken exclusively from the results of Uhler and Patterson.[8] The

[1] See Baly, *Spectroscopy*, pp. 564–567 (1912 ed.).

[2] *Abhandl. der Berliner Akademie*, 1889; also Kayser's *Handbuch*, **2**, 479, and **5**, 230.

[3] *Astrophysical Journal*, **20**, 237, 1904.

[4] *Ibid.*, **42**, 434, 1915. [5] *Zeitschrift für wiss. Photographie*, **15**, 293, 1916.

[6] This is really a series of doublets, but the two components, at the head of the series, apparently coincide in position. From this point the separation increases to a maximum of about 0.07 A, remains roughly constant for some time, and then rapidly diminishes. The first and last resolved doublet is about $m=60$ and $m=154$, respectively.

[7] In order to obtain a convenient 1-to-1 scale for plotting, frequency is given in terms of $10^9/A$, instead of the usual $10^8/A$. In this article frequency and first frequency-difference in $10^9/A$ are denoted by ν and $\Delta\nu$, respectively, while the same quantities in $10^8/A$ are denoted by ν' and $\Delta\nu'$.

[8] The frequencies and first frequency-differences, reduced to *vacuo*, are given by these authors for the A_1 series only. Professor Uhler has asked me in this connection to call attention to the fact that in reducing the wave-lengths to *vacuo* the index of refraction of air at 20° C. was used by mistake instead of that at 15° C. As a result, the frequency for $m=0$ is 0.124 ν' too large, while that for $m=168$ is 0.136 ν' too large. But the *relative* error thus introduced is only 0.012 ν' (= .0018 A) and therefore is negligible, as far as series relationships are concerned.

author, however, has a series of very fine exposures of the entire arc spectrum of carbon, taken with the 21-foot concave grating of the University of Wisconsin. Portions of these plates have been studied with an 8-inch Gaertner comparator, which the author was enabled to purchase through the generosity of the Rumford Fund Committee. The immediate object was to attempt to extend the various series of the 3883 band, and also to clear up

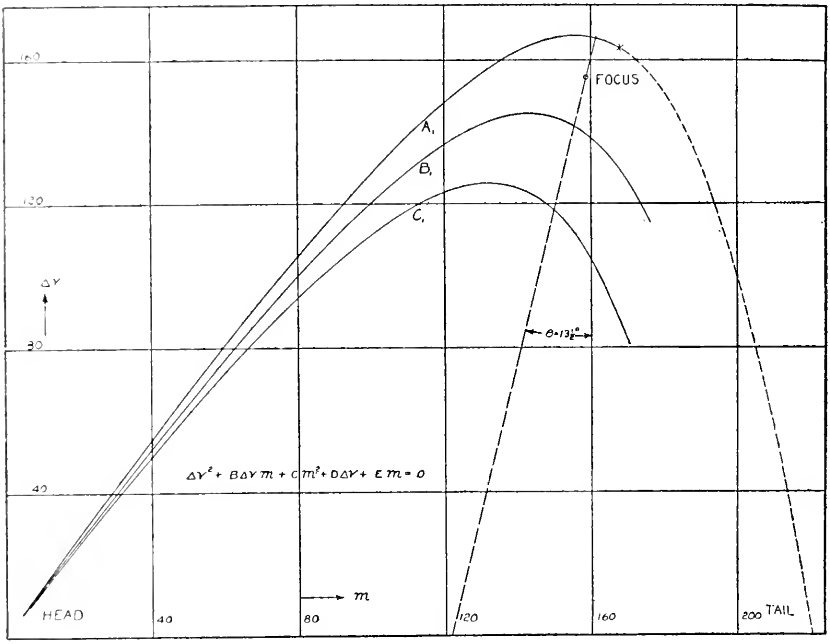

Fig. 2

certain perplexing points in the structure of the known portions. This investigation is still in progress, and the results will be presented in full in a separate paper.

It has not been possible to extend the A_1 series beyond $m=168$, the point at which all previous investigators have stopped. There is, however, no question as to the identity of the lines up to this point. From $m=160$ on, the intensity is rapidly diminishing, but at $m=168$ it is still great enough to indicate the possibility of following the series for several more lines. There is, in fact, a

series of lines which roughly forms a consistent extrapolation of the curve given in Fig. 2, but the intensities and appearances of these lines are consistent neither with each other nor with the last known A_1 lines. It must therefore be concluded that the A_1 series ends at $m = 168$. This is rather unexpected, for in the case of the B_1 and C_1 series (the corresponding series of the second and third heads) the intensity falls off uniformly. The author has succeeded in extending the C_1 series several lines beyond the previous limit, and the last two or three lines are so faint as to be barely visible. On the other hand, both the A_1 and B_1 series end at about the *same* wave-length (3640 A), while the C_1 series ends 20 A previous to this point. Beyond 3640, as other investigators have noted, the band loses all appearance of regular structure, while the background is filled with a great number of seemingly unrelated lines.

When the $\Delta\nu$-m curves for the B_1 and C_1 series had been computed and plotted (as shown in Fig. 2[1]), it was immediately evident that they would furnish a far more critical test of any new formula than would the A_1 series. For, while this series stops just past the maximum $\Delta\nu$ (which occurs at $m = 156.5$), the B_1 and C_1 series run very much farther. On the other hand, the A_1 series is distinctly the strongest in the band, and has the smallest "perturbations," so that the data for this series are more complete, more accurate, and more consistent than for any other series. The work on the B_1 and C_1 series has now been carried to the point where it is evident that the theory of band structure here presented applies with equal accuracy to these series. The actual results will be presented in another paper. The present contribution deals exclusively with the A_1 series, using the mean position of each doublet.

Kayser and Runge[2] concluded that this series could be represented satisfactorily by the four-constant formula

$$\nu = A + Be^{am} \sin (\beta m^2), \tag{6}$$

to which, however, they attached no importance, because of its

[1] The portion of C_1 between $m = 6$ and $m = 47$ is based on the author's own measurements, the series not having previously been identified over this interval. The same statement is true for the portion $m = 164$ to $m = 171$.

[2] *Ibid.*

complicated form. But, since the differences (O.–C.) amount to as much as 0.4 A in at least one place, the formula can scarcely be called satisfactory. The probable relative errors in the data of Uhler and Patterson are considered by the authors to be ±0.002 A for sharp lines and ±0.01 A for hazy lines. A smooth curve, such as is given in Fig. 2, shows that the average deviation of the experimental points is in practical accordance with this estimated error. (This is exclusive of the perturbations, which consist, usually, of a single line of exceptional character, displaced considerably from its expected position. The perturbations of similar series are similarly located, and all perturbations show exceptional behavior in a magnetic field.[1] They are undoubtedly destined to assume an important place in any complete theory of molecular structure.)

Kilchling,[2] in a painstaking investigation of which, unfortunately, only the first half has yet reached the writer, concludes that no three-constant formula can be used to represent this series. No attempt was made to investigate four-constant formulae, because of the great variety of possible forms. Instead he decides to use the purely empirical formula (as he himself calls it)

$$\nu = a + bm^2 - cm^4 + dm^6 - em^8, \tag{7}$$

in which the five constants are intrinsically positive. This corresponds directly to the Kayser and Runge formula for line series

$$\nu = a + bm^{-2} + cm^{-4} + \text{etc.},$$

which the authors carried to three terms only, although several more are necessary for real accuracy.

No hint as to how accurate Kilchling found formula (7) is given in the first half of his article. It is, however, possible to draw certain general conclusions as to the applicability of any general interpolation formula, such as (7). If repeated frequency-differences are computed, each line or, more conveniently, only every fifth or tenth line being used, then, if the pth order of frequency-differences is essentially constant, the series can cer-

[1] Fortrat, *Comptes rendus*, **156**, 1459, 1913; **157**, 991, 1913; **158**, 334, 1914; Deslandres and d'Azambuja, *ibid.*, **157**, 671, 814, 1913; **158**, 153, 1914; Deslandres and v. Burson, *ibid.*, **157**, 1105, 1913; **158**, 1851, 1914.

[2] *Loc. cit.*

tainly be satisfied by the general equation of the pth degree, having $(p+1)$ arbitrary coefficients. And it *may* happen that any one or more of the coefficients, *except* that of x^p, can be equated to zero without appreciably disturbing the agreement. Thus, if the differences of the 8th order are constant, formula (7) *may* be satisfactory, although more likely it will be found necessary to include also the odd-power terms. Now, the data taken from a smooth curve for the A_1 series show that the sixth differences are nearly enough constant so that a sixth-degree (7 constant) formula would be satisfactory to 0.01 A throughout. For the C_1 series similar computations show that it would be necessary to add one or more higher powers. Kilchling thus uses, for the A_1 series, a higher power (8th) than necessary, but the dropping of the odd powers necessitates this.

The author has tested the Kilchling formula on the A_1 series and finds that the deviations (O.–C.) cannot be brought within the limits of experimental error. This statement would be true in far greater degree for the B_1 and C_1 series. The Kilchling formula gives a "slope" curve which is too straight near the origin, but does not bend rapidly enough near the point of maximum first frequency-difference.

CALCULATIONS

The shape of the curves in Fig. 2 immediately suggests some sort of a conic, while the importance of this type of curve in all dynamic problems is an argument in favor of its use as an empirical formula.

In testing the A_1 series, I therefore first tried a parabola running through the origin and having its axis parallel to the $\Delta\nu$ axis (two undetermined coefficients). This was entirely unsatisfactory. Next I tried a hyperbola through the origin, with the axis parallel to $\Delta\nu$ (three coefficients). This came very near to being satisfactory, and a considerable amount of time was spent in a vain endeavor to get a perfect agreement. Finally, I tried a hyperbola through the origin, with the axis having any slope (four arbitrary constants), and this was found *entirely satisfactory*. The equation in $\Delta\nu$ and m is therefore

$$\Delta\nu^2+B\Delta\nu\cdot m+Cm^2+D\Delta\nu+Em=0. \tag{8}$$

For convenience the coefficient of $\Delta\nu^2$ is taken as the one to be made arbitrarily equal to unity. The fact that the curve goes through the origin eliminates the fifth constant. I wish, however, to lay stress on the fact that the absence of the constant term is not a material point. Both the A_1 and B_1 curves evidently head straight for the origin. But the C_1 curve, as recently determined, does *not*. It makes little difference in the actual calculations whether (8) or the complete equation of the second degree is used, since the elimination of the constant term between five simultaneous equations is a very simple process.

If (8) is solved explicitly for $\Delta\nu$, we have

$$\Delta\nu = -Bm/2 - D/2 \pm \sqrt{\left(\frac{B^2-4C}{4}\right)m^2 + \left(\frac{BD}{2} - E\right)m + D^2/4}. \qquad (8')$$

The branch of the hyperbola actually represented by the series corresponds to the negative sign before the radical. The best set of values of the constants for the A_1 series, in terms of the new unit of $\Delta\nu$, $(10^9/A)$, is

$$B = +\ 7.2383694 \qquad D = -2{,}101.1672$$
$$C = -13.258107 \qquad E = +2{,}929.5502$$

The broken lines in Fig. 2 give the extrapolation of the hyperbola obtained with these constants, and its axis.

No particular importance should be attached to the absolute values of the constants, since the slightest change in the shape of the hyperbola (especially near the vertex) causes a radical change in the absolute values, although but little change in the relative values. Because of this fact, it was not found expedient to use a least-square solution, since this does not in general give satisfactory results unless very approximate values of the unknowns have previously been determined. The procedure used was to pass the curve through four arbitrary points by solving the four simultaneous equations, and then to shift and reshift until the agreement was satisfactory. This is unfortunately a rather laborious process, for the fitting of the hyperbola, especially around the vertex, is a delicate operation. It is also unfortunate that the data in this region are poorer than for the lower part of the curve.

The conditions are just reversed in the case of line series, for which the procedure is perfectly straightforward.[1]

The accuracy with which the equation represents the experimental data is shown by the broken-line curve in the upper half of Fig. 3. Here the O.–C. value of each $\Delta\nu$ is plotted against the corresponding m. The center line of each three indicates the axis, while the other two lines are drawn on either side at a distance representing 0.01 A.

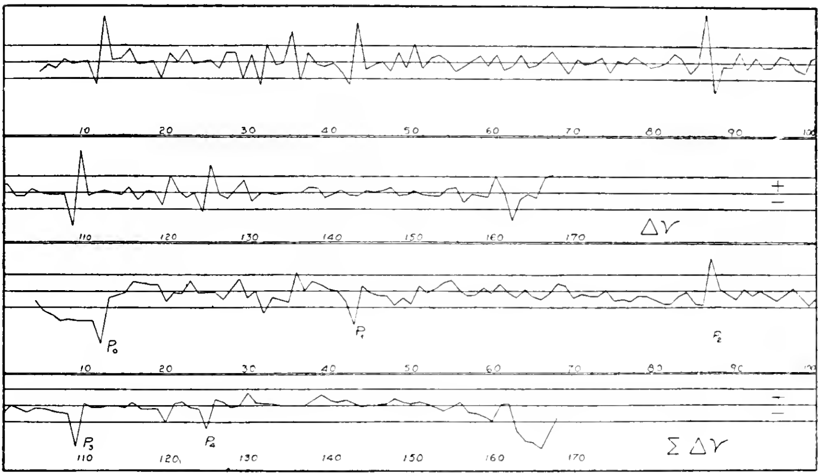

Fig. 3

It is, however, the final agreement between observed and computed frequency that is of real importance. As has been pointed out, the frequency is given by

$$\nu = \nu_0 + \Sigma\Delta\nu, \tag{9}$$

and this is *not* equal to $\int\Delta\nu$. For this particular curve I find that the difference between the summation and the integral amounts to about 0.005 A for the very first line (the $\frac{1}{2}\Delta\nu_{0.5}$ rectangle), and that this difference gradually diminishes to a value of about 0.0025 A at the next to last line and then jumps to 0.005 A for the last line (owing to the $\frac{1}{2}\Delta\nu_{n-0.5}$ rectangle). Thus, as far as fitting

[1] R. T. Birge, *Astrophysical Journal*, 32, 114, 1910.

the experimental data is concerned, the integral might equally well be used, instead of the summation.

But the integral of (8) is very complicated, having the form

$$\nu=C+am+bm^2\pm\{(cm+d)\sqrt{cm^2+fm+g}+h\sin h^{-1}(lm+k)\} \quad (10)$$

where a, b, ... k, are $f(B, C, D, E)$. I have therefore not tried to work with it, but instead have taken the successive $\Sigma\Delta\nu$. If one wishes to test at every point the agreement between observed and computed values, it is of course necessary to compute each $\Delta\nu$, and after that is done the actual frequencies follow immediately.[1]

The final agreement between observed and computed frequencies is given in the lower half of Fig. 3, the scale being the same as for the upper half. The two curves are so related that any point on the axis of the $\Sigma\Delta\nu$ curve indicates that in the $\Delta\nu$ curve the algebraic sum of the deviations, from $m=0$ up to the point in question, is zero. However, the $\Sigma\Delta\nu$ curve begins with a residual error of 0.006 A, which is not indicated on the $\Delta\nu$ curve. This arose from the particular value (ν_0) assumed for the head of the series. This value, in turn, was chosen so as to make the computed frequencies agree as well as possible with the observed frequencies for the first twenty or thirty known lines.

As Fig. 3 shows, the observed and computed frequencies agree to within 0.01 A throughout, with the exception of the perturbations (marked P_1 . . . . P_4) and slight irregularities at the two ends. In fact, the average residual (O.–C.) for the entire 164 known lines is 0.005 A, while, if the perturbations and irregularities mentioned are excluded, the average is only 0.003 A. This value is clearly within the limits of experimental error and constitutes a greater accuracy than has ever before been attained for any long band series. The only line-series formulae which have given better results are the three-constant formula with which W. E. Curtis[2] has fitted the hydrogen Balmer series and the six-constant formula used by Nicholson[3] for the helium series. The four-

[1] One method of computing any summation is that of finite differences. But the high order of differences required in this case, as shown in the discussion of the Kilchling formula, renders the method useless for extrapolation. It can, however, be used profitably for interpolation.

[2] *Proc. Roy. Soc.* (A), **90**, 605, 1914. [3] *Ibid.*, (A), **91**, 255, 1915.

constant formulae of Ritz and Hicks give deviations up to at least 0.1 A for all series save those just mentioned. It is therefore not unreasonable to expect that a formula with at least five constants, such as (9), will be necessary for the longest and most complex band series, when three to six constants are needed for the simplest line series.

The reasons for excluding the initial and final irregularities in computing the final accuracy are as follows:

1. The lines from $m=160$ to the end are broad and hazy. The experimental error is great enough to cover that shown in the curves of Fig. 3. I have tested this by taking several sets of readings on these lines and by obtaining error-curves entirely unlike those in Fig. 3.

2. The point marked P_0 at $m=12$ is not a true perturbation, and the errors to the left of this point are only apparent. A critical examination of the first three heads of this band has shown that they are essentially similar, and that each has the following structure: The head is composed of a *double line*, which can actually be detected as double on some of the negatives, and this doublet constitutes the *first member* of the "doublet" series (which has practically constant separation). The singlet series begins *in coincidence* with one component of the doublet series (the more refrangible, in the case of A_1 and C_1; the other, for B_1). The singlet series gradually leaves the doublet, drawing away in such a direction as not to cross the other component of the doublet.

The first line at which Uhler and Patterson resolve the A_1-A_2 coincidence (A_2-A_2 signifies the doublet series) is $m=13$. Therefore, from $m=12$ to $m=4$, the reading given is really the mean position of A_2-A_2, and not the true position of A_1. The $\Sigma\Delta\nu$ curve in Fig. 3 should therefore show an abrupt drop to about -0.03 A at $m=12$, as it does, and this apparent error should decrease linearly to zero at $m=0$.

The value of ν_0 that it was found necessary to use in (9) is $25{,}743.79\,\nu'$. This is $0.33\,\nu'$ ($=0.05$ A) higher than Uhler's measured value for the head of the band. But this difference is in perfect accordance with the constitution of the head. For the separation of the doublet series, at the head, is 0.05 A, and the A_1 series begins in coincidence with the *inside* component.

As has been mentioned, the actual constants given for equation (8) apply to the mean position of all resolved doublets of the A_1 series. The peculiar variation of the separation in these "singlet" series, from zero to a maximum and then back to zero, is, however, consistent with the formula just proposed, so that it would be quite possible to compute for each component a separate set of constants, yielding equally satisfactory results.

DISCUSSION OF RESULTS

The main objections (practical and theoretical) to this theory of band structure seem to the author to be:

1. The equations (8) and (9) involve a simple and definite relation between first frequency-difference and m, but *not* between frequency and m.

This may not be a true objection, for it is possible that the theoretical physicist is just as interested in a property of $\Delta\nu$ as he is in one of ν. In fact, Hasenöhrl,[1] using quantum theory, derives a simple expression for $\Delta\nu$ which apparently can be used to explain the structure of band series.

2. Equation (9) involves a summation, instead of an integral, and the integral which approximates it in value is too complex to be of any practical utility.

The summation is necessary from the very nature of the case, and, as is only too well known, the summation and difference seem destined to assume a very prominent place in the mathematical physics of the future. I have not thus far found any really satisfactory approximation for the summation. The first natural suggestion is to expand the radical of (8′) in ascending powers of m and integrate term by term. The series so obtained is convergent throughout the experimental range, but the rate of convergence is so slow that some forty or fifty terms would be necessary for the requisite accuracy.

Perhaps the best *rapidly* converging series which will approximately represent the hyperbola is

$$\Delta\nu = am + bm^3 + cm^5 + dm^7, \tag{11}$$

[1] *Physikalische Zeitschrift*, 12, 931, 1911.

for which both the integral and the summation will have the functional form of Kilchling's equation (7). The coefficients in the two cases, however, will be slightly different.

The accuracy with which the hyperbola fits the experimental data seems to indicate that we have here a true relation between $\Delta\nu$ and m, and in this respect the case is unique. For all other spectral series formulae have in them an unknown $f(m)$, which is taken usually as a semiconvergent series, and enough terms retained to give the required accuracy. Assuming, then, that the hyperbola *is* a true relation, we have the following structure for band series:

The series starts at $m=0$ (the head) and the first frequency-differences initially increase almost linearly. The values of $\Delta\nu$, however, finally reach a maximum and then decrease to zero. The point at which $\Delta\nu$ is again zero constitutes the tail of the series. It has the same structure as the head, hence a weak band might be the tail of a strong band degraded in the opposite direction.

In the case of all known band series the intensity has dropped to zero (or the series has been concealed by another band) before the tail has been reached, and for nearly all series this is true even before the point of maximum $\Delta\nu$ has been reached. In the case of the A_1 series the tail is given by $m=221$ $(=-E/C)$ and should appear at 3567.2 A. This position is in the 3590 band, and so the tail, if present, could not be observed. In the case of the C_1 series the tail is given (approximate calculation) by $m=191$, so that the last observed line $(m=171)$ is only *twenty lines* from the tail and at a distance of less than *twelve angstroms*. But no one of the observed tails is near the predicted position, and there seems to be no apparent reason why the intensity, after having dropped to zero, should increase again in the vicinity of the tail.

The values of $\Delta\nu$ and m for the vertex of the hyperbola and the slope of the axis (13°5 for A_1) can have no theoretical significance, since they depend upon the particular units chosen for $\Delta\nu$. For, if $\Delta\nu'$ $(=k\Delta\nu)$ is substituted in (8), a new hyperbola with a new vertex and new axis is obtained. The only object in stating such an obvious fact is to call attention to the following error which the author has already made.

A preliminary examination of the A_1, B_1, and C_1 curves seemed to indicate that the three hyperbolae were identical in size and shape and differed only in position. But the exact results for A_1 showed that this was not the case. It then appeared that the axes of the three hyperbolae were parallel, and this hypothesis was mentioned at the April 1917 meeting of the American Physical Society, the implication being that such a result might indicate some important relation between different series. But the simplest algebra shows that, if the axes are parallel for one $\Delta\nu$ unit, they will not be parallel for another, *unless* the B and C constants of (8) are the same for each series. And, if they are the same, the hyperbolae will be identical in size and shape—a condition known to be contrary to the facts. Therefore the parallelism of the axes, if true, is purely fortuitous and without meaning.

CONCLUSIONS

The only band series which can afford a real test for the hyperbolic formula (8) are those in which the first frequency-difference actually attains a maximum value. The ordinary band series forms only a short segment of the hyperbola and so without doubt can be satisfied equally well by many other four- or five-constant formulae. But the only band series, so far as the author is aware, which attain a maximum $\Delta\nu$ are those of the CN bands, and several series in the bands of phosphorus.[1] These latter are only fifty and sixty terms long; they have been measured with far less accuracy than the CN series; and, like the A_1 series, they run but a few terms beyond the maximum $\Delta\nu$. For these three reasons they would furnish but inferior material for a test.

It therefore seems that formulae (8) and (9) will be found satisfactory for *all* ordinary band series (Konen's bands of the "first type"), *with the single exception* of certain series in the 2370 band of air, for which Deslandres[2] states that the deviations from his law are in the direction *opposite* to that indicated by the hyperbola. These series therefore require special investigation.

[1] Geuter, *Zeitschrift für wiss. Photographie*, **5**, 1, 1907.

[2] Deslandres and A. Kannapell, *Comptes rendus*, **139**, 584, 1904.

SUMMARY

1. No satisfactory formula for long band series has thus far been proposed.

2. In the case of the main (A_1) series of the 3883 CN band—a series of 164 known lines—it has been found by the author that the relation between first frequency-difference ($\Delta\nu$) and a variable m is that of a hyperbola. The hyperbola cuts the origin, but does not lie parallel to the co-ordinate axes, so that it is given by a four-constant equation.

3. The actual frequencies are then given by the five-constant formula, $\nu_0+\Sigma\Delta\nu$, and the agreement between observed and computed values is of such accuracy as to indicate that the hyperbola represents a true relation and one, it is hoped, which will readily lend itself to a possible theory of molecular structure.

4. The $\Sigma\Delta\nu$ cannot conveniently be replaced by the $\int\Delta\nu$, and the formula has some troublesome features, but these are shown, in the light of the experimental facts, to be not unexpected.

5. According to this new formula, a band series should end in a tail having the same structure as the head of any band series, in contrast to Thiele's tail, the structure of which should be that of the head of a line series.

6. No series has been followed to its tail, and in the case of three series of the 3883 band there are no observable tails at the predicted positions, although in one case the series can be followed to within 12 A of the hypothetical tail. The intensity relationships, however, do not indicate that the tail should actually appear.

7. But few band series show sufficient deviations from the simple Deslandres' law to constitute a rigid test for the proposed hyperbolic formula, while in only one known band are the deviations of a nature contrary to that predicted by this formula.

8. The numerical results for the A_1 series only are presented in this paper. Those for the B_1 and C_1 series will be published later.

SYRACUSE UNIVERSITY
DEPARTMENT OF PHYSICS
May 26, 1917

A METHOD OF INVESTIGATING THE STARK EFFECT FOR METALS, WITH RESULTS FOR CHROMIUM[1]

By J. A. ANDERSON

The object of the present work was to find a method suitable for the study of the Stark effect for metals having a relatively high melting-point. The spectra of such metals as iron, nickel, chromium, vanadium, and titanium are of special importance in solar work, and if some of their lines show an effect due to an electric field it would at once become possible to determine whether electric fields exist in the solar atmosphere. In previous work on the Stark effect some of the lines of these metals have been observed, but in no case has an electric effect been found for them. Generally the source used has been of feeble intensity as compared to an arc or spark, necessitating the use of low dispersion, and hence the first problem which had to be solved in the present investigation was that of getting a source sufficiently bright to be studied with a spectrograph of moderate dispersion.

APPARATUS

The apparatus shown in Fig. 1 has given general satisfaction; most of the lines of the metals enumerated above can be recorded in exposures of from 10 to 30 minutes to a scale of about 4 A per millimeter, using a spectrograph such as that described in this paper.

A bell-jar, about 15 cm in diameter and about 30 cm high, stands on a glass plate in which there are two openings. The connection to the pump is sealed into one of these, while a larger glass tube, having a platinum electrode at its closed end, is sealed into the other. This larger glass tube is about 2.5 cm in outside diameter for about one-half its length, then tapers to about 1.5 cm in order to form a seat for the ground joint with the long silica tube *D*. *D* is about 20 cm long and 1.2 cm in outside diameter, and carries at its upper end the short silica tube *C*, to which it is

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 134.

fitted by a ground joint. The cathode is in two parts—*B*, made of iron, and *b*, of the metal which is to be studied—and it rests on top of the tube *D*. *C* has a vertical slot, about 1 mm in width, which extends down to the level of the top surface of *b*, the cathode proper. The tube *C* must be ground truly cylindrical inside and must be of a diameter about 0.1 mm larger than the cathode *b*. The necessity for this will be discussed below.

A fine platinum wire, 33 B. & S. gauge, is attached to *B* by means of a screw and carries at its lower end a short brass cylinder which makes contact with the platinum electrode in the larger glass tube. A mercury contact at this point was tried first, but the mercury vapor gave rise to so much trouble that it became necessary to resort to this form of metallic contact, which has worked admirably.

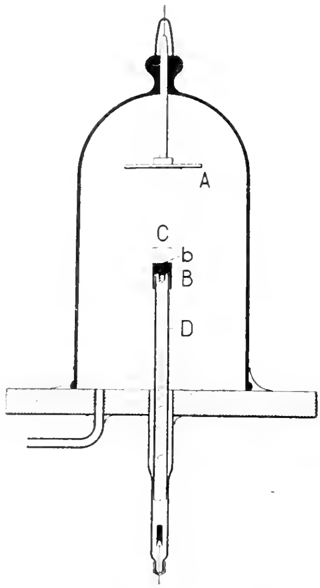

FIG. 1.—Diagram of apparatus. *A*, Anode; *Bb*, Cathode; *C*, Silica tube with slot for observing; *D*, Silica tube.

The anode is a massive disk of aluminium on the end of an aluminium rod. A thick platinum wire, fastened to the rod, is sealed into a glass tube, as indicated at the top of the diagram. The knob at the top of the bell-jar has a hole drilled through it, into which the glass tube is fitted by grinding; it is then sealed in with water-glass.

Direct current to produce the discharge is supplied by a set of high-potential generators connected in series. Each generator gives approximately 800 volts, and for most of the work 8 generators were used. Two spectrograms were obtained with the use of 16 generators, but so much difficulty was experienced in keeping the vessel operating satisfactorily at this high voltage that it was discontinued until some future time, when it will be tried again. A resistance consisting of distilled water with platinum electrodes was always used in series with the tube.

The resistance varied somewhat, but in general it was kept at about 32,000 ohms.

A high-vacuum oil pump is used to exhaust the bell-jar, the pressure employed being such that the Crookes' dark-space has a length of from 4 to 6 mm. At first hydrogen was used as residual gas, but later it was found that spectra of the metals are brighter in air or oxygen, and hence in the major part of the work commercial oxygen was employed. The gas was passed through concentrated sulphuric acid to remove most of the moisture, but, since no drying tube was connected to the vessel directly, there were always traces of water-vapor present. This was found to be a real advantage, as it gave the hydrogen lines with an intensity which was very suitable for determining the field-strength.

PHENOMENA OF THE DISCHARGE

With the use of 8 generators (about 6400 volts), a resistance of about 32,000 ohms in series, and pressure adjusted to give a dark-space of about 4 mm, the initial current, if one of the metals already mentioned is used as a cathode, will be about 100 milliamperes. With manganese the current is usually a little higher, while with magnesium it is about 200 milliamperes, the bell-jar acting as if it had practically no resistance. During the first minute the current usually decreases rapidly, and in about two minutes it becomes steady at approximately 40 milliamperes. In the meantime the cathode *b* has become almost white hot, its temperature being now from 800° to 1000° C. The cathode glow, Crookes' dark-space, and most of the negative glow are confined within the tube *C*, while a soft glow extends with decreasing intensity from the top of *C* up to the anode or even beyond it. On the anode may be a single bright point, or there may be nothing whatever visible. After the current has run about ten minutes, all of the tube *C*, excepting a few millimeters near the top, will be red hot, as will also the cathode *B* and the upper end of the tube *D*. The bell-jar will have warmed up considerably, too, and usually the middle portion of it will be too hot to be touched by hand, while the upper and lower parts will be relatively cool.

In order that the heating of the vessel may be equalized as much as possible, a hollow cylinder made of sheet brass is placed around the cathode. This cylinder, which is about 9 cm in diameter and 18 cm long, rests on the base-plate and reaches to within about 2.5 cm of the anode *A*. A suitable opening opposite the slot in *C* allows the light to pass out to the spectrograph, while the observer can see the cathode by looking obliquely down between *A* and the top of the cylinder. With this arrangement the heating of the bell-jar is much more uniform and also considerably slower, so that an uninterrupted run of from 30 to 60 minutes becomes possible. Since glass expands considerably with a rise in temperature, while silica expands very little, it is obvious that the ground joint between silica and glass must be located where there is little or no change in temperature. Hence this joint is placed well below the base-plate and is so arranged that it can easily be surrounded by a water bath if necessary.

If the cathode does not come in contact with the tube *C*, and if at the same time is not separated from it by too large a space, the discharge takes place only from the upper surface of *b*. If the separation between *b* and *C* is too large, the discharge tends to start from *B*, or even from the wire below *B*, and passing in a thin stream between *b* and *C* produces intense local heating, which soon causes little arcs to form. If, on the other hand, *b* and *C* are in contact, then, when the discharge has passed long enough to heat *C* to redness, *C* itself begins to act as a cathode, and no result can be obtained. Great care is therefore necessary in adjusting the cathode and the tube *C* so there shall be no contact, even after *b* has become white hot.

The only fault the writer has to find with the present design is that after several runs the inner surface of *C* becomes so thickly coated with the metal of the cathode that a contact or short-circuit is unavoidable. The only remedy is to let in air, remove the bell-jar, and clean the tube, which requires only ten minutes of actual manipulation, but really involves two or three days' delay in the experimental work, as will be more fully explained in connection with the results for chromium.

With several days' continued use the upper surface of the cathode *b* becomes hollowed out into a low, nearly flat-bottomed

dish, with a rim from 0.5 to 1 mm thick, which is not affected. In order to be able to observe the discharge at the surface of the cathode from the side, it is of course necessary to remove this rim occasionally. In the present work this has been always done when the depth of the dish reached about 0.5 mm.

As is well known, the electric field is by no means uniform in the Crookes' dark-space, but has its greatest intensity at the surface of the cathode, decreasing as the distance from the cathode increases, until it becomes zero at the edge of the negative glow. The rate of decrease of the field with the distance from the cathode—or, better, the rate of increase with the distance measured from the negative glow—is a matter of considerable importance, and it might be expected that this rate would be different for different apparatus.

In the ordinary Lo Surdo type of tube, which has a diameter of from 1 to 2 mm, Takamine and Yoshida[1] have found that the field-strength varies approximately as the square of the distance from the negative glow, so that the components of the hydrogen lines become approximately parabolic curves, the vertices being in the negative glow. In the present apparatus the field-strength varies almost linearly with the distance, as can be seen from Plate X*a* which shows $H\gamma$, the *n*-components above, the *p*-components below. The components are nearly straight lines. This is a very fortunate circumstance, since one can tell at a glance, when examining lines of other elements, whether the displacement varies linearly with the field-strength or not, merely by seeing whether the components are straight lines or curves.

THE SPECTROGRAPH

This is a plane-grating, two-lens type, with the entire mounting constructed of wood in order that changes might easily be made should experience show that the original design could be improved.

The collimator lens is a Voigtländer portrait objective of 38 cm focus and 10 cm aperture; the camera lens is a Cooke astrographic of $F/4.5$ aperture and 45 cm focus. The grating is a 4-inch Rowland, with 14,438 lines per inch, having a rather bright third

[1] *Memoirs of the College of Science* (Kyoto Imperial University), **11**, No. 2, pp. 137–146, 1917.

order. It is mounted in such a manner that its normal always makes an angle of 70° with the axis of the camera lens. The collimator is fixed in the wooden housing of the spectrograph, in order that the discharge vessel and projection system may remain in a fixed position. The camera and grating can rotate through an angle of about 30° around a vertical axis through the center of the grating surface, thus bringing the different portions of the spectrum on the photographic plate.

As set up, the third order can be covered from about λ 3000 to λ 5000, and the second order from λ 4500 to λ 7500. By giving the collimator another fixed position, of course it would be possible to observe all of the first order or all of the fourth order instead of the regions chosen. An advantage of this form of mounting is that the dispersion at any given point of the photographic plate is always the same, no matter what region of the spectrum happens to fall there. Hence, the dispersion for all points of the plate having been determined, either by a photograph of a known spectrum or by calculation, a scale in angstrom units may be ruled which will fit all plates closely enough for purposes of identification.

The plates used measure about 6×10 cm, and the plate-holder is movable in a vertical direction, so as to allow several exposures on the same plate. In the third order about 400 A are covered with one exposure, the dispersion varying from about 5.2 A per mm at the violet end to about 3.0 A per mm at the red end. In the second order the dispersion is, of course, just two-thirds of that in the third order.

With short exposures pairs of lines having a separation of 0.1 A can as a rule be resolved. With the longer exposures, which are subject to disturbances from temperature-changes and vibration, the practical limit of resolution is about 0.15–0.20 A. The accuracy of wave-length determinations has been found to be easily of the order of 0.01 A.

An image of the slot in *C*, Fig. 1, is projected with a magnification of one-third on the slit by using two achromatic lenses, each of 20 cm focus and 5 cm diameter. Before falling on the projecting lenses the light passes through a double-image prism, so that two images, one immediately above the other, appear on the slit, the

vibrations in the upper image being vertical, or parallel to the slot, in the silica tube C.

An exposure usually requires from 20 to 30 minutes, but the vessel has then become so hot that from one to one and a half hours are required to cool it before another exposure can be made. (Of course in future work this waste of time will be eliminated by the construction of a vessel which can be cooled artificially, thus making it possible to operate without stopping.)

About one hundred photographs have been taken up to the present time, nearly one-half of which were made with hydrogen as residual gas. These include spectra of iron, chromium, nickel, vanadium, and calcium. Only chromium has been at all carefully studied, however, and the present paper will be limited to a preliminary report on those lines in the spectrum of chromium which show an electric decomposition.

RESULTS FOR CHROMIUM

The chromium spectrum has been photographed from λ 3670 to λ 5410. In the earlier spectrograms very little of interest from the viewpoint of a Stark effect was observed; the normal arc lines were always present, but of the sensitive lines only faint traces of the groups at λλ 4098, 4111, and 4129 could be seen. These showed, however, a suspicion of an electric effect, and hence for some days this region was photographed repeatedly in the hope of bringing out the effect more clearly. One spectrogram happened to show these groups with an intensity something like one hundred times as great as the others, without, however, any marked increase in intensity of the normal arc lines such as λ 4254. Viewed with a small direct-vision pocket spectroscope, the triplet near λ 5200 is always quite intense, but, when the spectrogram just mentioned, No. 55, was made, it was noted that three lines on the red side of the green triplet had an unusually high intensity. These "lines" were later found to be the groups at λλ 5275, 5298, and 5329, and their appearance was always used to indicate the character of the discharge.

Considerable difficulty was encountered in obtaining another spectrogram like No. 55. The record showed that in making this

exposure the current was 30 milliamperes; the length of the dark-space, 5 mm; the voltage, approximately 6000; the exposure time, 20 minutes; and the residual gas, air. These conditions were exactly reproduced time after time, and still only mere traces of the sensitive lines were obtained. In these trials the apparatus was as a rule taken down and thoroughly cleaned between exposures. Since this did not lead anywhere, a number of successive runs were made without taking down the apparatus and without letting in any more gas than just necessary to keep the experimental conditions correct. Then it was found that the sensitive condition as a rule developed quite suddenly after about half a dozen successive runs, and, once formed, seemed to persist in successive exposures, until it became necessary to clean the apparatus as a result of a contact between the cathode and the metal deposited in the tube *C*.

Just what this sensitive condition of the cathode is, the writer is not prepared to say; it can be destroyed by letting in air to a pressure of about 1 cm of mercury or more, it seems to form equally well in air and in commercial oxygen, and a small amount of hydrogen or water-vapor does not seem to prevent it; whether it will form in pure hydrogen is not definitely known, but indications are that it will. An exactly analogous condition has been found for iron and nickel, and it would not be surprising if it should turn out to be quite general.

No doubt it is analogous to the condition of the cathode, which is familiar to those who have had experience with the cathode sputtering of such metals as aluminium, chromium, or even nickel. The discharge may be run for a considerable time without any deposition to speak of taking place. Then suddenly the metal begins to come down, and in a short time a good coating is produced. In the present work it has been observed that the deposition on the inner walls of the tube *C* is much more rapid when the sensitive condition exists than without it; also that the upper surface of the cathode is hollowed out quite rapidly under the same circumstances. The appearance of the dark-space and of the cathode itself is also very different. Without the sensitive condition and with a current of, say, 30 milliamperes, the cathode, as seen from above, is of a purplish-red color, and the dark-space is really very dark, while,

with the sensitive condition, the cathode appears greenish-white and the dark-space is bright green and really more luminous, at least near the cathode, than the negative glow.

The lines given in the table were all recorded when the cathode was in the sensitive condition, and no doubt many more could have been recorded with longer exposures; those actually used ranged from a few minutes in the ultra-violet up to about 40 minutes in the region λ 4800–λ 5100. Most of the lines are not to be found in published tables for chromium, hence the question naturally arises: Are they really chromium lines or are they due to some impurity? The writer believes them to be due to chromium, for the following reasons:

1. The stronger lines, such as the groups at $\lambda\lambda$ 4098, 4111, 4129, 5275, 5298, and 5329, appear regularly as hazy lines in the chromium arc in air and as groups of fine lines in the vacuum arc and vacuum furnace.

2. The weaker lines recorded in the table always appear when the stronger lines are present with sufficient intensity, and do not appear when the latter are weak or absent.

3. None of these lines has been observed when any other metal was used as a cathode, although otherwise the conditions were the same.

This does not exclude the possibility that the lines may be due to some compound of chromium and nitrogen or oxygen; if this be so, then, since all come and go together, it follows that the strong lines λ 4129 and λ 5329, which appear regularly in the ordinary arc, as well as a number of other lines, such as λ 4475.50, which show with great intensity on these spectrograms, must also be due to the same compound. Now, the lines of which λ 4475.50 is an example all appear in the chromium arc as regular, though rather weak, lines; in the present investigation they appear unduly strengthened, but they show no electric decomposition.

The spectrum given by the arrangement used is somewhat different from the arc, spark, or furnace spectrum of chromium. A number of arc lines do not appear at all or appear with a relatively very small intensity; others, again, seem to appear with about the intensity to be expected; while still others are stronger than might

TABLE I

AFFECTED CHROMIUM LINES

λ	Int.	*p*-component Δλ	*p*-component Int.	*p*-component Rate of Displacement	*n*-component Δλ	*n*-component Int.	*n*-component Rate of Displacement
3719.12 ..	2	−0.76	....	=	....	....	....
3720.74 ..	5	+0.48	....	=	Complex	....	....
3722.83 ..	2	+1.07	....	+	....	....	....
3730.09 ..	2	−0.96	....	=	....	....	....
3731.85 ..	5	+0.40	....	—	Complex	....	....
3735.13 ..	2	+0.91	....	=	....	....	....
3744.73 ..	1	−0.29	....	=	....	....	....
3747.41 ..	5	+0.32	....	=	−0.13*	....	=
3750.14 ..	1	+0.63	....	=	....	....	....
3862.07 ..	2	−0.42	....	—	−0.42	....	—
3863.11 ..	1	+0.29	....	?	+0.29	....	?
3865.02 ..	4	+0.25	....	=	+0.18	....	=
3870.28 ..	4	+0.11	....	?	+0.04	....	?
3896.14 ..	1	−1.54	....	+	....	....	....
3926.20 ..	1	−0.16	....	=	−0.16	....	=
3929.87 ..	1	+0.06	....	?	+0.06	....	?
4011.96 ..	4	−0.55	....	=	Complex	....	....
4013.26 ..	4	−0.56	....	=	0.00	....	....
					−0.44	....	=
4013.99 ..	3	−0.54	....	=	Complex	....	....
4096.94 ..	1	....	....	....	−0.66	1	=
4097.41 ..	1	....	....	....	+0.21	2	=
4097.80 ..	3	−0.68	8	+	....	....	....
4098.11 ..	4	....	....	....	−0.18†	7	—
					+0.19	7	+
4098.31 ..	4	+0.84	8	=	+0.84	2	—
4103.40 ..	1	+0.06	1	?	+0.06	1	?
4110.75 ..	1	−0.78	10	=	....	....	....
4111.02 ..	5	....	....	....	+0.23	10	=
4111.49 ..	4	+0.59	10	=	−0.69	2	+
4111.82 ..	3	....	....	....	−0.21	2	=
					+0.11	2	+
4112.18 ..	1	....	....	....	+0.58	2	=
4117.08 ..	2	+0.13	....	?	+0.13	....	?
4128.34 ..	1	−0.47	6	+	−0.51	2	+
4129.36 ..	7	+0.39	8	+	−0.06	5	=
					+0.06	4	=
					+0.20	4	=
					+0.33	4	=
					+0.46	3	=
4130.13 ..	3	+0.26	6	=	+0.26	2	—
4130.79 ..	1	+0.40	....	?	+0.40	....	?
4136.25 ..	1	+0.08	....	?	+0.08	....	?
4214.89 ..	3	−0.13	....	=	Complex	....	....
4215.20 ..	3	−0.20	....	=	Complex	....	....
4216.14 ..	3	−0.13	....	=	Complex	....	....

* Other weak *n*-components are present.

† This *n*-component is peculiar, as the displacement is constant throughout the entire dark-space.

TABLE I (*Continued*)

λ	Int.	*p*-component Δλ	*p*-component Int.	*p*-component Rate of Displacement	*n*-component Δλ	*n*-component Int.	*n*-component Rate of Displacement
4210.45 ..	3	−0.15	....	=	Complex	....	....
4309.05 ..	2	+0.71	....	+	+0.71	....	+
4353.97 ..	3	+0.86	....	=	+0.86	....	=
4389.37 ..	2	−0.11	....	?	−0.11	....	?
4422.68 ..	3	Complex	....	....	Complex	....	....
4423.15 ..	2	Complex	....	....	Complex	....	....
4423.44 ..	1	Complex	....	....	Complex	....	....
4453.96 ..	2h	+0.79	....	=	−0.43‡	....	=
4472.03 ..	2	+0.65	....	=	Complex	....	....
4490.86 ..	1	−0.32	....	=	....	....	....
4494.92 ..	2	+0.54	....	=	−0.25	....	=
4649.59 ..	3	+0.18	....	−	§	....	....
4900.81 ..	1	+0.30	....	=	....	....	....
4948.25 ..	2	+0.34	....	=	Complex	....	....
4983.84 ..	1	+0.54	....	=	Complex‖	....	....
4984.97 ..	1	+0.30	....	=	Complex	....	....
5005.81 ..	2	−1.41 +0.53 +1.50	4 1 3	= = =	−0.43 +0.31	3 3	= = ...
5027.74 ..	2	−1.60 +0.06 +0.41	3 2 2	= = =	.... +0.21	.... 4	.... = ...
5028.37 ..	1	+0.92	3	=	....	....	....
5055.69 ..	2	−2.19 +0.77	1 2	= =	+0.03 +0.54	2 2	= =
5056.93 ..	1	+0.42	....	=	....	....	....
5275.44 ..	1	−0.12	2	=	−0.20	2	=
5275.92 ..	2	−0.24	6	+	−0.18	3	+
5276.16 ..	2	0.00	3	....	−0.03 +0.45	6 3	? =
5280.81 ..	1	+	....	....	....	....	....
5296.88 ..	1	−0.04	2	?	−0.03	4	?
5297.50 ..	3	−0.34	10	=	−0.27 +0.17	1 2	? ?
5298.18 ..	2	−0.28	1	?	−0.12	8	−
5298.43 ..	2	−0.06	3	?	−0.03	5	?
5298.92 ..	1	+0.49	8	+	....	....	....
5324.19 ..	1	+0.43	3	?	....	....	....
5328.52 ..	4	+0.16	10	+	0.00 +0.14	5 5	.... +
5329.34 ..	1	+0.11	4	+	−0.32	4	+
5329.94 ..	1	−0.16	2	?	....	....	....
5330.18 ..	1	....	....	....	+0.38	1	=

‡ There is also an *n*-component to the red.

§ This *p*-component is peculiar, as it seems to run without displacement from the negative glow about half-way through the dark-space, where it stops, but a line appears at this point immediately on the red side which continues without further displacement to the surface of the cathode. Breaks of a similar nature have been observed in the *n*-components of λλ 4097.80, 4111.82, and 4128.34, and the *p*-component of λ 4129.36. Some of these may show in the reproductions.

‖ There is also a *p*-component to the violet.

be expected, the group including λ 4475.50 and the stronger lines given in the table being merely those which are strengthened most. A careful study of the general spectrum given with the present arrangement is of course in order, but until a considerable number of elements have been studied in this way, so that conclusions of general validity can be drawn, it would not add materially to our knowledge, and hence a discussion of those lines of chromium which do not show any electric decomposition will not be undertaken in the present communication.

In the table the first column gives the wave-lengths as determined from the photographs, using as standards the unaffected chromium lines which appear with what is considered normal intensity; the second column gives the relative intensities on a scale of 1–10; the third column contains the displacements of the *p*-components reduced to a field-strength of 12,000 volts per centimeter; the fourth column, the intensity of the components in the stronger part of the field; while the fifth column indicates how the displacement varies with the field-strength: a plus sign (+) signifies that the displacement is proportional to a higher power of the field-strength than the first; an equality sign (=), that it is roughly as the first power; while a minus sign (−) indicates that it varies slower than the first power of the electric intensity. In a few cases where the displacement is very small, or the displaced component happens to overlap a neighboring nitrogen band-line, making measurement difficult, a question mark is placed in this column. The sixth, seventh, and eighth columns give similar data for the *n*-components.

Plate X*c* shows the groups at λλ 4098, 4111, and 4129; X*a*, the groups at λλ 5005, 5027, and 5055, while the groups λλ 5275, 5298, and 5329 are given in X*d*. The *n*-components are above, the *p*-components below, in all cases. A drawing made from the results of the measures of the groups reproduced in *c* and *d* appears in the lower half of the plate.

It will be noticed that to the violet of λ 3900 are seven lines showing *p*-components only—in other words, lines which are apparently plane-polarized in the electric field. They are all weak lines, and hence it is possible that, if the *n*-components belonging

to them are complex, they might fail to be registered on account of their very low intensity; and indeed the stronger lines in this region appear to have complex *n*-components. There are, however, several clear cases of lines with only one kind of component—for example, λλ 4097.80 and 4110.75—which have only *p*-components, and λλ 4096.94, 4097.41, 4098.11, 4111.02, 4111.82, and 4112.18, which have only *n*-components.

The lines λλ 4098.11 and 4111.02 are especially interesting; referring to Plate X we notice that the *p*-component of λ 4098.11 can be traced for only a very small fraction of the distance through the dark-space, while λ 4111.01 can be traced faintly nearly all the way across to the surface of the cathode, although it is clear that the intensity diminishes progressively as we pass down through the dark-space. This shows that in a very weak field both these lines are almost completely unpolarized; as the field increases, they gradually become plane-polarized, λ 4098.11 reaching complete polarization in a much weaker field than λ 4111.02.

A few of the lines belonging to the strong groups near λ 4100 and λ 5300 appear in the solar spectrum, and λ 5297.50 and λ 5329.34 seem well suited for an investigation of possible electric fields in the solar atmosphere.

SUMMARY

1. An apparatus suitable for investigating the Stark effect for metals having a relatively high melting-point is described.

2. With this apparatus the relation between field-intensity and distance from the negative glow is approximately linear.

3. The condition of the cathode is of vital importance.

4. A preliminary survey of the spectrum of chromium from λ 3670 to λ 5410 has been completed and a total of 74 affected lines recorded.

Mount Wilson Solar Observatory
May 1917

A DETERMINATION OF THE GALACTIC CONDENSATION FROM CERTAIN ZONES OF THE *ASTROGRAPHIC CATALOGUE*[1]

By FREDERICK H. SEARES

In a series of articles recently appearing in the *Monthly Notices*[2] Professor Turner has brought together much valuable material relating to the numbers of stars observed on the photographs for several zones of the *Astrographic Catalogue*. It was his purpose to show that important conclusions may be derived simply by counting the stars within each interval of brightness, even when the scale of luminosity is arbitrary. The result of such counts he has discussed mainly from the standpoint of systematic variations in the distribution of the stars with respect to the Galaxy; but they are perhaps equally important for a determination of the galactic condensation itself, which at present is affected with much uncertainty because of the widely divergent values found by different investigators.

Kapteyn's discussion in 1908[3] of all the material then available led to the conclusion that the concentration toward the Galaxy, which is an easily demonstrated characteristic of the brighter stars, becomes more and more pronounced as increasingly fainter objects are considered. According to his figures the galactic condensation,[4] which for stars of the ninth magnitude is 2.8, rises to 5.7 at the twelfth magnitude; at the fourteenth magnitude its value is 11.5 and at the sixteenth, 27.7. Counts to the limiting magnitude of photographs of 88 Selected Areas made at Mount Wilson with the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 135.

[2] "A Proposal for the Comparison of the Stellar Magnitude Scales of the Different Observatories Taking Part in the Astrographic Catalogue," *Monthly Notices*, **69**, 302, 1909; **72**, 464, 700, 1912; **75**, 57, 143, 465, 601, 1914–1915; **76**, 2, 140, 1915; **77**, 35, 1916. An article by Pocock in the same journal, **77**, 432, 1917, relates to the same question.

[3] *Groningen Publications*, No. 18.

[4] Ratio of number of stars per unit area at 5° galactic latitude to number at 80°.

60-inch reflector confirm substantially the relatively large value for the faint stars.[1]

The recent investigation by Chapman and Melotte,[2] on the other hand, shows nothing of the rapid increase in the ratio which is so noticeable a feature of Kapteyn's results. From the fifth to the ninth magnitude Chapman and Melotte are in agreement with Kapteyn, but at the sixteenth magnitude their value for the condensation is only 4.2, which is not greatly in excess of that for the brighter stars.

The data collected by Professor Turner contribute directly to the question of this unexplained divergence. In his earlier papers he failed to find any evidence of relatively large numbers of faint stars in the Galaxy,[3] but later the phenomenon was brought to light, and values of the coefficient of condensation were found which are of the same order as that shown by Kapteyn's tables.[4] The matter merits a closer examination, however, for, as will appear later, the Astrographic data afford a striking confirmation of Kapteyn's results as far as the limiting magnitude of the *Catalogue*, which is about 12.5 on the scale of *Groningen Publications*, No. 18.

The numerical data upon which the present discussion is based are given in the upper part of Table I as logarithms of the stellar density N_m—that is, of the total number of stars per square degree to the limiting magnitude m. The limits adopted for Table I are provisionally designated by A, B, C, and D. They are different for the different zones, and later are found to have the values given in the last line of the table. The declination of each zone appears at the head of the table; the hours of right ascension are at the left, while the galactic latitudes corresponding to the middle of each hour are alongside the logarithms of the densities. The quantities below the horizontal line will be explained later.

Comparison with the original tables in the *Monthly Notices* shows that the derivation of the quantities in Table I under the headings A, B, etc., has usually required the formation of the cumulative sums

[1] *Mt. Wilson Communications*, No. 43; *Proceedings of the National Academy of Sciences*, **3**, 217, 1917.

[2] *Memoirs of the Royal Astronomical Society*, **60**, 145, 1914.

[3] *Monthly Notices*, **75**, 607, 608, 1915.

[4] *Groningen Publications*, No. 18, p. 54, 1908.

of the numbers of stars for various intervals of brightness within each hour of right ascension. Further, it has been necessary to reduce the results to the square degree as the unit of area. For this purpose the area of an Astrographic plate has been assumed to be 4 degrees. The actual value is 169 144 times this amount,[1] but, as much work had been done before the precise value was discovered, the approximate area was used throughout. The conclusions concerning the galactic condensation are in nowise affected, and, moreover, the totals for the faintest limit in each zone, assuming them to refer to an area of exactly 1 square degree, are probably more nearly correct as they stand, because of the loss of faint stars near the corners of the plates.

The mass of material involved is considerable, the real amount being concealed under the summarized form in which it is presented; it represents the results of observations extending over several years at each of nine different observatories. The total area of the zones is nearly 6500 square degrees (that for the Vatican extends from 59° to 65°; the others are 2° wide), or approximately a sixth of the entire sky. The number of stars per zone of course varies with the declination, but ranges from about 30,000 to 80,000 or 90,000; the total is little short of 600,000, although some allowance must be made for the overlap of plates.

The following paragraphs include such details as are necessary for the derivation of the data in Table I from the tabulated counts in the *Monthly Notices.*

Vatican.—Mean counts for zones +64°, +62°, +60°; *M.N.*, **75**, 602, 1915. The original unit is one-tenth of a star per plate of 4 square degrees. Cumulative totals were formed for limiting scale-readings 30 and 0 (A and B in Table I). To reduce to densities (number of stars per square degree), 1.60 must be subtracted from the totals for each hour. Thus, for 0^h the totals are 441 and 9568; their logarithms are 2.64 and 3.98, and the corresponding densities, 1.04 and 2.38, respectively; these appear under the headings A and B in the Vatican section of Table I.

Oxford.—Zone +28°; *M.N.*, **75**, 466, 1915. The original tabular values for 0^h, 1^h, and 2^h are the numbers of stars on 7, 7, and 6 plates, with a repetition of this sequence for the remaining groups

[1] *Monthly Notices*, **72**, 466, 1912; **75**, 603, 1915.

TABLE I

LOGARITHM OF OBSERVED STELLAR DENSITY, LOG N_m

R.A.	Vatican +62°			Oxford +28°				Bordeaux +17°				Toulouse +9°				Algiers −1°		
	Lat.	A	B	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B
0h.....	− 1°	1.04	2.38	−34°	0.49	1.42	1.83	−44°	0.60	1.27	1.58	−52°	0.28	0.99	1.37	−62°	0.88	1.44
1.....	0	1.19	2.31	−33	0.50	1.34	1.78	−44	0.48	1.30	1.56	−51	0.27	0.95	1.32	−60	0.77	1.36
2.....	+ 2	1.02	2.13	−28	0.64	1.33	1.78	−38	0.44	1.22	1.49	−45	0.41	0.96	1.40	−53	0.84	1.47
3.....	+ 6	1.00	2.07	−21	0.67	1.39	1.85	−29	0.55	1.20	1.62	−36	0.33	0.96	1.48	−42	0.92	1.55
4.....	+10	0.89	1.95	−12	0.37	1.14	1.59	−19	0.47	1.13	1.42	−24	0.58	1.16	1.61	−30	1.14	1.67
5.....	+16	0.78	2.04	− 1	0.66	1.55	2.12	− 7	0.96	1.75	2.07	−12	0.81	1.42	1.88	−16	1.31	1.80
6.....	+22	0.74	2.02	+10	0.65	1.80	2.39	+ 6	1.06	1.83	2.20	+ 2	1.34	1.69	2.10	− 3	1.61	2.28
7.....	+29	0.68	1.92	+22	0.36	1.45	2.15	+18	1.19	1.76	2.08	+15	1.08	1.49	1.96	+10	1.67	2.33
8.....	+36	0.56	1.69	+35	0.64	1.62	1.94	+32	1.00	1.55	1.81	+28	0.60	1.19	1.70	+24	1.44	2.03
9.....	+43	0.52	1.59	+48	0.49	1.65	2.02	+44	0.98	1.48	1.69	+42	0.48	1.06	1.58	+36	1.00	1.59
10.....	+49	0.55	1.64	+61	0.44	1.39	1.78	+58	0.85	1.35	1.52	+54	0.55	0.96	1.35	+48	1.05	1.53
11.....	+53	0.52	1.53	+75	0.58	1.44	1.76	+71	0.78	1.31	1.59	+64	0.53	0.94	1.35	+56	1.13	1.60
12.....	+55	0.34	1.54	+87	0.49	1.48	1.87	+79	0.69	1.23	1.62	+70	0.24	0.75	1.22	+60	1.01	1.47
13.....	+54	0.38	1.54	+79	0.52	1.45	1.75	+75	0.59	1.27	1.58	+68	0.44	0.88	1.31	+58	1.01	1.49
14.....	+51	0.49	1.57	+66	0.45	1.26	1.69	+62	0.78	1.26	1.50	+58	0.50	0.92	1.30	+51	0.98	1.55
15.....	+46	0.45	1.63	+53	0.40	1.35	1.72	+50	0.69	1.30	1.70	+46	0.55	1.03	1.52	+40	0.99	1.52
16.....	+39	0.54	1.72	+40	0.47	1.42	1.82	+36	0.74	1.33	1.73	+34	0.88	1.25	1.63	+28	1.15	1.66
17.....	+32	0.55	1.69	+28	0.70	1.58	1.92	+24	1.15	1.71	1.97	+20	0.89	1.31	1.79	+16	1.02	1.55
18.....	+25	0.71	1.87	+15	1.16	1.98	2.41	+10	1.07	1.74	2.05	+ 6	1.30	1.68	2.08	+ 2	1.08	1.51
19.....	+18	0.68	2.08	+ 3	0.88	1.90	2.45	− 2	1.16	1.84	2.17	− 6	1.12	1.52	1.99	−12	1.24	1.85
20.....	+12	1.00	2.07	− 8	0.85	1.86	2.31	−14	1.03	1.74	2.09	−19	1.22	1.59	1.95	−24	1.40	1.99
21.....	+ 7	1.06	2.13	−18	0.58	1.68	2.24	−26	0.73	1.56	1.94	−31	0.75	1.20	1.66	−37	1.26	1.76
22.....	+ 4	0.96	2.07	−26	0.51	1.60	2.05	−35	0.49	1.41	1.77	−42	0.82	1.20	1.62	−48	0.98	1.48
23.....	+ 1	1.06	2.22	−32	0.39	1.34	1.82	−42	0.47	1.35	1.71	−49	0.66	1.11	1.46	−58	0.91	1.37
....	0	1.01	2.15	0	0.79	1.81	2.35	0	1.09	1.82	2.18	0	1.27	1.64	2.10	0	1.39	2.02
....	10	0.94	2.10	10	0.75	1.76	2.20	10	1.05	1.77	2.12	10	1.12	1.53	2.00	10	1.37	1.97
....	20	0.76	2.00	20	0.66	1.65	2.12	20	0.92	1.63	1.97	20	0.89	1.36	1.82	20	1.31	1.87
....	30	0.62	1.82	30	0.59	1.51	1.97	30	0.77	1.46	1.78	30	0.72	1.19	1.67	30	1.18	1.74
....	40	0.53	1.67	40	0.54	1.45	1.87	40	0.67	1.33	1.63	40	0.59	1.08	1.54	40	1.05	1.59
....	50	0.49	1.58	50	0.49	1.42	1.82	50	0.62	1.27	1.56	50	0.53	1.00	1.42	50	0.96	1.49
....	60	0.47	1.52	60	0.46	1.38	1.78	.....	.....	.....	.....	60	0.49	0.94	1.32	60	0.95	1.46
....	.....	.....	.....	70	0.45	1.37	1.76	.....	.....	.....	.....	70	0.46	0.89	1.27	.....	.....	.....
Lim. mag.....		9.4	12.0	.....	9.2	11.4	12.5	.....	9.7	11.2	12.0	.....	9.6	10.6	11.6	.....	10.5	11.7

TABLE I—*Continued*

R.A.	Hyderabad −17°				Perth −32°				Cape −41°					Cape −42°			Melbourne −65°		
	Lat.	A	B	C	Lat.	A	B	C	Lat.	A	B	C	D	Lat.	A	B	Lat.	A	B
0h	−77°	0.37	1.36	1.69	−85°	0.61	1.16	1.51	−76°	0.76	1.08	1.48	1.59	−75°	1.21	1.52	−52°	0.81	1.48
1	−75	0.18	1.22	1.61	−78	0.53	1.11	1.45	−73	0.72	1.02	1.41	1.59	−72	1.36	1.59	−51	0.83	1.54
2	−64	0.12	1.26	1.65	−66	0.43	1.10	1.52	−64	0.72	1.02	1.36	1.62	−63	1.27	1.65	−48	0.78	1.42
3	−51	0.17	1.26	1.62	−54	0.52	1.16	1.48	−53	0.83	1.03	1.35	1.61	−53	1.38	1.70	−44	0.89	1.51
4	−38	0.37	1.32	1.70	−40	0.45	1.13	1.58	−42	0.71	1.11	1.50	1.71	−42	1.56	1.85	−38	1.06	1.67
5	−24	0.79	1.66	1.91	−28	0.62	1.26	1.73	−31	0.94	1.47	1.85	2.06	−31	1.75	2.06	−32	1.08	1.76
6	−11	0.77	1.71	2.06	−16	0.86	1.50	1.94	−20	1.07	1.45	1.89	2.05	−20	1.81	2.18	−26	1.20	1.94
7	+ 1	0.61	1.89	2.43	− 4	1.07	1.76	2.16	− 9	1.30	1.78	2.15	2.37	−10	1.94	2.30	−20	1.15	2.05
8	+14	0.49	1.59	2.12	+ 6	1.12	1.88	2.37	0	1.50	1.84	2.24	2.30	0	2.06	2.39	−15	1.22	2.02
9	+25	0.30	1.34	1.84	+15	0.99	1.70	2.12	+ 8	1.34	1.70	2.14	2.33	+ 7	1.88	2.22	−10	1.34	2.17
10	+35	0.11	1.14	1.69	+22	0.94	1.56	1.97	+15	1.00	1.52	2.01	2.23	+14	1.89	2.27	− 6	1.46	2.35
11	+42	0.11	1.16	1.77	+28	0.85	1.52	1.89	+20	0.93	1.43	1.83	2.09	+19	1.55	2.07	− 4	0.93	2.09
12	+45	0.24	1.19	1.70	+30	0.75	1.42	1.81	+22	1.00	1.40	1.88	2.04	+21	1.43	1.90	− 2	1.54	2.40
13	+44	0.40	1.30	1.85	+29	0.76	1.46	1.88	+21	0.93	1.45	1.86	2.13	+20	1.60	2.06	− 3	1.43	2.33
14	+39	0.43	1.47	2.00	+25	0.84	1.58	2.04	+17	0.91	1.39	1.89	2.21	+16	1.60	2.12	− 5	1.48	2.08
15	+31	0.43	1.46	2.03	+18	0.87	1.56	2.01	+11	1.00	1.49	1.98	2.29	+10	1.76	2.30	− 8	1.47	2.41
16	+20	0.48	1.31	1.86	+10	1.02	1.64	2.19	+ 4	1.12	1.51	1.92	2.26	+ 3	1.86	2.33	−12	1.41	2.28
17	+ 8	0.66	1.53	2.17	0	1.32	2.00	2.49	− 6	1.13	1.80	2.25	2.53	− 6	2.07	2.61	−18	0.98	1.93
18	− 4	0.44	1.48	2.19	−12	1.10	1.83	2.36	−16	1.07	1.61	2.20	2.20	−16	2.18	2.71	−24	1.23	2.17
19	−17	0.36	1.42	2.12	−24	0.87	1.48	2.00	−26	0.82	1.31	1.93	1.98	−26	1.73	2.25	−30	0.90	1.72
20	−30	0.35	1.48	2.01	−36	0.63	1.33	1.86	−37	0.70	1.22	1.75	1.84	−37	1.54	1.98	−36	0.84	1.74
21	−43	0.43	1.46	1.82	−48	0.47	1.02	1.55	−49	0.52	1.06	1.43	1.72	−49	1.42	1.87	−42	0.88	1.74
22	−57	0.34	1.31	1.74	−62	0.33	1.00	1.42	−60	0.76	1.12	1.51	1.77	−60	1.37	1.64	−47	0.86	1.65
23	−69	0.21	1.30	1.67	−74	0.69	1.35	1.62	−70	0.84	1.17	1.45	1.66	−70	1.20	1.56	−50	0.60	1.31
....	0	0.60	1.69	2.24	0	1.10	1.82	2.32	0	1.22	1.69	2.10	2.42	0	1.97	2.39	0	1.45	2.28
....	10	0.57	1.61	2.16	10	1.04	1.74	2.22	10	1.12	1.59	2.10	2.33	10	1.87	2.32	10	1.35	2.20
....	20	0.48	1.52	2.01	20	0.90	1.57	2.01	20	0.98	1.45	1.93	2.15	20	1.74	2.15	20	1.17	2.04
....	30	0.39	1.44	1.89	30	0.74	1.38	1.83	30	0.87	1.30	1.77	1.97	30	1.64	2.02	30	1.02	1.82
....	40	0.32	1.38	1.80	40	0.60	1.24	1.69	40	0.81	1.17	1.62	1.82	40	1.53	1.90	40	0.89	1.64
....	50	0.27	1.32	1.73	50	0.55	1.15	1.58	50	0.77	1.10	1.49	1.72	50	1.42	1.77	50	0.82	1.52
....	60	0.24	1.27	1.67	60	0.54	1.11	1.53	60	0.74	1.06	1.44	1.65	60	1.32	1.67	60	0.77	1.46
....	70	0.21	1.24	1.62	70	0.54	1.10	1.51	70	0.73	1.04	1.43	1.62	70	1.27	1.59	.....	.....	.....
....	80	0.20	1.23	1.60	80	0.54	1.10	1.50	80	0.72	1.02	1.42	1.60	80	1.26	1.55	.....	.....	.....
Lim. mag.	.....	8.8	11.2	12.2	.....	9.6	11.0	12.1	.....	9.9	10.8	11.9	12.4	.....	11.5	12.4	.....	10.2	12.0

of three hours. Cumulative totals were formed for limiting scale-readings 30, 12, and 3 (A, B, and C in Table I), the last also including the values marked "Not measured." The logarithms of the totals were reduced to uniformity by adding log (7/6) = 0.07 to the results for 2^h, 5^h, etc. The constant to be subtracted to reduce the densities is log $(4\times7) = 1.45$. The results for the other Oxford zones included in the discussion in *M.N.*, **75**, 468, cannot be used here, inasmuch as the counts have not been given.

Bordeaux.—Zone $+17°$; *M.N.*, **72**, 465, 1912. The original numbers are for each half-magnitude interval of the Bordeaux scale. The total number of plates is 180, an average of $7.°5$ per hour; the actual number is probably 7 and 8 per hour, alternately. There is no statement as to whether the results for the various hours have been reduced to the mean area of $7.°5$, but, since the correction to the logarithms is only 0.03, alternately positive and negative, the uncertainty is of no consequence and the data can be used as they stand. Cumulative totals were formed for the limits 8.5, 10.0, and 12.0 (A, B, and C in Table I). The constant to be subtracted from the logarithms to reduce to densities is log $(4\times7.5) = 1.48$.

Toulouse.—Zone $+9°$; *M.N.*, **76**, 150, 1915. The original values are cumulative totals. The limits here adopted are 9.5, 11.0, and 12.3 of the Toulouse scale (A, B, and C in Table I). The totals for 13.3 are not used because of incompleteness. The total number of photographs is 180; the remarks concerning the distribution of the Bordeaux plates also apply here, and the constant is again 1.48.

Algiers.—Zone $-1°$; *M.N.*, **72**, 700, 1912. In the original tabulation the declination is erroneously given as $+1°$ (*M.N.*, **76**, 155). Cumulative totals were formed for the limits 10.0 and 12.0, inclusive, of the Algiers scale (A and B in Table I). The total number of plates is 180, in groups of 7 and 8, alternately, for the successive hours of right ascension. From the remark under (*b*) *M.N.*, **72**, 702, 1912, it is inferred that the tabulated counts on p. 700 have not been referred to the mean area of 7.5 square degrees per hour. To reduce to densities, 1.45 and 1.51 were therefore subtracted alternately from the logarithms of the cumulative totals for the successive hours.

Hyderabad.—Zone $-17°$; *M.N.*, **77**, 434, 1917. The original tabulation by Pocock gives cumulative totals. Those for N_{50}, N_{20}, and N_8 (A, B, and C in Table I) were adopted. The number of plates is alternately 8 and 7 for the successive hours; but the published totals are for a constant area of 40 square degrees. To reduce to densities, 1.60 has been subtracted from the logarithms.

Perth.—Zone $-32°$; *M.N.*, **75**, 144, 1915. Cumulative totals have been formed for the limits D, G, and M, inclusive, of the Perth scale (A, B, and C in Table I). The number of plates is alternately 6 and 7, but the tabular values refer to an area of 40 square degrees and the constant is 1.60.

Cape.—Zone $-41°$; *M.N.*, **75**, 59, 1914. Cumulative totals have been formed for the limits 149, 79, -3, and -5, inclusive, of the Cape scale (A, B, C, and D in Table I). The number of plates is 6 per hour, with a total area of 24 square degrees. To reduce to densities, 1.38 has been subtracted from the logarithms of the totals.

Cape.—Zone $-42°$; *M.N.*, **76**, 3, 1915. Cumulative totals for "Measured" and "All" stars are given in the original tabulation. Their logarithms, minus the constant 1.38, appear in Table I under the headings A and B. The remaining Cape zones discussed in *M.N.*, **76**, 151, cannot be used for the present purpose.

Melbourne.—Zone $-65°$; *M.N.*, **77**, 39, 1916. The original tabulation gives cumulative totals. Those adopted are for the limits 15 and "All" (A and B in Table I). The number of plates is 3 per hour, except for hours 1, 4, 7, etc., where the number is 4. The constants for reduction to densities are, respectively, 1.08 and 1.20.

The reliability of the data collected in Table I depends mainly upon two things: first, the constancy of the limiting magnitude for the photographs of any given zone; second, the consistency of the scale of luminosity in which the results for the brightness are expressed. Thus given designations of brightness—for example, scale-readings 30 and 0 for the Vatican zone, which are the limiting values used here—must represent the same degree of brightness wherever they occur in that zone; otherwise the only method which can now be applied to these data for the investigation of the distribution of the stars with respect to the Galaxy will not give

dependable results. We have not at present the information necessary for the reduction of the brightness, referred usually to a more or less arbitrary scale, to an absolute scale of magnitudes. Consequently it is vital that the adopted limits should always mean the same thing within any given zone, and that the enumeration of the stars to that limit should be complete.

It is too much to suppose that the conditions thus implied have been rigorously satisfied. Constancy of exposure-time for all the photographs of a zone is of course presupposed, although occasional variations are known to have occurred. But, even with equality in the exposures, absolute uniformity in the limiting magnitude is not to be expected; more or less systematic fluctuations with the seasons are almost certain to occur. The sky is more transparent at one time of year than another, and, even when transparent, the atmosphere, during the winter months, is less tranquil, with a resulting diffuseness of definition in the photographic images that leads to errors of measurement. Seasonal differences in temperature and humidity modify both the sensitiveness and the gradation of the photographic plate, which in turn affect both the limiting magnitude and the consistency of the scale.

It is impossible to specify in advance the extent to which these and other disturbing factors have influenced the data with which we are concerned. Two or three circumstances affecting the completeness of the counts may be mentioned, however: As a rule, photographs for the Oxford zones were rejected unless the number of stars showing two images was at least three times the number recorded by Argelander, but this practice was not followed in the Milky Way. Only three of the plates for the zone at $+28°$ fall below this limit, however.[1] Again, the scale of brightness for the Melbourne zone exhibits a peculiarity caused by the use of different machines for the measurement of the images.[2] A considerable inconsistency affects the results, but apparently only those for the brighter stars. The difficulty has largely been avoided by choosing scale-reading 15, which is beyond the point where the large irregularity begins, as the first limit for the data in Table I. Finally, in

[1] *Astrographic Catalogue*, Oxford Section, Vol. 4, p. ix, 1908.

[2] *Monthly Notices*, 77, 36, 1916.

the case of the Hyderabad zone, there were large differences in the exposure-times and considerable variations in the sensitiveness of the plates;[1] but apparently an attempt was made to adjust these two factors in such a way as to approximate a constant limiting magnitude. The results at least seem to be as consistent as those of some of the other zones.

We shall later find evidence of irregularities in all of the zones, but for the present we assume that these exercise upon each other a compensation which reduces to a minimum their influence upon the calculated mean distribution of the stars with respect to the Galaxy.

For the derivation of the mean distribution from the values in Table I we first eliminate as completely as possible the systematic irregularities, whether observational or real in origin, as well as the accidental errors that may have affected the counts. This is accomplished by plotting the numbers in each column with their corresponding galactic latitudes as abscissae. From the smooth curves thus obtained are derived the ordinates for equidistant intervals of galactic latitude which appear below the horizontal line in the lower portion of Table I.

The dispersion of the plotted points with respect to the curves, especially when their sequence in right ascension is borne in mind, reveals at once the more conspicuous irregularities. Thus the Vatican results are closely accordant; we find nearly the same value of the density corresponding to a given galactic latitude, irrespective of the right ascension in which the region is located. The same is true of the Cape zone at -41°, and to a less degree of the C group for Toulouse. On the other hand, the A group in the Toulouse zone and the results for Oxford, Bordeaux, and Algiers show large variations of density with right ascension, which obviously are not directly a function of the galactic latitude. The densities for Melbourne are relatively free from systematic irregularity, but their accidental deviations seem rather large.

Later, when it has become possible to refer the data to an absolute scale of magnitudes, we shall be able to distinguish in advance many of the irregularities that are observational in origin from those

[1] *Ibid.*, 77, 433, 1917.

which represent real variations in the stellar distribution; and we shall then be in a better position to derive results for the mean distribution. At present we can only assume that the counts for regions in right ascensions differing by twelve hours have been systematically influenced in opposite directions by equal amounts. This at least is a plausible supposition so far as any seasonal effect is

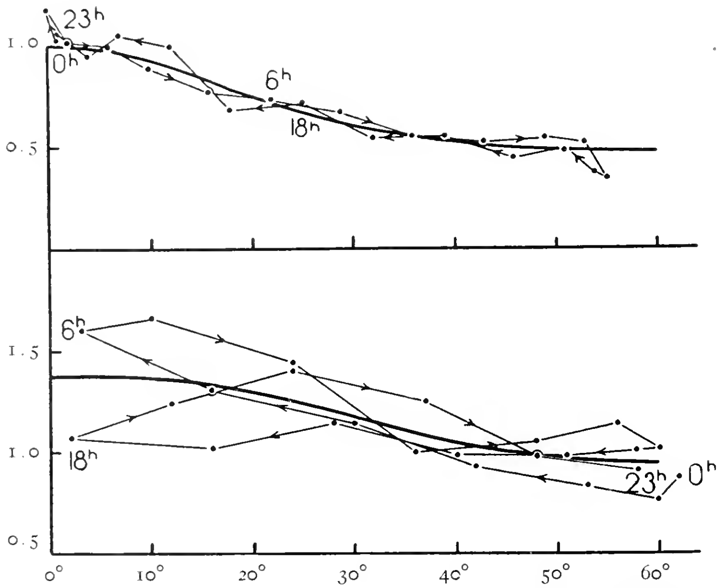

Fig. 1.—Stellar density and galactic latitude. Stars of Group A, Vatican zone, +62° (upper curve), and of Group A, Algiers zone, −1°. Note in lower curve the systematic deviations depending on right ascension.

concerned. It is on this basis that the curves have been drawn for those zones which show large variations with right ascension.

By way of illustration, two of the curves, those for the A groups of the Vatican and Algiers zones, are reproduced in Fig. 1. The broken line connects the points for adjacent hours of right ascension. For the Vatican curve there are no conspicuous irregularities, but for the Algiers results a different state of affairs exists. Note, for example, the systematic deviation in the plotted densities for 5, 6, and 7 hours from those for 17, 18, and 19 hours; yet the adopted

curve probably represents, for the zone, the mean variation of density with latitude with a tolerable degree of precision. The mean densities in the lower part of Table I should therefore be comparatively free from systematic irregularities which depend upon the right ascension.

The material has now to be combined into a final result. It may be remarked parenthetically that the mean data for each zone give directly a clear indication of the relatively high galactic condensation for the faint stars. In every case but one the difference between the values of log N_m standing in adjacent columns opposite 0° is larger than it is for the higher galactic latitudes in the last lines of Table I; in other words, with decreasing brightness the densities increase faster in the Galaxy than they do near the poles of the Milky Way.

The limits of brightness designated by A, B, C, and D were arbitrarily chosen, and thus far their precise relative values have not been determined; but for a combination of the data they must be referred to a numerical scale. We may use either that of Chapman and Melotte or that adopted by Kapteyn in *Groningen Publications*, No. 18. The latter has been chosen, since one of the purposes of the discussion is a detailed comparison of the results from the Astrographic material with the distribution tables of Kapteyn.

The transformation of the limits A, B, etc., into magnitudes depends upon the assumption that, for a given galactic latitude, equal mean densities correspond to the same limiting magnitude, whatever the position of the regions involved. Experience has proved that this supposition is reliable to a high degree, provided the areas compared are not too small; in the present case they are more than ample. With the mean densities (more precisely, the means of the logarithms of the densities) in the lower part of Table I as argument, we have only to interpolate from Kapteyn's tables,[1] for the proper latitude, the corresponding value of the magnitude. The results of this operation appear in the last line of Table I, which contains the mean of the limiting magnitudes thus derived from each of the densities standing immediately above.

[1] *Groningen Publications*, No. 18, p. 54, 1908.

In case the observed changes in density with latitude for any limit of brightness—for example, limit A of the Vatican data—were the same as that indicated by Kapteyn's tables, the result of the interpolation would naturally be the same for all the latitudes. The agreement of the individual values of the interpolated magnitudes is shown by Table II. This gives the deviation (individual *minus* mean), in tenths of a magnitude, from the mean limits at

TABLE II

DEVIATIONS FROM MEAN LIMITING MAGNITUDE

(Unit = 0.1 mag.)

Gal. Lat.	+62°		+28°			+17°			+9°			−1°	
	9.4	12.0	9.2	11.4	12.5	9.7	11.2	12.0	9.6	10.6	11.6	10.5	11.7
0°	0	−2	−2	−3	−3	−1	−1	−2	+4	+1	+1	−3	−2
10	+1	−1	−1	−2	−3	0	0	0	+3	+1	+1	−1	−1
20	0	+1	0	0	−1	+1	+1	+1	+1	+1	+1	+1	+2
30	0	+1	+1	0	0	0	+1	0	0	0	+1	+1	+2
40	−1	0	+1	+1	0	−1	0	−1	−1	0	+1	0	+1
50	−1	−1	+1	+1	+1	−1	0	−1	−2	−1	−1	−1	0
60	−1	−1	+1	+1	+1	....	....	....	−2	−2	−2	0	+1
70	....	....	+1	+2	+2	....	....	....	−3	−2	−2	....	....

Gal. Lat.	−17°			−32°			−41°				−42°		−65°	
	8.8	11.2	12.2	9.6	11.0	12.1	9.9	10.8	11.9	12.4	11.5	12.4	10.2	12.0
0°	−2	−4	−2	0	+1	0	0	0	0	−1	−1	−1	+1	0
10	−1	−3	−2	+1	+1	+1	0	0	0	0	−1	0	+1	+1
20	0	−1	0	+1	+2	+1	0	+1	+1	+1	+1	+1	+1	+2
30	+1	0	+1	0	+1	0	0	+1	+1	+1	+2	+2	+1	+1
40	0	+1	+1	−1	0	0	+1	0	0	0	+2	+2	−1	−1
50	0	+1	+1	−2	−1	−2	0	−1	−2	−1	0	0	−1	−2
60	0	+1	+1	−1	−1	−1	0	−1	−2	−1	−1	−1	−2	−2
70	−1	+1	+1	−1	−1	−1	+1	−1	−1	−1	−1	−1	....	....
80	−1	+1	+2	−1	0	0	+1	−1	−1	0	−1	−2	....	....

the head of the table which have now replaced the provisional designations A, B, etc. Though generally systematic, the residuals are remarkably small and show at a glance that the galactic condensation from the Astrographic data is substantially that found by Kapteyn; the excess of negative signs for 0° denotes a slightly brighter limit for this zone than the average, and consequently a small excess of density in the Galaxy for Kapteyn's results as compared with those from the Astrographic counts.

We are not concerned with these differences for the moment, however; the further combination of the results depends only on the mean limiting magnitude of each zone as a whole. Since these quantities are now referred to the same scale of brightness, the densities to which they correspond are directly comparable and may be united for a further elimination of the errors. Here again a graphical method is convenient. The entire series of mean densities in the lower part of Table I is plotted with the corresponding limiting magnitudes as abscissae. The smooth curve for the points of any given latitude represents the finally adopted variation of density with magnitude for that latitude. The ordinates of these curves have been collected in Table III, which for a limited

TABLE III

LOG N_m, NUMBER OF STARS PER SQUARE DEGREE BRIGHTER THAN m

Mag.	0°	10°	20°	30°	40°	50°	60°	70°	80°
8.5	0.50	0.45	0.32	0.24	0.18	0.14	0.12	0.11	0.10
9.0	0.75	0.70	0.56	0.46	0.39	0.34	0.31	0.31	0.30
9.5	1.03	0.96	0.82	0.70	0.61	0.55	0.52	0.51	0.50
10.0	1.29	1.20	1.06	0.94	0.83	0.77	0.73	0.72	0.72
10.5	1.52	1.44	1.30	1.16	1.04	0.98	0.93	0.92	0.92
11.0	1.75	1.67	1.53	1.38	1.25	1.19	1.14	1.12	1.11
11.5	1.99	1.90	1.75	1.60	1.47	1.40	1.35	1.32	1.29
12.0	2.22	2.13	1.97	1.82	1.68	1.60	1.55	1.51	1.46
12.5	2.45	2.36	2.19	2.03	1.90	1.81	1.74	1.69	1.63

range in magnitude is the equivalent of Kapteyn's table of densities. The differences between the two tables, in units of the second place of decimals, are given in Table IV. The slightly smaller densities for latitude 0° resulting from the Astrographic material are indicated by the persistent negative sign in the second column of Table IV.

A reference to the first series of curves, whose ordinates are summarized in the lower part of Table I, shows that most, if not all, of the systematic differences in Table IV are within the uncertainty of the reduction, and that the curves could be redrawn, without doing any violence to the original data, in such a manner as to remove almost entirely the systematic effect. We may therefore regard the results from the *Astrographic Catalogue* as affording

a practically complete confirmation of Kapteyn's values of the galactic condensation for the interval of four magnitudes between 8.5 and 12.5.

TABLE IV

COMPARISON WITH THE DENSITY TABLE OF KAPTEYN
(Table III *minus* Kapteyn, Unit 0.01)

Mag.	0°	10°	20°	30°	40°	50°	60°	70°	80°
8.5	−3	0	0	+2	+2	+1	+1	+1	0
9.0	−3	0	0	+1	0	−2	−2	−1	−2
9.5	−1	+1	+3	+2	0	−3	−3	−3	−1
10.0	0	+1	+3	+3	0	−2	−3	−2	−1
10.5	−1	+1	+4	+3	0	−2	−4	−2	+1
11.0	−3	0	+4	+3	−1	−2	−3	−2	−1
11.5	−3	−1	+4	+4	+1	−1	−1	0	−1
12.0	−4	−1	+3	+4	+1	0	0	+1	−2
12.5	−5	−1	+4	+5	+3	+2	+2	+1	−1

Since the value of the condensation is often expressed as the ratio of the number of stars at latitude 5° to the number at 80°, the results are also given in this form in Table V, alongside the corresponding ratios for Kapteyn and for Chapman and Melotte.

TABLE V

GALACTIC CONDENSATION. RATIO OF NUMBER OF STARS AT 5° TO NUMBER AT 80°

	Magnitude								
	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5
Astrographic Catalogue	2.4	2.7	3.2	3.5	3.8	4.2	4.7	5.4	6.2
Kapteyn	2.6	2.8	3.1	3.4	3.8	4.3	4.9	5.7	6.7
Chapman and Melotte	2.6	2.7	2.8	3.0	3.1	3.2	3.3	3.4	3.5

It is scarcely necessary to remark that the values here found for the condensation are entirely independent of the fact that we have used Kapteyn's tables for the determination of the limiting magnitudes; the result would have been the same had the different zones been reduced to a homogeneous system by the tables of Chapman and Melotte.

It is now of interest to examine the outstanding irregularities in the original data of Table I when compared with the mean densities of Table III. For this purpose the mean densities and

latitudes were first formed for 0^h and 1^h, 2^h and 3^h, etc., for the groups of each zone. Since the values in Table I refer to the middle of the hours of right ascension, the results thus derived correspond to the exact hours—1^h, 3^h, etc. The comparison with Table III gives the residuals O.−C. shown in Table VI, in units of the second

TABLE VI

DEVIATIONS OF OBSERVED DENSITIES FROM THE MEAN DISTRIBUTION
(Unit=0.01 in the logarithm)

R.A.	+62°		+28°			+17°			+9°			−1°	
	9.4	12.0	9.2	11.4	12.5	9.7	11.2	12.0	9.6	10.6	11.6	10.5	11.7
1^h	+15	+12	−4	−13	−18	−14	−3	−8	−30	−4	−9	−11	−3
3	+7	−8	+3	−20	−31	−25	−16	−20	−29	−12	−7	−11	+1
5	−3	−8	−30	−55	−54	−20	−28	−34	−19	−9	−8	−4	−6
7	+2	+9	−22	−15	+1	+10	+7	+4	+19	+8	+6	+15	+20
9	−3	−4	+9	+22	+10	+27	+15	+4	−16	−2	+6	+6	+12
11	+4	−2	+12	+13	+7	+22	+12	+3	−2	−3	−5	+12	+9
13	−13	−4	+12	+22	+19	+5	+6	+13	−21	−14	−10	+9	+4
15	−4	−2	+3	−1	−4	+12	+4	+3	−6	−3	−2	−2	+3
17	−6	−4	+5	−1	−11	+15	+5	+3	+10	+4	+3	−19	−21
19	−5	+3	+21	+8	+6	+2	−2	−7	+13	+3	0	−32	−36
21	+13	−3	−4	−5	−3	−3	+3	+5	+18	−12	+8	+17	−19
23	+5	−6	−13	−12	−12	−24	+1	+3	+12	+12	+7	−2	−4
Syst. dev.	+1	−1	−1	−6	−8	0	0	−3	−4	−1	−1	−2	0
Av. dev.	7	5	12	16	15	16	8	9	16	7	6	12	12

R.A.	−17°			−32°			−41°			−42°			−65°	
	8.8	11.2	12.2	9.6	11.0	12.1	9.9	10.8	11.9	12.4	11.5	12.4	10.2	12.0
1^h	+5	+10	+10	+3	+3	0	+7	+1	−2	−5	−3	−8	−3	−8
3	−10	+3	0	−8	−1	−9	+9	−4	−16	−10	−4	−4	−4	−17
5	+22	+3	−6	−17	−13	−14	−1	+8	−2	−3	+14	+5	+10	−3
7	+7	0	−2	−4	−4	−12	+9	+10	0	−3	+6	+2	+6	+8
9	−6	−16	−7	+6	+12	+7	+22	+14	+6	−4	+2	−6	+1	0
11	−20	−22	−7	+10	+8	−1	−8	0	−4	−1	−9	−4	−14	+4
13	+4	−7	+5	+2	+6	−2	−2	+1	−3	−3	−23	−10	+12	+10
15	+9	+6	+18	+2	+7	+4	−13	−8	−9	+1	−18	−5	+15	+7
17	+3	−28	−13	+13	+11	+12	−11	0	−8	+1	−1	+9	−3	+5
19	−20	−31	−6	+9	+10	+14	−6	+4	+16	−3	+22	+30	−1	+8
21	+6	+8	+9	−10	−6	−1	−11	0	−3	−5	+3	+9	−6	+5
23	+5	+9	+8	−4	+6	−4	+12	+10	−2	+4	−6	−8	−14	−14
Syst. dev.	0	−5	+1	0	+3	0	+1	+3	−2	−3	−1	+1	0	+1
Av. dev.	10	12	8	7	7	7	9	5	6	4	9	9	7	8

place of decimals. The means, both with and without regard to sign, are at the bottom, the former being the systematic deviations

of the groups, the latter the average deviation of the mean of a pair of densities, from the adopted values in Table III.

The systematic differences are unimportant and attract attention only in those cases (mainly in the Oxford zone at $+28°$) in which two or three abnormally low values were more or less ignored in drawing the original curves. Had the corresponding residuals likewise been disregarded in forming the means for Table VI, the systematic deviations would have been small throughout. The small average deviations for the Vatican zone at $+62°$ and the Cape zone at $-41°$ recall the remark of an earlier paragraph concerning the internal consistency of the results for these zones.

Examining the individual residuals, we find numerous instances of persistence of sign, which show clearly a more or less regular variation in the observed densities with right ascension. Moreover, the similarity in the sequence of signs for the different limiting magnitudes within a given zone suggests that the variations in the observed density have the same general character for stars of all degrees of brightness.

To exhibit more clearly the nature of the irregularities, the left-hand set of curves shown in Fig. 2 has been prepared. Their ordinates are the means, for each zone, of the deviations shown in Table VI for each hour of right ascension; the abscissae are right ascensions. To avoid confusion, the results for the two Cape zones at $-41°$ and $-42°$, which are much alike, have been combined. There are occasional similarities in the curves which are suggestive —for example, the first 12 hours for the zones at $+28°$, $+17°$, $+9°$, and $-1°$; but, so far as the present data alone are concerned, it is impossible to say whether the irregularities represent real variations in the distribution of the stars or nothing more than a seasonal influence upon the limiting magnitude and gradation of the photographic plates, superposed upon accidental errors of observation.

If now we examine the relation of the results thus derived to the spiral[1] found by Turner to characterize the distribution of the stars of these same zones of the *Astrographic Catalogue*, we must consider the deviations in Table VI from a different standpoint. The curve in question, whose equation is

$$\alpha+3.66\,\delta=247°,$$

[1] *Monthly Notices*, **76**, 7, 152, 1915.

represents the median line of a region of "obscuration" in which the ratio of the density of the faint stars to that of the bright stars is relatively small. Within any given zone—that is, for a constant declination—Turner has found the value of this ratio to vary with

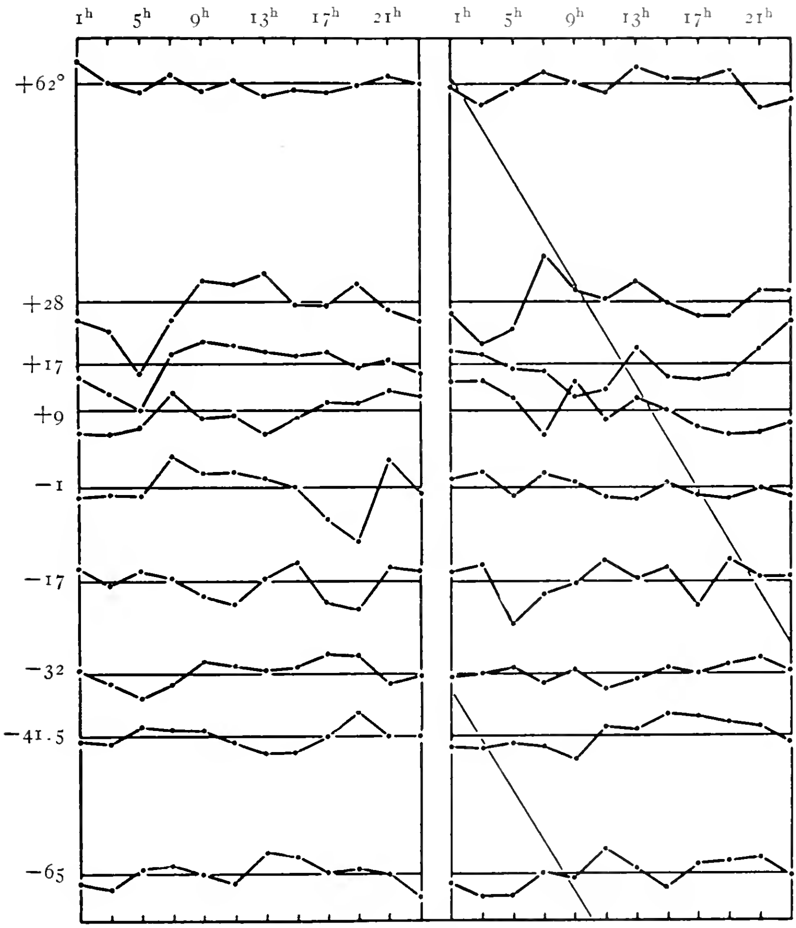

FIG. 2.—Irregularities in observed densities for various Astrographic zones. Vertical spacing of curves proportional to declination of zones given at the left; abscissae are right ascensions.

Left: Deviations from mean distribution shown in Table III. Ordinates are the means, for each hour, of the quantities in Table VI.

Right: Deviations in ratio of faint stars to bright. Ordinates are differences, last group *minus* first group, of quantities in Table VI. The median line of Turner's spiral of obscuration is indicated by the inclined lines.

right ascension between limits which depend upon the limiting magnitudes.

For a comparison with this result we may begin by forming the differences of the logarithms of the densities in Table I for the extremes of limiting magnitude there shown. It is simpler, however, to form the differences, for the extreme magnitudes in each zone, of the residuals given in Table VI. Since the deviations themselves have the form O.−C., the difference of any pair, for the limiting magnitudes F and B, may be written

$$\delta=(\log D_{\mathrm{F}}-\log N_{\mathrm{F}})-(\log D_{\mathrm{B}}-\log N_{\mathrm{B}}) \qquad (1)$$

or

$$\delta=\log\frac{D_{\mathrm{F}}\,N_{\mathrm{B}}}{D_{\mathrm{B}}\,N_{\mathrm{F}}} \qquad (2)$$

in which the values of D and N are the observed and the mean tabular densities, respectively (Tables I and III), for the two limits.

It is the variation of δ with right ascension in which we are interested; $D_{\mathrm{F}}/D_{\mathrm{B}}$ is the ratio of the density of the faint stars to that of the bright stars, and $N_{\mathrm{B}}/N_{\mathrm{F}}$ a factor, varying with galactic latitude but not longitude,. which neutralizes the influence of the relatively high galactic concentration of the faint stars toward the Galaxy. In other words, as is evident at once from Table VI, the difference in the deviations for a faint and a bright limit is the logarithm of the ratio of the number of faint to bright stars in excess of the normal ratio corresponding to the mean distribution shown in Table III.

The variation of δ with right ascension is shown by the right-hand set of curves in Fig. 2, whose ordinates were obtained by subtracting the first column for each zone in Table VI from the last, after correction for the systematic deviations at the bottom. The fluctuations are of the same order of magnitude as those shown in the other part of the figure; some similarities of form are vaguely suggested, although there is nothing that cannot be accounted for on the basis of inherent uncertainties in the data; but, whether the fluctuations are real or not, there is obviously no relation between them and Turner's spiral, which is represented by the inclined lines crossing the figure. An agreement with his results would require a well-defined minimum in each of the curves at the point of inter-

section with the spiral. Moreover, the amplitude of the variations shown in Fig. 2 is much less than that required, for his difference between the "unobscured" and "obscured" regions amounts to about 0.4 in the logarithm of the ratio of faint stars to bright.[1]

In seeking an explanation for this divergence, we may consider equation (2). Unless I have misunderstood his method of reduction, Professor Turner has treated the variations in the quantity log D_F/D_B as though they were independent of the galactic condensation; this indeed could be done, were the condensation not dependent upon brightness. Thus in *M.N.*, **76**, 3, 1915, Table I, zone -42°, he has given the deviations of log A/M (another notation for log D_F/D_B) from its mean value; and on the following page he discusses the sequence of signs for these deviations, and for a similar series for zone -41°, from the standpoint of the "obscured" region. Later, these same deviations, modified by a constant, reappear in Table IV, *M.N.*, **76**, 153, 1915, where they are subjected to the analysis which leads to the spiral whose equation is given above. Again, the deviations given in *M.N.*, **75**, 468, 1915, Table III, for the differences of the logarithms of the densities corresponding to two limiting magnitudes enter unchanged into Table VI, p. 156 of Vol. **76**.[2] In neither case has there been any allowance, apparently, for the galactic phenomenon in deriving the spiral; and, similarly, in the preliminary description of the obscured patches shown in *M.N.*, **75**, 480, 1915, the discussion seems always to be based upon the unmodified ratio of the number of faint stars to the number of bright stars. In fact, only traces of the relatively high condensation of the faint stars had been detected in the Astrographic counts when the first account of the obscured areas was prepared.[3]

The quantity whose variations have been studied by Professor Turner, and upon which he has based his conclusions, is, therefore, by equation (1),

$$\delta' = \log D_F - \log D_B = \delta + \log N_F - \log N_B \qquad (3)$$

[1] *Monthly Notices*, **75**, 480, 1915.

[2] Here, as in Table IV of the same article, the first difference is for the hour of right ascension which stands at the head of the column.

[3] *Monthly Notices*, **75**, 481, 1915.

which differs from the quantity δ whose values are shown graphically in Fig. 2 by the amount $\log N_F/N_B$. But this latter quantity is not independent of the galactic latitude. Its variation is a consequence of the relatively high galactic concentration of the faint stars. Were the condensation the same for stars of all magnitudes, its value would be a constant for any pair of limits F and B; but, as we approach the Galaxy, the faint stars increase in numbers more rapidly than the bright ones, with a resulting increase in $\log N_F/N_B$.

There is a wide range in the galactic latitudes of the Astrographic zones, and the changes in the factor $\log N_F/N_B$ accounts for a large part of the variation in $\log D_F/D_B$ studied by Professor Turner. When their influence is removed, there remain only the differences δ illustrated in Fig. 2. It is thus evident that high latitudes will contribute toward the formation of an "obscured" region; a reference to the diagram in *M.N.*, **75**, 480, 1915, shows that in general the centers of these regions are remote from the Galaxy. Again, the sequence of numbers in the last column of Table IV, *M.N.*, **76**, 153, 1915, which represent the amount of the obscuration phenomenon per magnitude, is approximately duplicated by the variations in the values of $\log N_F/N_B$ interpolated from Table III for the appropriate latitudes and for limiting magnitudes differing by $1^M.0$; for example, 12.5 and 11.5. Since the curves in Fig. 2 reveal no trace of the spiral of obscuration, it seems necessary to attribute the origin of that phenomenon to the influence of the relatively high concentration of the faint stars in the galactic regions.

The question under consideration is really that of the uniformity of the stellar distribution in galactic longitude. The evidence presented here suggests that the irregularities cannot be large, at least for the fainter stars, and this conclusion is strengthened by an examination of the densities derived from the photographs of the Selected Areas made with the 60-inch reflector.[1] The limiting magnitude for these plates is about 17.5; a noticeable variation of density with right ascension is present, but it is of the character to be expected from seasonal influences. The counts on the

[1] *Mt. Wilson Communications*, No. 42; *Proceedings National Academy of Sciences*, 3, 188, 1917.

Franklin-Adams plates,[1] limiting magnitude about 17, show even smaller systematic irregularities than those affecting the Mount Wilson photographs. Neither the Mervel Hill nor the Johannesburg series of plates reveals any variation in right ascension or galactic longitude that cannot reasonably be attributed to accidental errors.

SUMMARY

Counts collected by Professor Turner of nearly 600,000 stars on the photographs for 10 zones of the *Astrographic Catalogue*, extending from declination $+62^\circ$ to -65°, have been discussed from the standpoint of stellar distribution with respect to the Galaxy. The galactic condensation found is very nearly that derived by Kapteyn (Tables III, IV, and V). The deviations of the observed densities (Table VI) from the mean distribution show systematic variations in right ascension which are definite, but not larger than the uncertainties affecting the results. They are not in agreement with the spiral of obscuration derived by Turner from the same data; the origin of this latter phenomenon is to be found in the failure to allow for the high galactic concentration for the faint stars.

Mount Wilson Solar Observatory
June 1917

[1] *Memoirs of the Astronomical Society*, 60, 171, 1914.

THE ELIMINATION OF POLE-EFFECT FROM THE SOURCE FOR SECONDARY STANDARDS OF WAVE-LENGTH[1]

By CHARLES E. ST. JOHN and HAROLD D. BABCOCK

I. INTRODUCTION

Since the publication of the list of secondary standards of wave-length by the International Committee,[2] the attention of several spectroscopists has been devoted to the problem of adding to the number of such spectral lines. It has been sought to extend the range of wave-length covered and also to provide standards at more frequent intervals than was at first thought necessary. Determinations of wave-lengths by interference methods have been published by Burns, Meggers, and Merrill,[3] Meissner,[4] Burns and Meggers,[5] Eversheim,[6] Burns,[7] Werner,[8] and still other investigations are in progress. During the past year preliminary work in this field has been carried on at this observatory, and it is the purpose of the present paper to contribute some of the results which are now at hand.

A primary requisite which a spectral line must have in order to serve as a standard of wave-length is reproducibility. In practice this quantity depends upon two factors which determine the errors involved in locating the line in the spectrum, namely, the absolute constancy of the wave-length under the range of working conditions permitted, and the characteristics of the curve defining the intensity distribution in the line. But quite aside from the question of reproducibility, it is equally important from a practical point of view that the adopted value of a standard should be its

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 137.

[2] *Astrophysical Journal*, **32**, 215, 1910, and **39**, 93, 1914.

[3] *Scientific Papers of Bureau of Standards*, No. 274, 1916.

[4] *Annalen der Physik*, **51**, 95, 1916.

[5] *Scientific Papers of Bureau of Standards*, No. 251, 1915.

[6] *Annalen der Physik*, **45**, 454, 1914.

[7] *Journal de Physique* (5), **3**, 457, 1913.

[8] *Annalen der Physik*, **44**, 289, 1914.

fundamental wave-length, that is, the wave-length free from displacements due to disturbing conditions in the arc itself that are associated primarily with proximity to the poles but are extended in range by increased current strength. It is not the purpose by this term to distinguish between a possible effect due to changes in the relative intensities of the components of a complex vibrating system and that of perturbing influences acting upon the emitting centers, or to suggest an explanation of pole effect. When, as will appear later in the paper, arc conditions are such that sensitive lines yield their fundamental wave-lengths, these lines then behave like normally stable lines. The absolute measure of pole-effect in any source is obtained by comparing the wave-lengths given by it with the corresponding fundamental values as defined above.

The introduction of the pressure-effect into the values of the secondary standards, by operating the iron arc at atmospheric pressure, is undesirable from this point of view. But while for special purposes we do, indeed, employ the arc *in vacuo* as a source of reference lines, it can hardly be recommended as a general source of secondary standards on account of its greatly diminished brightness and of the inconvenience attending its use. Moreover, the revisions of the values of the pressure-effect which are now in progress will soon supply precise data for the elimination of this factor when necessary, and the correction, being dependent in amount only upon the barometer reading, can be applied with certainty. Pole-effect, on the other hand, does not lend itself readily to such simple treatment, chiefly on account of its rapid variation from point to point in the arc and its sensitiveness to change in arc conditions. These practical considerations permit a distinction to be drawn between pole-effect and pressure-effect in regard to their bearing upon the establishment of a system of standard wave-lengths.

The phenomena of pole-effect have been studied by Goos,[1] Gale and Whitney,[2] Royds,[3] Whitney,[4] and St. John and Babcock[5]

[1] *Astrophysical Journal*, **38**, 141, 1913. [2] *Ibid.*, **43**, 161, 1916.

[3] *Kodaikanal Bulletins*, Nos. 38 and 40, 1914.

[4] *Astrophysical Journal*, **44**, 65, 1916.

[5] *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

under a variety of conditions, but hitherto none of the suggested explanations has met with general acceptance. While it is possible that some of the relations between pole-effect, arc-length, and current-strength, which are now quantitatively measured, may throw light upon the nature of the effect, it is not our purpose to discuss here the theories that have been proposed to account for it.

The present paper consists in an examination for pole-effect of an iron arc operated according to the specifications of the International Committee, and an experimental study of the conditions under which the error introduced by polar influence may be avoided in another form of arc.

2. APPARATUS AND METHODS

The observations discussed here were made both with the plane-grating spectrograph and with interference apparatus. The former instrument is the same as that used in a previous investigation on the pole-effect.[1] When employed in conjunction with the totally reflecting prisms and rotating sector over the slit, it has proved a powerful means of measuring minute changes in wavelength. Interference methods are also especially applicable to problems of this character; certain distinct advantages are made available by this means—namely, the possibility of obtaining a great extent of spectrum upon each photograph under strictly uniform observing conditions, the integrating action over a specified portion of a source, and the comparative insensibility to instrumental displacements. The errors inherent in this method and those which affect results derived from gratings enter in different ways, and accordance in the values reached by the two methods, used independently, is a strong indication that both are free from systematic inaccuracy.

The interference apparatus which has recently been installed in this laboratory is in general similar to that described by Fabry and Buisson.[2] The auxiliary dispersion is provided by a Rowland concave grating of 635 cm radius of curvature, which receives parallel light from a collimating mirror of nearly the same focal length, thus giving a spectrum free from astigmatism. The grat-

[1] *Loc. cit.* [2] *Journal de Physique*, **9**, 929, 1910.

ing has approximately 87,500 lines, each 50 mm long, spaced 590 to the millimeter, and provides a scale of 5 A per millimeter in the first order. Flat plates 25 cm long are used in the camera, covering 1200 A in excellent focus in the first-order spectrum, which can be observed into the infra-red as far as λ 9000. The etalon consists of fused quartz plates 40 mm in diameter, held parallel and at a fixed distance by invar separators. Silver films cathodically deposited are used for the portion of the spectrum thus far studied. The etalon is contained in a small, double-walled wooden box, and this, with the mirrors which project the rings upon the slit, is protected by an outer wooden case. The whole system stands upon the large cement pier which supports the slit and grating of the spectrograph.

Fig. 1 shows in diagram the arrangement of the parts in the optical train. The mirror M_1, of 20 cm diameter and 60 cm focal length, projects a fourfold enlarged image of the source upon the diaphragm in front of the quartz plates E. The rings are projected upon the slit S by means of the plane mirror M_2 and the concave mirror M_3. The latter has a diameter of 10 cm and a focal length of 63.3 cm. Behind the slit a plane mirror M_5 can be moved on vertical ways into or out of the beam of light which ordinarily passes to the collimator M_4, the grating G, and the plate P_1. If it is desired to photograph the red cadmium rings when the camera

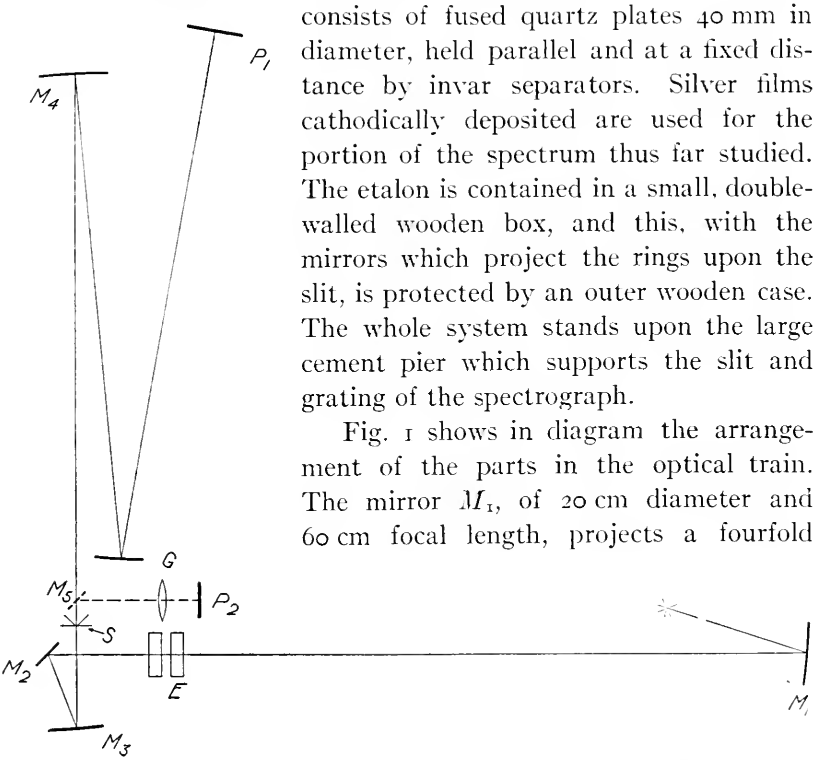

FIG. 1.—Optical system and arrangement of interference apparatus

is in a position which does not include that wave-length, the mirror M_5 is brought into position behind the slit so that the rings can be received in the auxiliary camera as shown. Important advantages are found in the precise achromatism of the system, in the fact that no essential part of the apparatus has to be touched between successive exposures, and in the comparative freedom from temperature disturbances. The gain in accuracy to be expected with the use of large-scale interference rings and increased auxiliary dispersion, pointed out by Pfund,[1] is realized in this apparatus, and it is doubtful whether further increase in the focal length of the projecting mirror M_3 is desirable. The focal length adopted permits the use of low magnification in the measuring microscope, with the consequent advantages in regard to the grain of the plate.

It is of interest to calculate the plate factor in angstroms per millimeter at any point in the interference system for a given set of conditions, in order to compare what one sees on photographs taken with this apparatus with the images of spectral lines formed by gratings or other instruments. It can easily be shown from the elementary theory of the interferometer that

$$\frac{pr^2}{2F^2}-E-(n-1)=0 \tag{1}$$

where

p = order of interference at center of ring system.

F = effective focal length of lens or mirror which projects the rings on the slit, as actually affected by any magnification in the spectrograph.

E = fractional order of interference.

n = number of ring, counting innermost ring as the first.

r = radius of n^{th} ring.

Now, if wave-length λ is measured from the center of the ring system in units of spectral range, we have at the n^{th} ring

$$\lambda=n-1+E.$$

But from equation (1) we find

$$r^2=\frac{n-1+E}{4K},$$

[1] *Astrophysical Journal*, 28, 211, 1908.

where K is written for $p/8F^2$, so that we may write

$$\lambda = 4Kr^2 .$$

Accordingly

$$\frac{d\lambda}{dr} = 8Kr ,$$

and in order to express this quantity in angstroms per millimeter it must be multiplied by the value of the spectral range, which is λ/p. This gives

$$\frac{d\lambda}{dr} = \frac{8K\lambda r}{p} = \frac{\lambda}{F^2} r .$$

For the first ring, with the aid of equation (1), this is readily transformed into

$$\frac{d\lambda}{dr} = \frac{\lambda}{F}\sqrt{\frac{2E}{p}} .$$

If we assume

$\lambda = 5000$ A, $F = 633$ mm, $p = 40{,}000$ (10 mm etalon), $E = 0.5$,

we obtain

$$\frac{d\lambda}{dr} = 0.040 \text{ A per mm.}$$

For comparison it may be added that for λ 5000 in the fourth order of a grating having 590 lines to the millimeter, used at 9.1 meters focus, the plate factor is 0.370 A per mm, i.e., the dispersion is about one-tenth of that calculated above for the interference method.

Attention may be called to the necessity for accurate focusing of the enlarged image of the source upon the diaphragm in front of the etalon when it is desired to select any specified portion of it for study. A large relative aperture of the projecting mirror makes it possible to focus with precision, especially when the following method is used: On looking through the arc toward the projecting mirror it is easy to locate the image of the diaphragm, since the light normally incident upon the first silver film is largely returned over the path which it traversed and greatly augments the illumination. The image of the diaphragm appears, then, to float in the arc and is so much brighter than its surroundings as to be easily

observed. It can also be seen when the arc is not burning, provided the general illumination in the room is sufficient, and its position can be very satisfactorily located by means of a hand magnifier. The electrodes of the arc, with the current cut off, can be observed simultaneously, and freedom from parallax insures a precise focus. In Plate XI are shown images of the arc, to which reference will be made later, with the diaphragm placed as in use. Owing to the fact that the arc and its image are somewhat off the axis of the projecting mirror, a small amount of astigmatism is introduced. By taking care to make the focus perfect for horizontal lines, we find that this feature becomes of advantage rather than otherwise, since it tends to smooth out the variations in intensity which exist in a horizontal section of the arc and to make the illumination of the diaphragm more uniform while leaving the extent of the arc-axis in use sharply defined.

The observations to be described were obtained for the most part with a diaphragm measuring 5×12 mm, but for special purposes others of circular shape are employed, having diameters of 5, 9, and 12 mm. The rectangular diaphragm is especially useful when it is necessary to economize light while using only a limited extent of the axis of the arc.

To test the auxiliary spectrograph for distortion of the field of view in a direction parallel to the spectral lines, a plate was placed over the slit, having a series of ten narrow openings arranged to simulate the relative positions of the sections of interference rings formed by the slit. With the etalon removed an exposure was made to the spectrum which reproduced each line broken into ten dots, with much the same appearance as though produced by interference. An examination of the distances separating these dots for lines in different portions of the negative and a comparison with the distances of the slots in the occulting plate showed constant magnification and freedom from distortion in all parts of the field.

The method of observation is essentially that of Fabry and Buisson. The etalon is usually adjusted with the aid of the green mercury line by observing the rings in a telescope while a very small exploring diaphragm is moved about in the beam

of light. When the adjustment for parallelism is complete with large aperture, the diaphragm is inserted very close to the first plate without touching any sensitive part of the instrument, and the protecting cover is put in place. The exposures are so distributed as to provide a check upon the constancy of the etalon.

The use of standard distance-gauges over the slit for the purpose of reducing linear diameters to angular measure is avoided, the observed diameters themselves furnishing all the necessary data as follows: The diameters are measured upon a machine provided with two precision screws whose axes are at right angles, from two to five rings being observed, according to the nature of the problem at hand. When four or five diameters are observed, the fractional order may be most accurately found from a least-squares solution of the observational equations, which are written in the form of equation (1) above. For differential observations upon the wave-length of the same line under slightly different conditions two rings are generally observed, and a plate-constant, $K\lambda$, is determined. It is the average of all the values obtained by dividing each approximate wave-length by the difference in the squares of the diameters corresponding to it. The value of K for any line is then taken by redivision, and the fractional order is quickly obtained from each diameter.

The integral order of interference and the correction for change of phase are readily found by well-known methods, a convenient arrangement of which is described by Meggers.[1]

The arc lamp is supported upon a small wooden platform, which is provided with a coarse screw for moving it at right angles to the line of sight. This permits easy correction of position without disturbing the setting for focus. Details of the mounting are shown in the lower half of Plate XI. The guiding screw referred to is marked A. The base of the lamp is heavy enough to give sufficient stability, and the moving parts are well defined in position. Screw B provides a vertical motion of the entire upper part of the lamp, read on a millimeter scale S_1. The upper electrode is moved up and down relatively to the lower by screw C, the scale S_2 serving to show

[1] *Scientific Papers of Bureau of Standards*, No. 251, 1915.

the arc-length. For centering the electrodes screw D provides a separate horizontal motion for the upper terminal. Current for the arc is supplied by direct-current generators at a maximum of 300 volts. The voltages commonly used are 110 and 220, with lamp resistance in series with the arc for control. An ammeter is kept in circuit and the current is read to 1/5 ampere.

3. OBSERVATIONS

a) *Provisional standard source.*—In order to examine a given source for the presence and amount of pole-effect contained in it, a reference source practically free from this disturbing influence is

TABLE I

POLE-EFFECT IN PFUND ARC WITH CARBON NEGATIVE POLE
IRON LINES OF GROUP *d*

Plate	Negative Pole *minus* Center		Plate	Positive Pole *minus* Center	
	Number Lines	Mean Shift		Number Lines	Mean Shift
		A			A
1	29	+0.003	1	38	+0.007*
2	31	.000	2	38	.008*
3	29	+ .001	3	37	.002
4	32	.000	4	36	.003
5	32	− .001	5	38	.003
6	36	.000	6	37	.002
7	36	− .001	7	37	+0.003
8	36	0.000			
Mean	....	0.000	Mean	....	+0.004

* No sector used.

essential. Our former conclusion, that the center of a 6-mm, 6-ampere Pfund arc is free from pole-effect,[1] is incorrect, as will be shown below, and a substitute was sought in the Pfund arc having the negative pole of carbon. Such an arc has been tested for pole-effect and found suitable for the purpose. The length of arc was maintained between 6 and 8 mm, and the current was 6 amperes on the average. The measurements summarized in Table I were made on plates obtained with the 30-foot plane-grating spectrograph, using sector and prisms over the slit. The

[1] *Astrophysical Journal*, 27, 298, 1906.

lines observed all belong to group *d*. For these lines no difference is found between wave-length at negative pole and at center, while a comparatively small displacement appears at the positive pole. A narrow central zone of this arc was accordingly adopted provisionally as a standard source, assumed to be free from pole-effect.

b) *Pole-effect in center of international arc.*—In accordance with the specifications of the International Committee on Wave-Lengths,[1] an arc 6 mm long between iron rods 7 mm in diameter, fed by a current of 6 amperes, under a pressure of 220 volts, having

TABLE II

CENTER INTERNATIONAL ARC *minus* CENTER PFUND CARBON-POLE ARC
IRON LINES OF GROUP *d*

Plate	Number Lines	Mean Shift	Plate	Number Lines	Mean Shift
		A			A
1	21	+0.004	7	34	+0.005
2	19	.006	8	33	.006
3	14	.010	9	34	.006
4	33	.007	10	32	+0.004
5	23	0.008			
6	38	+0.006	Mean. . .	. . . .	+0.006

Mean pole-effect in 6-mm, 6-ampere Pfund arc for the same lines is +0.010 A.

its positive pole above, will be referred to in this paper as the international arc. Light is taken from a central zone not exceeding 2 mm in width.

The slit of the 30-foot spectrograph was placed at right angles to the axis of this arc at its middle point, as recommended for finest definition in the specifications of the committee, and by means of a small totally reflecting prism simultaneous exposures were made upon this source and upon the corresponding part of the provisional standard source. A rotating sector equalized the effective intensity upon all parts of the slit, and special care was taken to see that each light-source completely illuminated the grating. As shown in Table II, ten exposures give for some thirty sensitive lines in the region λ 3840–λ 4200 a mean value of +0.006 A for the difference international *minus* Pfund carbon-pole arc. The

[1] *Trans. International Union for Co-operation in Solar Research*, **4**, 59, 1914.

conclusion indicated is that the international arc has pole-effect, even in its central section, at least 0.006 A in amount.

c) *Distribution of pole-effect in international arc.*—Spectra from near the negative pole of the international arc and from its center were compared by means of the 30-foot spectrograph, using the method of simultaneous exposures already referred to. The difference in brightness of the two parts of the arc was compensated by a rotating sector; control measurements made upon stable lines

TABLE III

INTERNATIONAL ARC, NEGATIVE POLE *minus* CENTER

Plate	Normal Polarity, Pos. Pole Above			
	Group *d*		Group *e*	
	Number Lines	Mean Shift	Number Lines	Mean Shift
		A		A
1	34	+0.021	3	−0.012
2	40	.019	3	.012
3	40	.018	3	.013
4	37	.014	3	.009
5	34	.018	3	.012
6	33	.019	3	.011
7	33	+0.017	3	−0.010
Means	......	+0.018	......	−0.011

Pole-effect in 6-mm, 6-ampere Pfund arc for the the same lines: Group *d*, +0.011 A; Group *e*, −0.009 A.

Plate	Inverted Polarity, Neg. Pole Above			
	Group *d*		Group *e*	
	Number Lines	Mean Shift	Number Lines	Mean Shift
		A		A
1	19	+0.018	1	−0.004
2	21	.013	2	.004
3	39	.010	3	.001
4	38	.008	2	.004
5	39	.013	3	.008
6	41	.010	3	.009
7	41	.011	3	.012
8	41	.013	3	.010
9	40	.013	3	.009
10	40	.010	3	.007
11	40	+0.013	3	−0.005
Means	......	+0.012	......	−0.007

showed freedom from instrumental displacement; the distinguishing feature of the pole-effect appeared in the opposite displacements given by lines of groups *d* and *e*. From 35 to 40 sensitive lines between λ 3800 and λ 4200 were observed upon each of seven plates, the mean values obtained being shown in the left-hand part of Table III. The polarity of the international arc was then reversed, making the upper pole negative, as it is in the Pfund arc, and the difference negative pole *minus* center was again observed in the same manner. The results of these measurements are collected in the right-hand part of Table III, which is self-explanatory. The mean pole-effect measured in the Pfund arc for the same list of lines is added at the bottom of the table for comparison. These

observations leave no doubt as to the existence of pole-effect in the international arc in somewhat greater amount than in the 6-mm, 6-ampere Pfund arc. The amount of material discussed in the case of the *d* lines is thought sufficient to give real meaning to the difference in the two mean values in Table III; the investigations bearing upon this point are still in progress.

The interference method was next employed for comparing integrated wave-lengths over the central half of the international arc with the corresponding values from the provisional standard source described above. A suitable diaphragm, placed in front of the etalon, selects the desired portion of the source, and since with proper precautions the instrument integrates all the light falling upon the diaphragm it is well suited to such a purpose. Under these conditions it was found that the central half of the international arc gave wave-lengths for 30 sensitive lines greater on the average by 0.005 A than the corresponding portion of the Pfund arc having the negative pole of carbon, although 59 stable lines showed no change in wave-length between the two sources. Furthermore, a comparison of the central half of the international arc with the central third of the same arc was made by a suitable exchange of diaphragms. In this case a list of 35 lines known to be free from pole-effect showed no variation, but 35 sensitive lines of group *d* showed a mean decrease of 0.003 A when the smaller portion of the arc was used. These facts are most readily explicable on the assumption that the central half of the international arc exhibits pole-effect, and a comparison of the two mean differences given above indicates that even the recommended central third of the international arc is somewhat affected by the polar influence. Although these data are not sufficient to afford a reliable measure of the amount of pole-effect in the central third of the arc, they tend to confirm the more exact quantitative measures presented above in section *b*.

Tests made toward the close of the investigation by the method described in the following section show that when the international arc is lengthened to 12 mm a central zone $1\frac{1}{4}$ mm wide still contains measurable pole-effect. The lines of a well-known list belonging to group *e* are, on the average, displaced

toward the violet 0.004 A from the positions which they have when free from pole-effect. The lines of group *d* which were tested show less displacement, but, on the whole, a tendency to shift toward the red.

d) *Development of the Pfund arc for standard purposes.*—The source described above as a provisional standard, the Pfund arc with carbon negative pole, is for many purposes all that could be desired. It is especially useful in the ultra-violet, where it yields most of the strong iron lines unreversed and very narrow. No difficulty is experienced from the so-called carbon bands when only the central region of the arc is used, and, as Pfund says, it yields iron lines "rivaling in homogeneity and sharpness the red and green lines of cadmium produced in a vacuum arc."[1] But its intensity, especially in the region of longer wave-lengths, is relatively feeble. The international arc, on the other hand, has been found characterized by a troublesome amount of pole-effect, which necessitates the introduction of uncertain correction terms before it can be safely employed as a reference source in many kinds of spectroscopic problems. A study has been made of the Pfund arc, accordingly, in order to develop its possibilities as a source suitable for a standard.

The interference method is so well suited to such a problem that extensive use has been made of it. For the most part a 5-mm etalon was used, this rather short length being chosen in order that the widest lines, which are in some cases most sensitive to pole-effect, might give measurable interference fringes. A diaphragm in front of the etalon selected a horizontal section through the center of the arc $1\frac{1}{4}$ mm wide.

The method usually employed for differential interference observations was abandoned, partly on account of the prolonged exposures sometimes necessary, and also because of the saving of labor and greater accuracy made possible by the plan adopted. The procedure followed consisted in determining the wave-lengths of the sensitive lines of groups *d* and *e* from each source in terms of selected stable lines occurring upon the same plate. Three sharp *a* lines in the green, which by comparison with the green mercury

[1] *Astrophysical Journal*, **27**, 298, 1906.

line have been found to be constant in wave-length under varying arc conditions, were adopted as reference standards.

It should be noted that the existence of pole-effect in small or moderate amounts is not necessarily accompanied by any suggestive appearance of the spectrum lines, such as unsymmetrical widening. The only certain test for the effect is the actual measurement of the wave-length. The differences shown in the accompanying tables cannot be attributed to unsymmetrical widening dependent upon increase of intensity, or to an "intensity equation" directly, since the individual lines exhibit a much greater range of intensity upon each plate than a given line shows from one plate to another, while the differences of wave-length are substantially the same for all lines of a given group. Burns, Meggers, and Merrill conclude[1] that "interferometer wave-lengths are comparatively unaffected by any intensity equation," and, so far as the present data bear upon this question, they seem to confirm that conclusion.

While the method employed eliminates systematic instrumental errors from these measurements, the objection may be raised that the observer tends to set the cross-wire of his microscope in a systematically different way upon lines belonging to different groups. All of the photographs at present under discussion, however, resemble one another very closely; they were taken with the same etalon, had approximately equivalent exposures, and were measured by the same observer, so that if such a source of error is present at all it is constant upon this series of plates. The differences that are shown in the behavior of these sensitive lines appear to arise, therefore, from actual changes in the wave-lengths of the lines.

Reference has already been made to a former paper in which the pole-effect was studied under certain conditions of arc-length and current-strength for this type of arc. In that investigation the center of such an arc was used for the working standards, but the tests made to ascertain whether it was itself entirely free from polar influence are now seen to be inadequate. In Table IV are given the results of a comparison of the center of the 6-mm,

[1] *Scientific Papers of Bureau of Standards*, No. 274, 1916.

6-ampere Pfund arc with the same region of the Pfund arc having the negative pole of carbon. A small amount of pole-effect is clearly indicated for lines belonging to both groups *d* and *e*.

A visual examination of the Pfund arc through a dense neutral screen shows a clearly defined structure consisting of the outer layer, which owes its luminosity principally to the band spectrum of iron oxide, a darker space within, which emits very little of the red end of the spectrum, and two brilliant streams of vapor in the axis of the dark space, of which the one proceeding from the negative pole is by far the brighter. Traces of

TABLE IV

POLE-EFFECT IN CENTER OF 6-MM, 6-AMPERE PFUND ARC

Group *d*			Group *e*		
Carbon Neg. Pole	Pfund	Shift	Carbon Neg. Pole	Pfund	Shift
5232.950....	.952	+0.002	5383.377...	.371	−0.006
5266.568....	.570	.002	5404.146...	.140	.006
5324.190....	.192	.002	5410.918...	.912	.006
5569.628....	.631	.003	5415.204...	.196	.008
5572.852....	.855	.003	5424.073...	.065	−0.008
5586.766....	.767	.001			
5615.653....	.656	.003			
5624.552....	.557	+0.005			
Mean..	....	+0.003	Mean..	....	−0.007

this structure are shown in Plate XI*a*. This photograph of the Pfund arc was made with light from the spectral region λλ 5800–7000, and the original contrasts have been considerably reduced by means of ammonium persulphate. Plate XI*b*, on the other hand, shows the international arc photographed in the same way with no reduction of the scale of contrast. The original exposure in this case was very much less than for the Pfund arc. If such photographs are taken with blue and violet light, the outer mantle is found much reduced, and the inner "dark" space, referred to above, is very much increased in relative intensity. Our observations upon pole-effect have led us to associate it with the bright streams of vapor which issue from the poles, and it was therefore natural

to choose the comparatively dark region between the adjacent ends of these vapor streams as a source presumably free from pole-effect. In the 6-mm, 6-ampere Pfund arc, however, as was shown above, this procedure is not permissible, but the observations next to be described show that simple modifications of the arc attain the desired result.

A normal Pfund arc 12 mm long was operated at varying current-strengths under a pressure of 220 volts. It was found that with currents of from 4 amperes to 7 amperes the arc was remarkably steady, and any desired portion of it could be selected

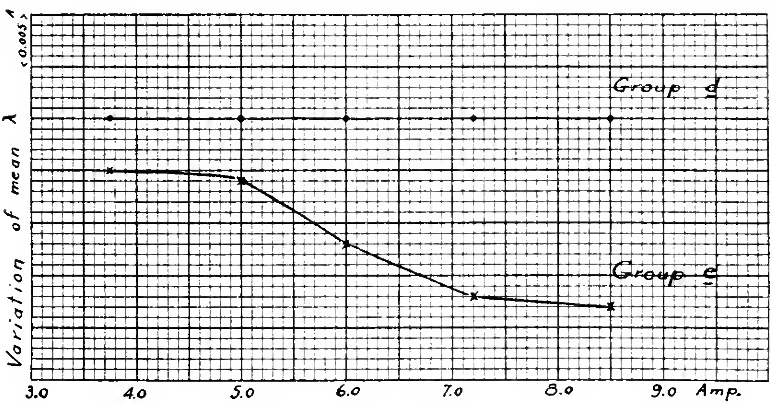

FIG. 2.—Relation between wave-length and current strength for sensitive lines

with considerable accuracy; exposures of an hour's duration could be made without restriking the arc. With further increase of current, intervals of unsteadiness appeared, but satisfactory plates could be taken up to 8.5 amperes. For each current-strength employed the wave-lengths of a series of *d* lines and of certain *e* lines were determined in terms of the same three standard *a* lines according to the method described above. The results are displayed in Table V and in the diagram, Fig. 2. The wave-lengths have had applied to them the necessary small corrections for phase-change.

A comparison of the means at the bottom of the columns shows striking uniformity for the *d* lines, but for currents above 5 amperes very definite displacements for the lines of group *e*. Special

attention should be called to the fact that, for currents of 5 amperes or less, even the sensitive *e* lines show practically constant values. The obvious interpretation of these results is that for the arc specified a central zone at least $1\frac{1}{4}$ mm wide is free from pole-effect when the current is less than 5 amperes, and wave-lengths derived from it have their fundamental values.

TABLE V

RELATION OF WAVE-LENGTH AT CENTER OF 12-MM PFUND ARC TO CURRENT-STRENGTH STANDARDS λλ5397.132, 5405.781, 5434.528

λ	Current in Amperes					Means
	3.75	5.00	6.00	7.20	8.50	
			Group *d*			
5232	.950	.950	.951	.950	.952	.951
5266	.568	.567	.565	.567	.563	.566
5324	.190	.188	.189	.188	.186	.188
5569	.629	.628	.629	.627	.627	.628
5572	.852	.851	.851	.852	.851	.851
5586	.767	.765	.765	.767	.766	.766
5602	.954	.955	.953	.954	.954	.954
5615	.652	.653	.655	.655	.655	.654
5624	.552	.556	.549	.552	.553	.552
5488	.457	.457	.457	.457	.457	.457
			Group *e*			
5383	.379	.377	.372	.370	.371	....
5404	.148	.146	.138	.135	.132	....
5410	.918	.920	.912	.910	.908	....
5415	.205	.205	.198	.188	.190	....
5424	.075	.073	.070	.062	.059	....
5407	.545	.544	.538	.533	.532	....

Confirmation of this conclusion is afforded in Table VI, in which the second column presents for group *d* means taken from all the measures in Table V, and for group *e* the means from the two lowest currents given in that table; the third column gives the results for the same lines in the carbon-pole Pfund arc, in terms of the same standards. The succeeding columns add corresponding data from Pfund arcs having negative poles of copper, brass, and chro-

mium steel, respectively. The arc-length was nearly 12 mm, and the current was approximately 5 amperes in each case. The variations in the means are undoubtedly within the errors of measurement, as it must be remembered that the lines concerned are among the most difficult of all iron lines to measure. Since the pole-effect is found to have its greatest value in the vicinity of the negative pole, it seems improbable that such accordance would be found if the values in the second column obtained with the iron pole were affected by it, as this hypothesis would require exactly equal amounts to appear in the other arcs considered.

TABLE VI

SENSITIVE LINES INDEPENDENT OF CONSTITUTION OF NEGATIVE POLE
CENTER OF 12-MM, 5-AMPERE PFUND ARC

λ	Negative Pole					Wtd. Mean
	Iron	Carbon	Copper	Brass	Cr. Steel	
			Group *d*			
5232	.951	.950	.950	.949	.950	.950
5266	.566	.568	.566	.566	.565	.566
5324	.188	.190	.190	.190	.189	.189
5569	.628	.628	.628	.630	.627	.628
5572	.851	.852	.852	.851	.852	.851
5586	.766	.766	.765	.764	.768	.765
5615	.654	.653	.653	.655	.652	.654
5624	.552	.552	.554	.552	.552	.552
5474	.144	.145	.145	1.45	.144	....
			Group *e*			
5383	.378	.377	.377	.378	.378	.378
5404	.147	.146	.145	.144	.143	.145
5410	.919	.918	.917	.917	.920	.918
5415	.205	.204	.206	.205	.206	.205
5424	.074	.073	.073	.074	.074	.074
5407	.545	.544	.544	.544	.544	....

It is found by examining monochromatic images of the Pfund arc, formed by various spectrum lines, that the distribution of luminosity in the iron vapor is practically independent of the constitution of the negative pole, as far as such observations have been

made. Under these conditions there is a slight increase in the brightness of the iron vapor close to the pole in question, but it is not prominent. The brilliant stream issuing from this electrode, on the other hand, shows in general the same characteristic distribution for the various substances which we have used for cathodes in our mixed arcs. In each case there is great increase of intensity close to the negative pole. Since it has been shown above that, in the case of the Pfund arc having the negative pole of carbon, the pole-effect disappears in the vicinity of the cathode, we have evidence here which suggests that it is not mere proximity to the negative pole which produces the effect, but that an essential condition is that the emitting centers originate with this stream. Our experience in obtaining photographs for measurement on pole-effect amply confirms this idea, for when the prism selecting the light at the pole is given a slight lateral displacement out of this narrow stream, the observed pole-effect is greatly diminished.

Furthermore, in a former investigation, already referred to, it was found that when the Pfund arc is operated *in vacuo* the pole-effect disappears and that the brilliant vapor stream which issues from the negative pole at atmospheric pressure is lacking, the arc appearing as a soft nebulous glow, brighter, indeed, near the negative pole, but without definite structure. In the light of these observations it is difficult to see how pole-effect can be directly dependent upon anything in the nature of electronic bombardment of the metal vapor by particles from the negative pole.

The gradual appearance of pole-effect in the center of an arc as its length is decreased may be seen in Table VII. The current was kept constant at 5 amperes, and arc-lengths of 12, 10, 8, and 6 mm were employed, the method of observation being the same as that referred to in the preceding paragraphs. The lines of group *d* are shown to remain practically constant in wave-length under these conditions, but the *e* lines, which offer a more sensitive test for the presence of pole-effect, show it definitely for lengths less than 8 mm. By a combination of the data presented in Tables V and VII it is found that, in order to be free from pole-effect in the center of the Pfund arc, the length must be at least 8 mm and the current not over 5 amperes. Under these condi-

tions a horizontal zone at least $1\frac{1}{4}$ mm wide may be used with safety, even for observations upon the most sensitive lines of group *e*. For *d* lines, on the other hand, the arc-length may be as short as 6 mm for a current of 5 amperes, or the current may be as much as 8.5 amperes for a length of 12 mm without introducing an appreciable amount of pole-effect.

TABLE VII

RELATION OF POLE-EFFECT TO ARC-LENGTH FOR 5-AMPERE PFUND ARC
CENTER OF ARC USED

λ	Arc-Length in Millimeters			
	6	8	10	12
Iron Lines of Group *d*				
5232	.951	.950	.949	.950
5266	.566	.569	.566	.567
5324	.189	.191	.189	.188
5569	.626	.628	.629	.628
5572	.850	.852	.852	.851
5586	.763	.765	.765	.765
5615	.652	.653	.654	.653
5624	.552	.552	.552	.556
5474	.144	.145	.144	.145
Iron Lines of Group *e*				
5383	.372	.375	.376	.377
5404	.141	.144	.144	.146
5410	.912	.916	.916	.920
5415	.197	.205	.204	.205
5424	.063	.071	.072	.073
5407	.537	.542	.542	.544

A set of conditions under which the Pfund arc may be used without introducing pole-effect having been developed, it is of interest to examine the wave-lengths of lines in the adopted list of international secondary standards known to exhibit pole-effect. In Table VIII, the first column gives the value published by the International Committee, the second shows the fundamental value obtained under these conditions, and the third contains the differences. The Mount Wilson values are not final, but represent our best knowledge of these lines at present. Definitive wave-lengths

covering a wide range of spectrum will appear in a later paper. The mean difference shown in Table VIII is practically coincident with the mean difference obtained above in Table II, although the observations involved in the two cases are entirely independent.

TABLE VIII

POLE-EFFECT IN INTERNATIONAL SECONDARY STANDARDS

I.A.	Mt. Wilson	I.A. *minus* Mt. Wilson	I.A.	Mt. Wilson	I.A. *minus* Mt. Wilson
4707.288....	.282	+0.006	5232.957...	.949	+0.008
4736.786....	.782	.004	5266.569...	.566	.003
4859.758....	.752	.006	5302.315...	.309	.006
4878.225....	.219	.006	5324.196...	.188	.008
4903.325....	.318	.007	5569.633...	.627	.006
4919.007....	.001	.006	5586.772...	.765	.007
4966.104....	.096	.008	5615.661...	.653	+0.008
5001.881....	.873	.008			
5192.363....	.356	+0.007	Mean...	....	+0.0065

In Table IX we compare our values with the determinations by Burns, Meggers, and Merrill[1] for lines of group *d*. The mean indicated makes it clear that pole-effect played a part in their

TABLE IX

POLE-EFFECT IN RECENT INTERFEROMETER OBSERVATIONS—GROUP *d*

Mean λ free from Pole-Effect	Burns, Meggers, Merrill	Difference	Mean λ free from Pole-Effect	Burns, Meggers, Merrill	Difference
5232.949....	.957	+0.008	5586.765...	.772	+0.007
5266.566....	.572	.006	5615.653...	.661	.008
5302.309....	.315	.006	5624.552...	.559	+0.007
5324.188....	.195	.007			
5569.627....	.633	.006	Mean...	....	+0.007
5572.851....	.859	+0.008			

investigation, and that its amount was substantially the same as that existing in the international secondary standards. This latter fact is sufficient to account for the failure of these observers to find evidence of it in their results, as they made no comparisons with a source free from this influence. They say in their paper:[2]

[1] *Scientific Papers of Bureau of Standards*, No. 274, 1916. [2] *Ibid.*

"Thus it seems probable that the measurement of additional lines on the plates from which the I.A. standards were obtained would lead to the same values of the wave-lengths as though the plates had been taken under normal conditions." This conclusion is confirmed by the observations presented above, but it must be sharply distinguished from the statement in the summary at the end of their paper to the effect that "interferometer observations under the conditions described were found to be reasonably free from the effects of pole shifts." No evidence was adduced by them in support of this statement, and in the light of the present data it is seen to be unwarranted.

e) Comparative sharpness of iron lines in different arcs.—An apparent superiority of the international arc over the Pfund type lies in its increased intensity for unreversed lines, but this increase of intensity is accompanied by a corresponding increase in width of the lines themselves, as quantitatively measured by Nutting,[1] so that the advantage gained in exposure time is offset by the disadvantage of lower reproducibility in the international arc. The relative sharpness of certain lines in the region λλ 5167–5455 in the international arc and in the Pfund arc when free from pole-effect is shown in Plate XII, in which *a* is taken from the center of the international arc and *b* from the Pfund arc. The etalon used was 15 mm long, i.e., the order of interference is about 55,000. The diaphragm admitted from the center of the arc an equatorial section $1\frac{1}{4}$ mm wide, while the specifications permit the use of 2 mm in the case of the international arc. If this larger amount had been used, the difference between the two arcs would have been still further increased. The prominent lines of group *a* are shown reversed in the international arc, and there is practically no material suitable for measurement upon the plate from which this illustration was made. Upon a plate from the Pfund arc free from pole-effect, however, we are able to measure all lines of sufficient intensity, and a still higher order of interference could be used. The number of lines which reverse in the international arc, but which are unreversed and suitable for measurement in the Pfund arc, is still greater in the region of shorter wave-lengths.

[1] *Bulletin of Bureau of Standards*, 2, 439, 1906.

4. THE IMPORTANCE OF FUNDAMENTAL WAVE-LENGTHS

The main purpose of the new determinations is to obtain extensive lists of wave-lengths that may be used for deriving the constants and testing the accuracy of series formulae, for comparing wave-lengths in terrestrial and cosmic sources, and for studying the changes in wave-length due to varying experimental conditions. For most purposes it is not a sufficient criterion that a wave-length be reproducible; it should also represent the fundamental vibration under standard conditions. Evidence that this end may be attained has been furnished by three lines of investigation: change of current strength, of arc length, and of constituents of negative pole. The basal assumption is that when no further variation in wave-length accompanies a change of any one of the three variables, the other two being held constant, the fundamental condition has been reached.

An entirely independent demonstration of pole-effect in the measures of Burns, Meggers, and Merrill and of the misleading interpretations that may follow from their use in statistical investigations is shown in Table X. The first column contains their

TABLE X

POLE-EFFECT SHOWN BY FICTITIOUS SUN-ARC DISPLACEMENTS

Burns, Meggers, Merrill	Mt. Wilson		B, M & M *minus* Mt. Wilson Arc	Mt. Wilson Sun *minus* B, M & M	Mt. Wilson Sun *minus* Mt. Wilson Arc
	Arc	Sun			
5192.362....	.356	.359	+0.006	−0.003	+0.003
5205.609....	.606	.613	.003	+ .004	.007
5232.957....	.950	.954	.007	− .003	.004
5281.807....	.798	.802	.009	− .005	.004
5324.195....	.189	.193	.006	− .002	.004
5379.579....	.576	.578	.003	− .001	.002
5393.183....	.177	.182	+0.006	−0.001	+0.005
Means..........................			+0.006	−0.002	+0.004

For 17 flame lines, group *a*, mean difference Burns, Meggers, and Merrill *minus* Mount Wilson is −0.0004 A.

wave-lengths; the second and third, our determinations in the center of a 6-mm Pfund arc and in the sun made from high-dispersion spectrograms taken with a plane-grating spectrograph. The wave-lengths of these sensitive lines were referred to well-

established standards belonging to group *a*. It is obvious from the last two columns of Table X that opposite conclusions may be drawn from such comparisons, depending upon whether the arc wave-lengths used are contaminated with pole-effect, as in the Burns, Meggers, and Merrill measures, or whether they represent the more nearly fundamental values of the Mount Wilson data. In early work at Mount Wilson upon sun-arc comparisons the iron lines in the solar spectrum of groups *c* and *d* were displaced to the violet, and the lines of group *e*, abnormally to the red. It is now recognized that this arose from residual pole-effect in the center of the arc. Recent determinations with the 12-mm arc yield results of the same sign and magnitude for lines of the sensitive groups as for stable lines on the same plates as shown in Table XI. The

TABLE XI

SUN-ARC DISPLACEMENTS FOR UNSTABLE LINES BECOME NORMAL WHEN POLE-EFFECT IS ELIMINATED

λ	Fe Lines, Group *d*		λ	Fe Lines, Group *e*	
	Old, Pole-Effect	New, No Pole-Effect		Old, Pole-Effect	New, No Pole-Effect
5393	−0.006	+0.001	5364	+0.021	+0.011
5476	.004	.008	5367	.020	.008
5569	.002	.005	5364	.012	.008
5572	.002	.007	5383	.018	.006
5586	.006	.006	5404	.015	.006
5615	.000	.006	5410	.016	.012
5624	.008	.006	5415	.026	.005
5638	−0.008	+0.016	5424	+0.028	+0.008
Means	−0.004	+0.007	Means	+0.019	+0.008
9 stable lines, group *a*				+0.008	+0.008

disappearance of the abnormal behavior of such lines furnishes confirmatory evidence that pole-effect has been eliminated from the iron arc and indicates that the solar lines are produced under conditions in which pole-effect plays no part.

Not only is there evidence of pole-effect in the international iron wave-lengths, but it appears also in the determinations for other elements. In any given spectral region a "Standard R−I," Rowland *minus* international, for lines whose wave-lengths in sun

and arc are equal, may be obtained from stable iron lines by taking account of their sun-arc displacements. In Table XII are shown some results obtained by applying this criterion. The apparent displacement of the solar lines to the violet for lines showing positive pole-effect, indicated by negative values in the last column, and the abnormally large displacement to the red for lines with negative pole-effect are so similar to the results found for iron before the elimination of this disturbing influence that the abnormal sun-arc displacement shown in Table XII must be referred to errors in the arc wave-lengths rather than to conditions in the sun.

TABLE XII

POLE-EFFECT IN THE PUBLISHED WAVE-LENGTHS OF OTHER ELEMENTS

	Rowland λ	Pole-Effect	R−F	Standard R−I	Sun-Arc
Manganese... Fuchs*......	4783.613	+0.012	0.181	0.187	−0.006
	4823.694	+0.011	0.175		−0.012
			R−Holtz		
Calcium..... Holtz†......	6102.937	+0.017‡	0.201	0.204	−0.003
	6122.434		0.202		−0.002
	6162.390		0.194		−0.010
	3624.410	−0.037‡	0.152	0.136	+0.016
	3647.707		0.152		+0.016

* *Zeitschrift für Wissenschaftliche Photographie*, 14, 263, 1914.

† *Ibid.*, 12, 101, 1913.

‡ Whitney, *Astrophysical Journal* 44, 65, 1916.

A similar conclusion relative to the errors in the calcium wave-lengths of Holtz is reached when they are compared with the measurements *in vacuo* made by Crew and McCauley,[1] as in Table XIII. In this case the difference, Holtz *minus* Crew and McCauley, should be the pressure-displacement per atmosphere. Here again abnormal values are found bearing a relation to pole-effect similar to that observed for iron before its elimination from the iron data. As pole-effect practically disappears for iron *in vacuo*, it may be assumed with a high degree of probability that the

[1] *Astrophysical Journal*, 39, 29, 1914.

abnormal pressure-shifts indicated are due to errors in the wave-lengths of Holtz at atmospheric pressure rather than in those of Crew and McCauley *in vacuo*.

Data are not at hand for testing other published determinations. The tests here applied are simply illustrations of statistical comparisons already undertaken that have misled the investigator and of the uses to which such data may properly be applied. If the great labor expended upon the redetermination of wave-lengths is to be justified by results, due consideration should be given to the

TABLE XIII

FICTITIOUS PRESSURE-DISPLACEMENTS FOR CALCIUM DUE TO POLE-EFFECT

Series	Holtz	Crew and McCauley	Holtz *minus* C. and McC.	Pos. Pole *minus* Center Whitney
t..........	4092.690	.649	+0.041	
t..........	4094.983	.944	+0.039	+0.055
t..........	4098.575	.552	+0.023	
			+0.034	
T_2.........	6102.736	.716	+0.020	
T_2.........	6122.232	.216	+0.016	+0.017
T_2.........	6162.196	.177	+0.019	
			+0.018	
T_1.........	3624.106	.107	−0.001	
T_1.........	3630.739	.749	−0.010	−0.037
T_1.........	3630.958	.973	−0.015	
T_1.........	3644.757	.760	−0.003	
			−0.007	

elimination of pole-effect, a necessary preliminary to wave-length determinations. It is not too much to hope that the new data will be valuable for a generation at least, as the Rowland measures have been in the past, but, as the matter now stands, it is a question whether any of the published determinations of wave-lengths or of pressure-displacements for the more sensitive lines are sufficiently free from pole-effect to comply with the more and more exacting demands that will be made upon such fundamental data.

5. CONCLUSIONS

From the data presented above, the superiority of the Pfund arc 12 mm long, carrying a current of 5 amperes, as a source of standard spectrum lines appears to be established. Whether viewed from the standpoint of higher reproducibility, owing to increased sharpness of lines, or from that of attaining the fundamental wave-length, owing to the freedom from pole-effect, the advantage lies with this type of arc in preference to the arc specified by the International Committee. It is true that from λ 5650 to the red the decrease in brightness in the Pfund arc is sufficient to require rather long exposures when this source is used with high-dispersion apparatus; but from that point in the spectrum to the ultra-violet no difficulty is experienced. The international arc, on the other hand, is lacking in stable lines in the region λλ 5600–6000. To the red of λ 6027 there is a sufficient number of lines belonging to group *b*, which, being free from pole-effect, may be used as standards when observed in any form of arc.

The uniformity in the wave-lengths of the stable lines under varying conditions of arc is especially to be noted, along with the fact that even the most sensitive lines are unaffected in position by change of constitution of the negative electrode, while, under the set of working conditions developed, their wave-lengths exhibit remarkable constancy.

Attention should be directed also to the advantage of using the Pfund arc, under conditions similar to those described in this paper, as a source for working standards of intensity. The same conditions that favor constancy of wave-length also make for uniformity in the system of relative intensities, and the comparatively large range of working conditions permitted with this form of arc makes it much better suited to the purpose than an arc in which the intensity-gradient of a given line from center to pole is much steeper, as it is in the international arc. Obviously, if the intensity of a sensitive line is 5 or 10 times as great at the negative pole as it is at the center of an arc, it is highly important to reproduce with the greatest care the conditions under which the line is observed, if it is to be used as a reference standard of intensity. A slight change in the position of a slit which is at right angles to

the axis of the arc introduces an entirely new system of relative intensities in the case of a short arc, and a comparatively small error in focusing the image upon the slit will produce a similar result. Ample enlargement of the image, an achromatic projecting system accurately focused on the slit at a definite position and azimuth, preferably 90° to the axis of the arc at its midpoint, and a source in which the intensity-gradients from center to pole of even the sensitive lines are low, are some of the conditions which should be observed if the systems of relative intensity for spectrum lines in terms of the "arc" intensities used by different investigators are to have definite meaning.

It is a pleasure to acknowledge our indebtedness to the careful measurements and reductions carried out for us by Miss M. O. Burns, of the computing staff.

6. SUMMARY

1. The interference apparatus now in use at this observatory is described, and the methods of observation and reduction are explained.

2. By means of this instrument and a powerful plane-grating spectrograph quantitative data have been collected relating to the relative merits of three forms of iron arc when used as sources for standards of wave-length and of intensity.

3. The international arc has been shown to exhibit large pole-effects, not only at the negative pole, but in the central one-half, central one-third, and even in the central plane section.

4. The displacements at the negative pole as compared with the center are greater in this arc when the negative pole is below than when it is above.

5. In the Pfund arc with a carbon negative pole the wave-lengths of sensitive lines are the same at the negative pole and at the center.

6. The ordinary Pfund arc 6 mm long, carrying 6 amperes, has measurable pole-effect in its central region.

7. The Pfund arc operated with 5 amperes or less, at a length of 12 mm, is free from pole-effect in a central zone at least $1\frac{1}{4}$ mm wide.

8. The poorest lines in the iron spectrum, those of group *c*, when free from pole-effect, can be measured with high precision.

9. When pole-effect is eliminated, the wave-lengths of iron lines are found to be independent of the nature of the negative pole, whether this be of iron, carbon, copper, brass, or chromium steel.

10. The adopted wave-lengths of international standards belonging to groups *c* and *d* are shown to be influenced by pole-effect.

11. This influence enters to about the same extent into the values of wave-length recently published by other observers.

12. In the green part of the iron spectrum the pole-effect in Nos. 10 and 11 amounts to +0.006 A for lines of group *d*.

13. The sharpness of both stable and sensitive lines, when observed in a source free from pole-effect, is shown to be much greater than it is in the international arc.

14. The importance of eliminating pole-effect is shown by the misleading results obtained by using published data contaminated by this influence.

15. The abnormal sun-arc displacements formerly obtained for lines of groups *d* and *e* become of the same sign and order as for stable lines when pole-effect is eliminated.

16. Attention is directed to the importance of certain precautions to be observed when iron-arc lines are to be used for comparisons of intensity.

Mount Wilson Solar Observatory
June 1917

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE[1]

By PRENTICE REEVES

"A determination of the least quantity of radiant energy capable of exciting the sensation of light could and probably by preference should be made in the laboratory by a direct method." This statement by H. E. Ives[2] led the writer to start on the study of this subject. Ives computed the value for the *minimum visibile*[3] from data at hand, and since then H. N. Russell[4] has supplemented the work by introducing data inaccessible to Ives. The data used by Ives on the mechanical equivalent of light and the stellar magnitude of the standard candle were well established, and the data offered by Russell were on the pupilary area of an eye adapted to the dark, and on the stellar magnitude of the faintest visible objects. These values used by Russell are respectively twice and one-tenth those used by Ives, so that the corrected value of the *minimum visibile* is just one-fifth that of Ives. By using a direct method in the laboratory, where all physical stimuli were under the control of the experimenter, the uncertainties of stellar observations were largely overcome. Another advantage of this paper is the use of measurements taken of the observers' pupils. The luminosity-curves of the observers show each observer to have a normal area-curve.

The apparatus used was a modification of the visual sensitometer used by P. G. Nutting,[5] and a light-proof lamp-house with decimal neutral filters. The diagram, Fig. 1, shows the scheme used to control the intensity of the light in the visual sensitometer. The

[1] *Communication No. 51*, from the Research Laboratory of the Eastman Kodak Company.

[2] "The Minimum Radiation Visually Perceptible," *Astrophysical Journal*, **44**, 124–127, 1916.

[3] This term was used by Russell to indicate the threshold value for perceptible radiation.

[4] "The Minimum Radiation Visually Perceptible," *Astrophysical Journal*, **45**, 60–64, 1917.

[5] "The Retinal Sensibilities Related to Illuminating Engineering," *Trans. of the Illum. Eng. Soc.*, **11**, 1–21, 1916.

light-source (N) is focused on the slit (S) by means of the lens (L), and the stimulus is illuminated through the opal glass window (O). The size of the stimulus is controlled by screening the window and in this way any desired aperture is obtained. The intensity of the light is controlled by means of the absorbing wedge (W), which is operated by the experimenter and slides on machined metal ways over the slit. By reading the brightness of the window without the wedge and by knowing the density of the wedge at any setting the transmitted light is computed from the equation

$$\log I_0 = \log I - D,$$

where I is the brightness of the window with no wedge, D is the density of the wedge before the slit, and I_0 is the reduced brightness.

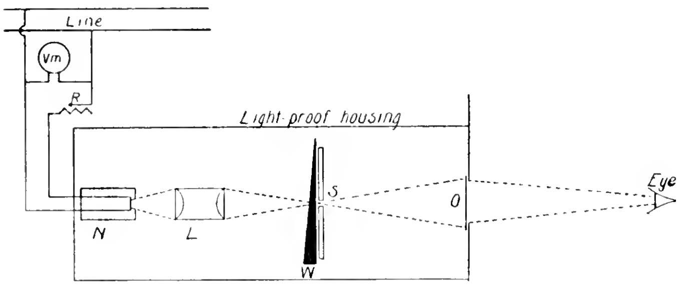

FIG. 1.—Stimulus control

The wedge was made by coating neutrally dyed gelatine on plate glass in a wedge-shape, so that the density varies from about 0.03 to 7.5, i.e., the percentage of the incident light transmitted varies from about 92 to 3.2×10^{-6}. When the dyed gelatine dried, a cover-glass was bound over it, as the unprotected gelatine soils with handling and is susceptible to moisture. The bound wedge is carefully calibrated and checked from time to time. A millimeter scale is mounted on the wedge, and the portion of the wedge before the slit is read from a pointer. A device was placed on the apparatus for recording the position of the wedge when work was done in darkness and the records could be read later.

The observer used a head-rest for all observations, so that the distance, fixation, etc., would be as nearly constant as possible. To approximate conditions of stellar observations, a stimulus 1 mm in

diameter viewed at a distance of 3 m was used in the first part of the experiment. With this arrangement of the apparatus readings were also taken at distances of 1.5 m and 35 cm. Several light-sources were used in this part of the work, but the results are very little different from one another. The results taken under the same conditions from day to day vary through wide ranges, and this characteristic has been found by the writer and other experimenters to hold for all threshold work with visual stimuli.

When observations were made with this apparatus, all adjustments were made and the observer then remained in total darkness for at least fifteen minutes before making any observations. The "star" was then exposed at a brightness just above the visual threshold and slowly dimmed by moving the wedge until the threshold was reached and the position of the wedge was recorded. The wedge was then set so that the "star" could not be seen and moved slowly until the "star" just became visible and this position recorded. This procedure was repeated several times, and the average of the several readings was taken as the threshold value for that series. (When the lamp in a lamp-house was employed, a 1-mm "star" was used and the intensity of the light was controlled by a set of decimal filters. By reading the brightness with no filters and knowing the filter-densities the investigator could compute the brightness of the star. The procedure in this case was the same as before.)

When the experiment was first started, the diameter of the pupil was measured by means of a glass wedge. The width of the wedge was 1 mm at the small end and 11 mm at the wide end and was 10 cm long, so that the wedge increased 1 mm in width for each centimeter of length. The wedge was also thicker at the wide end, making it prismatic, and an object viewed through it was displaced. If the wedge was held before the eye so that the pupil was only partly covered, two images were seen, one over the other, as one image came through the glass and was displaced and the other was the normal image seen around the wedge. By shifting the wedge a position could be reached where the sides of the wedge were both tangent to the pupil and only the displaced image was seen. A slight shift of the wedge introduced the second image, and

the pupilary diameter was obtained by measuring the width of the wedge where the second image just disappeared. To get the diameter of the pupil when adapted to darkness, the small "star" was exposed at an intensity slightly above the threshold and was used as an object for the wedge. The amount of light used was considered too small to cause a contraction of the pupil. The results from this wedge show the average maximum pupil to be 6.8 mm, a value too small, according to some experiments, although in agreement with other results when using faint sources of light. T. H. Blakesley,[1] by using a faint light-source, found that his pupil varied in diameter from 6.74 to 7.20 mm, the higher values being less frequent. A probable explanation for the failure of the method of the glass wedge may be that for the light-source to be seen around the wedge a relatively large area of the pupil must be uncovered, in order to allow sufficient energy to reach the retina to give rise to a sensation, and, as a result, the width of the wedge before the pupil is much less than the pupilary diameter. Or it may be a question of the distribution and sensitiveness of retinal elements at the extreme of the periphery.

In order to get a better method for recording the pupilary diameter, it was decided to take instantaneous flash-light pictures of the eye adapted to the dark. A millimeter scale was pasted near the eye in the same plane as the pupil, the subject was seated near the camera, so as to secure a large image, and after sufficient adaptation the picture was taken. The average maximum diameter of the writer's pupil as taken from several photographs is 8.3 mm. The results show that the diameter of the pupil is not constant, even when the level of brightness of the environment remains quite fixed and the eye is fully adapted to the dark. The photographs were taken on different days and also on the same day, with sufficient interval between exposures for the eye to return to normal conditions. Photographs of eyes of other observers show just as wide variations, and the average maximum diameters also show individual differences. These fluctuations in diameter were found to hold throughout the range of intensities from total darkness to the reflection from white drawing-paper in full sunlight. The

[1] "A Means of Measuring the Apparent Diameter of the Pupil of the Eye in Very Feeble Light," *Phil. Mag.*, **20**, 966–969, 1910.

writer and Mr. Julian Blanchard took photographs of the eye at the two extreme intensities and at six intermediate intensities for both eyes exposed to the sensitizing brightness and for only one eye exposed, the other eye being closed. (These data will be published as soon as the experiments can be completed.) The results do not agree with those published by W. H. Steavenson,[1] who used five normal subjects between the ages of twenty-one and twenty-five years and found an average diameter of 0.336 inches or 8.5 mm, with an average deviation from the mean of only 3 per cent. This average seems to be higher than the writer's results, and the variation much too small. The writer used three subjects trained in dark-room observations and expects to photograph several other subjects in the near future.

Nutting says: "The image of the pupil formed in front of the eye by the aqueous humor and the cornea is the entrance pupil of the optical system, and, since the position of the iris varies with the accommodation, so will this entrance pupil."[2] The relative diameter of pupil and image focused at 25 cm is 1.02, and, as this is about the distance used for the photographs, the correction may be safely neglected.

Table I shows the data obtained for the 1 mm "star" at a distance of 3 m with the different light-sources, and the variation from one source to another is seen to be no more than that between results from the same source. The average of all observations was used in computing the final results. Table II shows data for the "star" at 1.5 m and 35 cm. Table III shows the comparative results of three observers under the same conditions. Table IV contains a summary of results and the computed value of the *minimum visibile*.

If we let B=normal candle-power per square centimeter of source, assuming the inverse square law and a point-source, the flux through 1 sq. cm on the axis at the eye will be

$$\frac{SB}{R^2} \text{ lumens,}$$

[1] "Aperture of Eye-pupil," *Journal of B. A. A.*, **26**, 303, 1916.

[2] *Outlines of Applied Optics.* Philadelphia: P. Blakiston's Son & Co., 1912. Pp. xi+234.

and the flux through the pupil of area A will be

$$Fp = \frac{SBA}{R^2} \text{ lumens.}$$

Now $B = \text{Lumens}/\pi$ and $A = \pi r^2$, so

$$Fp = SL\frac{r^2}{R^2},$$

where Fp is the flux through the pupil; S, the area of the "star" in square centimeters; L, the brightness of the star in lamberts; r, the radius of the pupil; and R, the distance of the eye from the "star." If this equation is then multiplied by the mechanical equivalent of light, M (using Ives's value of 1.59 ergs per sec. per sq. cm), we get the equation for the *minimum visibile* as

$$\text{Least perceptible radiation} = SLM\frac{r^2}{R^2} \text{ ergs per sec.}$$

TABLE I

1 mm "star" at distance of 3 m

100-watt carbon filament lamp, 111 volts, 0.904 amps. Color-match with standard

Sensitometer with Wedge	Lamp-House with Filters
0.0065 ml*	0.0102
.0053	.0180
.0126	.0144
.0093	.0155
0.0074	.0161
	.0142
	.0153
	0.0072
Mean 0.0082	Mean 0.0064

Sensitometer with wedge

Nernst glower No. 1, 27 mm long, av. diam. 0.63 mm, 156 volts, 0.4 amp.:

Mean = 0.0088

Small carbon lamp, 116 volts, 0.530 amps for color-match:

Mean = 0.0095

40-watt tungsten, 116 volts:

Mean = 0.0045

Nernst No. 2, 36 mm long, av. diam. 0.66 mm, 220 volts, 0.2 amps.:

Mean = 0.0072

Mean of all = 0.0072

* All results are in millilamberts, one thousandth of a lambert. It has been officially adopted by the Illuminating Engineering Society and was defined by the Committee of Nomenclature and Standards (*Trans. Illum. Eng. Soc.* 10, 642, 1915,) as "the brightness of a perfectly diffusing surface radiating or reflecting 1 lumen per sq. cm," that is, in accordance with Lambert's cosine law: 1 ft.-candle = 1.076 ml, 10 meter-candles = 1 ml.

In Table IV the writer's threshold is seen to be 17.1×10^{-10} ergs per sec., the average for three observers is 19.5×10^{-10}, as compared with Ives's computed result of 38×10^{-10} and Russell's experimental result of 7.7×10^{-10} ergs per sec. (Nagel[1] reviews the earlier work done in this field and gives as his own result for the smallest quantity of energy sufficient for stimulation 31.6×10^{-10} ergs per sec. but does not give details of procedure for comparative purposes.)

TABLE II

1 mm "star" at distance of 1.5 m.
100-watt carbon lamp

	Sensitometer with Wedge	Lamp-House with Filters
	0.0029	0.0028
	0.0034	0.0030
	0.0026	0.0020
		0.0022
Mean=	0.0030	0.0016
		0.0026
Mean=		0.0024
Mean of all=		0.0026

1 mm "star" at distance of 35 cm
100-watt carbon lamp

	Sensitometer with Wedge	Lamp-House with Filters
	0.00015	Mean=0.00027
	0.00026	
	0.00029	
Mean=	0.00023	
Mean of all=		0.00024

TABLE III

1 Mm "Star" at 3 M., Nernst No. 2

Observer	Threshold	Diameter of Pupil
1	0.0072 ml	8.3
2	0.0100	8.4
3	0.0092	7.4
Means	0.0088	8.0

At first the wide variations in the separate threshold values in Table I seem too great, but let us consider some of the factors which influence the threshold determination. The threshold varies with

[1] Helmholtz *Handbuch der physiologischen Optik*, 3. Aufl., 1911, Band 2, p. 280.

the pupilary diameter, and this we found was not constant. When the stimulus is just at the threshold, we must consider the fluctuations due to attention and fatigue. Ideo-retinal light, probably caused by the retinal circulation, is a decided hindrance to accurate observations and at times compels the observer to stop all procedure. After-images often persist after the stimulus has been removed and are likely to recur so as to be confused with the stimulus itself. Involuntary movements of the eye make the conditions of visual fixation variable, and any change in the area of the retina used would alter the result obtained. Probably the greatest factor is

TABLE IV

	Threshold	Energy at Eye	Area of Pupil	Energy Entering Eye
			sq. cm	ergs/sec.
Writer...........	0.0072 ml	3.16×10^{-9}	0.54	17.1×10^{-10}
Mean of 3 observers	0.0088 ml	3.89×10^{-9}	0.50	19.5×10^{-10}
Ives's data.......	$6^{M}_{.}0$	1.35×10^{-8}	0.28	38.0×10^{-10}
Russell..........	$8^{M}_{.}5$	1.35×10^{-9}	0.57	7.7×10^{-10}

the physiological condition of the observer from one day to the next. To determine an extreme of this factor, the writer took observations when affected by a severe cold, and the threshold was raised so that the values could not be used at all. As these factors are not subject to control, the only method is to take as many observations as possible over a wide range of time and to accept the general average as the threshold value.

These difficulties make the problem of determining the length of time required for such luminous intensities to produce a sensation almost a hopeless one. The writer is now collecting data on this subject, as well as on the problem of the visibility of large and small images of the same total intensity, and they will be published later.

The writer wishes to acknowledge the assistance of Messrs. L. A. Jones, Julian Blanchard, and Milton Fillius in the preparation of this paper.

Research Laboratory, Eastman Kodak Company
Rochester, N.Y.
June 15, 1917

PRELIMINARY EXAMINATION OF THE PLANETARY NEBULAE FOR PREFERENTIAL MOTION

By C. D. PERRINE

As it is generally believed that the planetary nebulae belong to our stellar system and as their average velocities are high, the question is raised whether, like the stars, there may be a tendency among them to motion in the direction of the ellipsoidal axis.

The most valuable data for such a purpose, viz., radial velocities and proper motions, are meager indeed. However, it seemed worth while to make the attempt, as the total number of these objects appears to be very limited, and, were complete data available for all such objects known, the weight of any conclusions would probably be very inferior to corresponding results for the stars. We also have any evidence which may be furnished by the distribution of these objects in the sky.

For this investigation there were available individual radial velocities for but fifteen of these objects, the classical thirteen by Keeler[1] and two by Campbell.[2] As the two by Campbell are exceptionally high, the determination of prolateness was limited to Keeler's list.

But two proper motions were available, viz., by H. D. Curtis.[3]

The available data are collected in Table I. v' are the radial velocities corrected for a solar motion of 20 km toward $18^h, +30°$. The velocities of Nos. 6644 and 4732_2 are by Campbell.

It is interesting to note that the two with very high velocities by Campbell are very close to the southern vertex of the ellipsoid and that both are stellar. It is also worthy of note that all three stellar objects of Keeler's list have large positive velocities, two of them being the highest on his list, and that these two are also close to the same vertex. With the exception of these stellar objects, the others are large and bright.

[1] *Publications Lick Observatory*, 3, 201.

[2] *Proceedings of the National Academy of Sciences*, 1, 9, 1915.

[3] *Ibid.*, 1, 10, 1915.

PROLATENESS

As stated above, the excessive velocities of Campbell's two nebulae were not used in the determination of prolateness. If we adopt limits of 50° from the vertices for the major axis and of 60° to 90° from the vertices for the minor axis, there are 8 nebulae yielding a velocity of 32.0 km for the major axis and 5 yielding a value of 18.6 km for the minor axis. These values give a prolateness of 1.72, which agrees well with the value given by the stars of large proper motion.

TABLE I

DATA FOR PLANETARY NEBULAE

N. G. C.	Description	α 1900.0	δ 1900.0	Obs. v	v'
1535......	vB	$4^h\ 9^m 36^s$	$-13°\ 0'$	-10.2 km	$-\ 27.6$ km
3242......	vB	10 19 58	$-18\ \ 8$	$+\ 6.0$	$-\ \ 4.0$
6210......	vB	16 40 18	$+23\ 59$	-34.4	$-\ 15.6$
6309......	B	17 8 26	$-12\ 48$	-51.6	$-\ 37.8$
6543......	vB	17 58 35	$+66\ 38$	-64.8	$-\ 48.8$
6572......	vB	18 7 14	$+\ 6\ 50$	$-\ 9.7$	$+\ \ 8.3$
6644......	Stellar	18 26 24	$-25\ 13$	....	$+202$
47322......	Stellar	18 27 54	$-22\ 43$	....	-141
6790......	B Stellar 9.5	19 17 52	$+\ 1\ 19$	$+48.5$	$+\ 64.7$
6818......	B,vS	19 38 20	$-14\ 24$	-16.8	$-\ \ 4.2$
6826......	B,pL	19 42 7	$+50\ 17$	$-\ 5.3$	$+\ 12.5$
6891......	Stellar 9.5	20 10 25	$+12\ 26$	$+40.8$	$+\ 57.0$
6905*.....	B !!	20 17 56	$+19\ 47$	....	....
7009†.....	vB !!!	20 58 44	$-11\ 46$	-49.8	$-\ 39.8$
7027......	Stellar 8.5	21 3 18	$+41\ 50$	$+10.2$	$+\ 25.6$
7662......	vB !!!	23 21 5	$+41\ 59$	-11.5	$-\ \ 2.5$

* $\mu = 0\overset{''}{.}056$. † $\mu = 0\overset{''}{.}054$.

The amount of data is extremely limited, and the result is therefore of small weight. As far as it goes, however, it is definitely in favor of an excess of motion in the general direction of the ellipsoidal axis. Little information can be drawn from the only two measures of proper motion available, further than that they appear to be just within the limits at which preferential motion was found among the stars.

DISTRIBUTION

The distribution of these objects seems to furnish stronger evidence of a preference for the direction of the ellipsoidal axis. In

the diagram published by H. D. Curtis[1] are seventy-three of these objects north of 33° south declination. Two-thirds of these are within 50° of the ellipsoidal vertices, or about one-third of the total area of the sky.

Curtis points out in the same note that the smaller of the planetaries show the greater affinity for the galactic plane. His diagram shows that the smaller ones have an apparent preference also for the ellipsoidal vertices. This and the generally high velocities

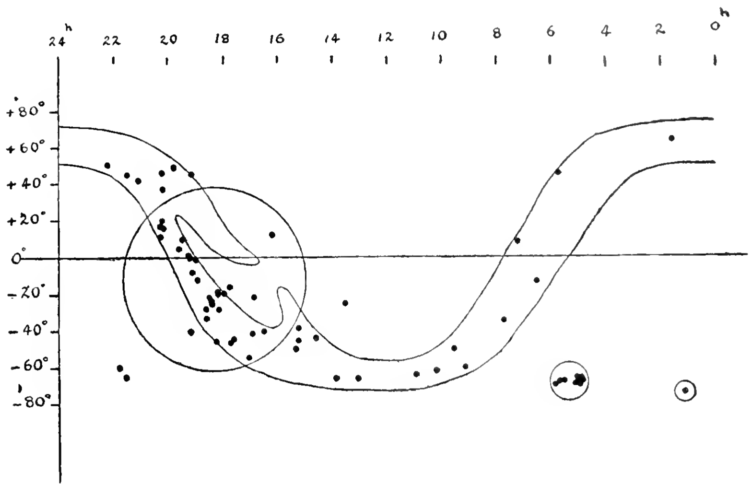

FIG. 1. Distribution of stellar planetary nebulae

of the five stellar planetaries of the foregoing short list caused me to chart the positions of the sixty-five stellar planetaries of the three *N.G.C.* catalogues for the entire sky. The resulting distribution as shown in the accompanying diagram is very suggestive. Not only do these objects show a striking affinity for the galaxy, as pointed out by Curtis, but they also show a most decided preference for the 18^h ellipsoidal region and an equal avoidance of the opposite region of sky. Their affinity for the galaxy is not confined alone to its plane, but extends to the actual branches. This is particularly striking in the Sagittarius-Ophiuchus-Aquila region (the region of the great bifurcation), where for a considerable space, and that richest in planetaries also, they are wholly confined to the southern

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 53, 1917.

branch, the northern, terminant streamer not having any so far as known.

Even more striking, if possible, are the groupings of these stellar planetaries in the Magellanic Clouds. Taken in connection with the appearance and known structure of the Clouds, this evidence of the stellar planetaries would seem to justify, not only the conclusion that the Magellanic Clouds are similar in constitution to the Milky Way, but the strong presumption that they are in reality more or less isolated parts of the galaxy.

Considering the evidence as a whole, there seems no room to doubt that the planetary nebulae are members of our own stellar system.

CONCLUSIONS

1. The planetary nebulae show a preference for the regions of the vertices of ellipsoidal motion, particularly that near 18^h. The few radial velocities of these bodies which are available also show greater motion in the direction of this axis.

2. The planetary nebulae undoubtedly belong to our stellar system.

3. The restriction of the stellar planetary nebulae to the Milky Way and Magellanic Clouds indicates, if in connection with the appearance and other known facts of the constitution of these several bodies it does not establish, a very close relationship between the Clouds and the Milky Way.

Observatorio Nacional Argentino, Córdoba
April 8, 1917

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THE

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BAND AND LINE SPECTRA OF IODINE

By R. W. WOOD and M. KIMURA

Iodine vapor is of peculiar interest spectroscopically, in that it is one of the few substances which can be caused to emit line spectra of an almost infinite variety, of definite types, and very regular structure, by excitation with monochromatic light, as has been shown by one of us. These resonance spectra are very intimately associated with the complicated banded absorption and emission spectra, on which account it has seemed very desirable to make a comprehensive study of the spectra of this element excited by other means under the highest dispersion possible.

The present paper will deal chiefly with the electrical excitation of the vapor in vacuum tubes. In the course of the investigation we made the discovery that many of the lines of the line spectrum are complex under high dispersion, appearing as doublets, triplets, quadruplets, and quintuplets, the total width of the group in the case of the quintuplets being about 0.35 A. These complex lines behave in a most remarkable manner in the magnetic field, and we made a very exhaustive study of the Zeeman effect which they exhibit. This subject will be taken up in a subsequent paper.

The different types of spectra emitted by iodine vapor under different conditions of excitation were studied by Konen[1] nearly

[1] *Annalen der Physik*, **65**, 257, 1898.

twenty years ago. In vacuum tubes excited electrically he found a band spectrum and a line spectrum, the relative intensities of which depended upon the diameters of the tubes, current-densities, vapor-densities, and other circumstances. Konen gives the wave-lengths of about 350 lines in the range of spectrum from λ 3030 to λ 5787, but makes very little mention of the band spectrum, stating that it was so feeble that it could be photographed only with a direct-vision prism-spectrograph (exposure 8 to 9 hours) which gave a spectrum less than 3 cm in length for the range 5700–3000. We have, however, so improved the conditions of excitation that we have been able to photograph this band spectrum in the fifth-order spectrum of a large plane grating, with an objective of three meters focus—that is to say, with apparatus capable of completely resolving the absorption spectrum, which, as has been shown by one of us, contains in the neighborhood of 40,000 lines in the visible region. The spectrograms from which Konen's measurements of wave-lengths in the line spectrum were made were obtained with a small concave grating of one meter radius in the first-order spectrum, and he gives 0.04 A as his mean error. We are, however, in agreement with Kayser, who considers the limit of accuracy to be more nearly 0.1 A. Konen was unable to secure photographs in the second order on account of the faintness of the light. We have of course had no difficulty in photographing this spectrum in the fifth order, as the lines can be made very much brighter than the bands.

We found, in the early stages of the work that the insertion of a spark-gap or capacity in the circuit increased enormously the intensity of most of the lines and suppressed almost completely the band spectrum. There were, however, other lines which were reduced in intensity, and a few which showed little or no change. This circumstance has been mentioned briefly by Goldstein, and Stark has also alluded to it, classifying the lines which were increased in intensity as "spark lines," the others as "arc lines."

In the preliminary part of the work we had considerable difficulty in finding suitable electrodes. Platinum is very rapidly attacked by the ionized iodine vapor, and deposits in the form of a brownish coating of very low reflecting power, which is probably

a compound of the metal with iodine. We finally adopted tubes provided with external electrodes of tin foil. These tubes were of the form shown in Fig. 1. The bulbs were about 4 cm in diameter and 15 cm in length, joined by a capillary, which was blown out in the form of a thin bulb at *A*, for the emergence of the light. The process of exhaustion was as follows: A few flakes of iodine were introduced into the bulb through the tube *B*, which was then sealed. The flakes were then brought to the bottom of the tube *B*, and the tube *C* put in communication, through a U-tube immersed in liquid air, with a Gaede pump. If liquid air is not available, a

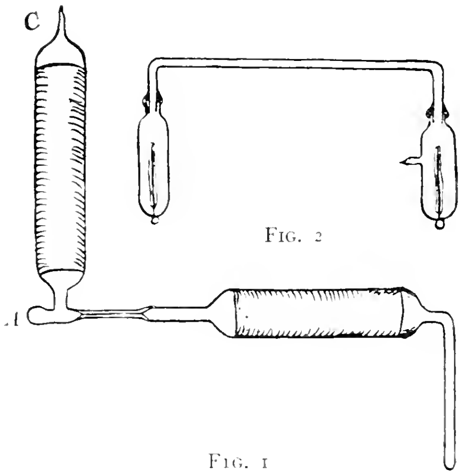

FIG. 2

FIG. 1

tube filled with fragments of caustic potash should be introduced between the tube and the pump, to hold back the iodine vapor. During the process of exhaustion the bulbs must be strongly heated with a Bunsen flame. Before the tube is sealed off from the pump a small flame should be applied cautiously to the bottom of the tube *B*, until the iodine has entirely sublimed to the upper part of the tube. It is also a good plan to test the vacuum in the following way: Wrap the bulbs with tin-foil electrodes and start the discharge, using a coil capable of giving a six- or eight-inch spark; then touch the walls of the bulbs with cotton wet with liquid air. If the capillary is very fine, the vacuum in the bulb nearest the pump will usually be found to be much higher than in the second. At very low pressures the color of the discharge in iodine vapor is chamois-yellow, and the exhaustion should continue until this condition obtains in *both* bulbs, when they are cooled with liquid air or solid CO_2. If neither of these substances is available, immerse the tube *B* in a mixture of ice and salt. A yellow discharge

in one bulb and a pink discharge in the other indicate that nitrogen is still present in the bulb beyond the capillary.

In our experiments we found that, to get the line spectrum at its brightest, the diameter of the capillary should be not over 0.15 mm, and considerable practice was required before suitable tubes could be produced. They were drawn down from 6 mm tubing, which was first heated until the walls nearly collapsed. In the latter part of the investigation we found that very satisfactory iodine tubes could be made by using electrodes of thin platinum foil enameled with a thin layer of soft glass, which was smeared on in the flame of a blast lamp. These electrodes last a long time, heavy currents can be used, and the capillary can be a millimeter or more in diameter.

The spectrum of the electrically excited iodine vapor is made up of a fluted band spectrum between wave-lengths 5200 and 7000, which in the fifth-order spectrum of the large plane grating shows a structure comparable to that of the absorption spectrum, and a continuous band between wave-lengths 4300 and 4800 A. This latter band becomes relatively feeble if the iodine is at a low pressure, as when the lateral tube is placed in a refrigerating medium, and we have only the fluted band, the integrated color of which is the chamois-yellow referred to above. As the pressure increases, the color becomes white and finally violet-blue, owing to the development of the continuous band.

Superposed on the band spectrum we usually have the line spectrum also, though it is nearly absent in wide tubes with small current-densities. It may be developed by constricting the tube or by increasing the current-density, as by the introduction of a condenser in the circuit. With the tubes provided with external electrodes it can be brought out strongly by means of a spark-gap placed in parallel with the tube. It may also be brought out, as we found, by merely heating the discharge tube to a high temperature by means of a burner. This indicates that it probably results from the dissociation of the iodine molecule. As the line spectrum develops in intensity, the band spectra fade away and finally disappear. Even in the tubes provided with external electrodes we found that traces of CO appeared after prolonged use. A yel-

low discoloration of the glass also developed, and it seems probable that this gas or CO_2 is liberated from the glass by the action of the ionized iodine.

A portion of the band spectrum with the lines superposed and the line spectrum alone are reproduced as negatives in Plate XIII, *a*. The latter was taken with a condenser in parallel (internal electrodes). The influence of the current-density is well shown by *d* and *e*, which reproduce the greater part of the visible spectrum. The lower portion of each was taken without spark, the upper with spark in parallel with the tube (external electrodes). For example, line 5234 is strong in the upper spectrum of *a* and the lower spectrum of *d*, while it has disappeared entirely in the lower spectrum of *a* and is relatively weak in the upper spectrum of *d*. Line 5245 behaves in exactly the reverse manner.

The disappearance of the band spectrum and the appearance of the lines can be brought about gradually by means of a variable capacity. We are of the opinion that it is the result of dissociation resulting from elevation of temperature, for we have succeeded in bringing about the same change by heating the narrow part of the tube with a Bunsen burner. The tube (Fig. 2) used in this experiment was provided with internal electrodes sealed into glass bulbs, which we joined by a quartz tube having a bore of about 3 mm, and cemented into the bulbs with sealing-wax. This tube, when excited by the coil, gave the band spectrum, with scarcely a trace of the lines (Plate XIV, *f* [upper spectrum]). The horizontal portion of the quartz tube was now strongly heated; the color of the discharge changed and the lower spectrum was obtained, with the band much weakened and the lines strongly developed.

The band spectrum has been photographed in the fifth-order spectrum of a seven-inch plane grating with a lens of three meters focus. A portion of the spectrum in the region of the 5626 line is reproduced on Plate XIV, *g*, in coincidence with the absorption spectrum. Since the photograph is reproduced as a negative, the absorption lines appear light instead of dark (upper spectrum). It is clear from the photograph that the spectra are not complementary, though every emission line of the band has an absorption line in coincidence with it. There are many absorption lines,

however, which are not represented in the emission spectrum. This very probably results from something in the nature of dissociation. We have not yet studied the possible changes in the fine structure of the emission band spectrum with varying current-density, but there are indications that such changes occur, for some of our plates show greater dissimilarity between emission and absorption than the one just mentioned—*h* for example.

The action of high temperature on the band absorption spectrum has been investigated, however, and changes have been noticed which throw some light on the matter.

EFFECTS OF HIGH TEMPERATURES ON THE ABSORPTION SPECTRUM

Evans has given, in the *Astrophysical Journal* (**32**, 1, 1910), an account of an investigation of the disappearance of the absorption spectrum of iodine at high temperatures. His investigations were made with a spectroscope of low dispersion, and only the gradual disappearance of the bands was recorded. His results would be perfectly explained on the supposition that the absorption spectrum results from diatomic molecules I_2, which, at high temperatures, break down into monatomic molecules devoid of absorbing power. He found that the denser the vapor the higher the temperature necessary to cause the complete disappearance of the spectrum.

We have studied the phenomenon with a spectroscope of the Littrow type of six meters focus, using the fifth-order spectrum of the seven-inch grating. This grating, which is the best ruled by Anderson, gives very nearly its full theoretical resolving power (450,000). Photographs were made with dense iodine vapor in a quartz bulb (previously exhausted) heated by two blast lamps, and with the same bulb at room temperature, with much less dense vapor. We also made a large number of plates with a tube of pyrex glass heated in an electric oven. In every case we attempted to secure pairs of plates which showed the absorption spectrum at about the same degree of intensity, as this condition brought out the changes in the minute structure to better advantage. On the assumption that diatomic absorbing iodine breaks down into a colorless monatomic gas, we should expect the spectrum to fade away precisely as it does when the amount of vapor is decreased

by lowering the density. This is not the case, however, as will be seen by comparing the photographs (positives) reproduced on Plate XIV, *i*, the lower one taken with the quartz bulb at high temperature (perhaps 1000° C.), the upper with the bulb at a temperature of 35°. The stem of the bulb was immersed in boiling water in the first case; consequently the iodine was at a density corresponding to 100°. If we compare the two photographs, we notice that some lines are much stronger in the upper spectrum of the cold vapor than in the lower. Some of these lines have been indicated by arrows. Many lines have about the same intensity in both spectra. Others, however, are distinctly stronger in the spectrum of the hot vapor. These also are indicated by arrows and small dots.

It appears then that the lines are affected in different degrees by an elevation of temperature. Those indicated by the arrows above the upper spectrum are the most readily quenched, and those indicated by arrows below the lower spectrum are the ones most resistant to temperature. Clearly we are dealing with something more complicated than the dissociation of a diatomic molecule. Similar differences were found in the case of the spectra made with the tubes of pyrex glass at temperatures ranging from 350° to 500° C.

It seems quite possible that the band emission spectrum will be found to be much more nearly the exact complement of the absorption spectrum of the vapor at a high temperature than in the case shown in Plate XIV, *g* and *h*. This matter will form the subject of a future investigation.

THE LINE SPECTRUM

Though the wave-lengths which we have redetermined are probably not much more accurate than those given by Konen, it appears to be worth while to give them, as we have divided the lines into two groups, the arc and spark lines previously alluded to. Moreover, we have determined the wave-lengths of some 50 of the lines from plates made in the fourth-order spectrum of the 3-meter spectrograph, and these are correct to 0.01 A. It is doubtful if the others can be relied on beyond 0.1 A, and the same is true of Konen's values.

In Table I the "arc" lines, or the ones which show a *decrease* of intensity as a result of increasing the current-density by a condenser or parallel spark-gap, are indicated by an asterisk. It should be noticed, however, that this effect is of variable magnitude, some lines being greatly weakened, others less so; some lines show no change at all, and others (the spark lines) are enhanced in varying degrees. On this account it is difficult to make a very sharp classification. All wave-lengths are given on the international scale and the values in italic were determined from plates made in the fourth-order spectrum and are correct to within about 0.01 A. The others are correct only to 0.1 A.

STUDY OF THE COMPLEX LINES WITH THE ECHELON

Instruments and methods.—The echelon used in this work was a new one by Hilger, with twenty plates, 1 cm thick, in optical contact. It was used in conjunction with a collimator and telescope, each of 9 feet focus. It may be well to point out that this is about the right focal length to furnish the full resolving power recorded on a photographic plate. The telescopes of shorter focus usually employed, while excellent for visual observations, give poor results when used for photographic work. A wooden box surrounded the echelon and the objectives of the collimator and telescope, and the temperature within this inclosure was kept constant to within 0°1 C. by a toluole thermostat. The slit of the collimator was removed and its place taken by the second slit of a Hilger constant-deviation spectroscope, used as a monochromator. To make settings with this instrument we simply swung the 9-foot collimator a little to one side, and viewed the slit of the monochromator with a lens of high power, opening it wide for the purpose of identifying the line, and then gradually closing it while keeping the desired line always within the aperture. This method is far simpler and more satisfactory than attempting to form an image of one slit on another by means of a lens, and we were able to study separately lines the distance between which was only 7 angstrom units. Some of the lines showed such irregular structure with the echelon that we suspected the presence of neighboring lines that were not removed by the monochromator. This was found to be

TABLE I

Wave-Lengths	Intensities: Without Spark	Intensities: With Spark	Wave-Lengths	Intensities: Without Spark	Intensities: With Spark
4632.4	4	10	4924.4	1	2
4634.8	1	8	4929.9	1	2
4640.7	2	10	4938.6	1	6
4657.4	1	6	4943.1	1	6
4663.8	1	6	4957.6	1	6
4666.5	4	12	4965.7	0	2
4675.5	6	10	_4968.33_	2	6
4676.5	1	8	*4974.5	1	0
*4687.3	1	0	4984.4	1	2
*4691.1	1	0	_4986.95_	2	10
4700.8	0	0	*4991.9	1	0
4702.5	1	2	4992.2	1	2
4707.9	1	2	5008.4	0	4
4711.7	1	2	5028.8	1	2
*4722.1	1	0	*5032.3	1	0
*4726.3	1	0	5036.1	1	6
4730.5	1	8	5046.4	1	4
*4734.1	1	0	*5048.1	1	0
4737.1	1	2	5057.4	1	4
4742.9	1	2	5061.9	0	4
4752.7	1	2	5065.5	2	6
4763.4	10	10	*5068.2	1	0
4765.7	1	4	5090.7	1	4
4768.2	0	6	5098.8	1	2
*4773.1	1	0	_5114.44_	1	8
*4775.8	1	0	*_5119.32_	20	15
*4782.5	1	0	5124.6	1	2
4784.8	1	4	*5130.5	1	0
4787.2	1	2	5131.3	1	2
4788.2	1	2	*5133.2	1	0
4790.9	0	2	*5136.1	1	0
4799.8	0	4	*5138.5	1	0
4800.2	2	4	*5145.2	1	0
4806.4	2	6	5147.4	0	6
4808.0	0	2	5149.7	0	6
4828.3	2	6	5154.9	1	2
4835.1	2	6	5156.4	1	8
4850.4	2	10	_5161.20_	8	30
4853.1	1	2	5174.6	1	2
*_4862.33_	20	16	5175.1	1	2
4864.5	1	6	5176.3	1	2
4881.6	1	4	5178.1	1	8
4883.7	0	8	_5185.14_	1	8
*4887.7	2	0	*5186.3	1	0
4891.3	1	6	5189.4	1	2
4893.8	1	4	5198.9	1	8
*_4896.72_	12	8	*_5204.08_	10	4
*4902.2	4	0	5205.5	1	2
4908.5	1	2	_5214.04_	1	4
4910.3	1	2	_5216.22_	2	10
*_4916.94_	16	10	_5228.93_	0	8

TABLE I—*Continued*

Wave-Lengths	Intensities: Without Spark	Intensities: With Spark
*5234.58	10	8
5245.65	4	15
5265.150 5265.266	2	10
5266.8	2	2
5269.36	2	10
5288.7	0	4
5296.7	1	2
5299.68	0	6
*5304.3	1	0
5309.0	1	8
5314.6	1	4
5322.71	0	6
5326.4	1	4
5336.6	1	0
5338.20	6	18
*5341.8	1	0
5345.17	6	18
5349.7	1	2
5351.9	1	2
5356.0	1	4
5367.5	2	4
5369.75	4	12
5372.5	1	4
*5374.5	1	0
5380.1	1	4
5405.11 5405.23 5405.38 5405.59	4	16
5407.35	2	12
5411.7	1	4
5415.0	0	4
5421.97	0	4
5422.71	0	4
*5427.4	6	4
5435.80	4	10
5437.97	2	8
5449.0	1	4
5457.1	2	4
5464.77	6	20
5468.1	1	2
5475.1	....	0
5479.55	1	6
5491.52	1	8
5493.45 5493.05	0	8
5497.08 5496.96 5496.85 5496.79 5496.73	2	15

Wave-Lengths	Intensities: Without Spark	Intensities: With Spark
*5501.00	2	0
5504.77	2	8
5522.1	0	4
5527.5	1	4
5546.4	2	2
5551.7	0	4
5568.7	0	0
*5586.3	4	2
5590.3	2	2
5593.09	0	4
5598.55 5598.68	2	6
5600.21	2	6
*5601.8	2	1
5603.2	2	4
5612.82	2	6
5625.66	4	15
5643.4	1	4
5673.7	1	4
5678.06 5678.15	2	10
5679.9	0	0
5690.89 5690.96	2	10
5702.07	0	2
5710.43	2	10
5723.5	0	0
5725.0	....	0
5738.5	2	10
5739.5	0	10
5734.8	0	1
5760.8	2	8
*5764.3	6	4
5774.7	2	10
*5780.4	2	1
5787.1	1	6
*5790.2	1	0
*5793.0	1	0
5819.6	1	2
5830.0	1	6
*5832.7	1	0
5875.1	1	4
*5893.8	8	6
*5908.5	1	0
5920.7	1	4
*5928.6	1	0
5950.1	4	10
*5956.6	2	0
*5960.0	2	1
5962.8	0	1
*5966.1	2	1
*5967.7	2	1

TABLE I—*Continued*

Wave-Lengths	Intensities		Wave-Lengths	Intensities	
	Without Spark	With Spark		Without Spark	With Spark
*5980.5	2	0	6257.4	1	4
*5984.2	2	0	6267.1	0	1
6007.6	1	2	6268.5	0	4
6015.8	1	4	*6276.8	1	0
*6023.9	6	2	*6280.3	1	0
*6036.5	1	0	6290.4	0	1
*6038.6	1	0	6291.3	1	2
*6041.4	1	0	*6293.9	6	2
6043.9	1	2	*6296.4	2	0
*6046.5	1	0	*6313.1	2	1
*6048.4	2	0	6320.9	1	4
*6053.0	2	0	6323.6	0	1
6068.8	1	4	*6330.2	2	0
6074.9	2	6	*6333.5	2	0
6078.2	1	2	*6337.9	4	2
*6082.3	10	6	*6339.5	6	2
6084.7	1	2	*6348.3	1	0
6086.8	1	2	*6350.9	1	0
6115.7	0	1	*6355.4	2	1
6125.4	1	2	*6359.1	4	2
6127.4	2	8	*6367.2	2	0
6132.9	1	2	*6371.6	2	0
6149.0	1	2	6375.8	0	1
6161.9	1	2	6378.2	0	1
*6187.0	1	0	*6411.1	2	1
*6191.6	4	2	*6415.2	2	1
6195.5	1	4	*6428.7	1	0
6200.4	1	4	6440.2	1	4
6204.7	0	6	6476.0	1	2
*6213.0	4	2	*6488.1	4	2
6229.2	0	2	6495.0	1	2
6232.9	1	2	6516.1	1	2
*6233.2	2	1	*6538.3	2	1
6236.3	1	4	*6560.3	4	2
*6240.2	2	0	6574.8	0	1
*6244.3	4	2	6578.0	0	4
6245.8	2	2	6579.8	0	1
6250.6	0	2	*6583.2	4	0
6255.5	1	2	6585.0	0	4

the case. In some cases, as we subsequently found, two complex lines were so close together that they were passed simultaneously through the slit of the monochromator. To overcome these difficulties we crossed the echelon with a plane grating of 15,000 lines to the inch, placing it with its glass plates horizontal, between the collimator and grating. The collimator lens was an ordinary telescope objective of 1 meter focus, and the spectrum was formed

by a Cooke photographic objective of four inches aperture and the same focal length as the collimator. The slit was reduced in length to about 0.1 mm by means of two strips of tin foil fastened on the inside with soft wax. The beveled edges were on the outside, which is the proper design for a slit, though many instrument-makers reverse matters and give us beveled edges on the inside, which sometimes causes spurious lines by reflection of oblique rays. If the slit of the spectroscope was opened wide, the echelon spectra of the complex lines could be seen in the broad images of the slit. The echelon was leveled and brought to the proper position by observing these spectra. The slit was then closed until it was reduced practically to a needle hole, and the echelon spectra contracted to vertical rows of minute dots or single dots, each row representing a complex line and each single dot a simple line. In this way it was possible to photograph with the echelon the entire iodine spectrum from violet to red on a single plate. We even succeeded in photographing the nitrogen band spectrum in this way. A photograph of a portion of the spectrum taken with the grating-echelon combination is shown in coincidence with *a*, the spectrum taken with the grating alone, on Plate XIII, *b*. A smaller portion of the spectrum more highly enlarged is shown by c', the spectrum formed by the grating alone lying between the spectra formed by the echelon-grating combination. We shall take up now the structure of the various lines studied thus far, designating in each case whether the observations were made with the echelon alone or were from the plates made with the echelon crossed with the grating.

LINE STRUCTURE

In the case of many of the complex lines we made accurate determinations of the wave-lengths of the principal lines correct to 0.01 Å from photographs made in the fourth-order spectrum of the plane-grating spectrograph of 3 meters focus. These photographs also served as a check in interpreting the results obtained with the echelon. A portion of one of these is reproduced on Plate XIII, *c*, showing the complex line λ 5497. The constants of the echelon were as follows:

$\mu_D = 1.57493$, $C-D = 0.00410$, $D-F = 0.00996$, $F-G = 0.00837$.

From these values we calculated the constants in the Hartmann formula,

$$\mu=\mu_0+\frac{C}{(\lambda-\lambda_0)}$$

$$C=141.09, \qquad \lambda_0=1519, \qquad \mu_0=1.54267.$$

The wave-length intervals corresponding to the distance of two successive orders were calculated from the formula

$$d\lambda_m=\lambda^2\left\{\frac{1}{t\left[(\mu-1)-\lambda\frac{d\mu}{d\lambda}\right]}\right\}$$

for various wave-lengths, and their values are given in Table II. A curve showing the relation between λ and $d\lambda_m$ was then drawn, from which the wave-length intervals between successive orders for any value of λ could be found.

TABLE II

λ	$d\lambda_m$
4632.4	0.3345
4666.0	0.3397
5016.2	0.4006
5162.0	0.4237
5345.0	0.4672
5464.6	0.4854
5625.0	0.5175
5691.0	0.5308
5875.0	0.5605

While some of the complex iodine lines showed an irregular structure, or appeared as close doublets, the majority exhibited a series of lines of four or five members decreasing in intensity and separation toward the region of short wave-lengths, thus . In the majority of cases the width of the group was less than the distance between the orders of spectra of the echelon. Obviously we cannot, on a single photograph, get a true record of the relative intensities of the lines making up the series, for, if we put the echelon in position of "single order" for the brightest line (first member of series), the last line will appear too faint in comparison, as it will be at or near the position of double order. We usually adjusted

the echelon so as to show the last or faintest line in "single order"; this made the first line relatively weak, but gave us a better record of the series for measurement. Two or three of the lines showed a series as wide as, or a little wider than, the distance between orders of echelon spectra. In this case the last member of the series falls upon or beyond the first member seen in the next order. In these cases, however, our photographs made with the grating in the fourth-order spectrum indicated the presence of the last member of the series, though it was not quite resolved, and by carefully comparing these photographs with those made with the echelon it was usually possible to determine the series. In the list which follows, the strongest line is designated by 0.000 and the distance of the other components from this zero position is indicated, the minus sign indicating of course the side of short wave-length.

Most of the lines in the violet prove to be single lines: the following showed structure:

λ *4404*. Four lines nearly equidistant, 0.000, −0.057, −0.105, −0.167 from plates made by echelon crossed with grating.

λ *4465*. Double lines; 0.000, −0.069.

λ *4474*. Typical series of five lines of decreasing spacing and intensity. Strongest or main line .000, others at +0.078, +0.137, +0.190, and +0.232. This series appears to point toward longer wave-lengths. The other similar series point in the opposite direction. The reversal of this series was checked by a photograph made in the spectrum of the fifth order.

λ *4632*. Structure similar to foregoing, but series turned the other way. Main line 0.000, others −0.085, −0.152, −0.196, −0.228. This line studied by the echelon alone.

λ *5065*. Three components, +0.130, 0.000, −0.095 (echelon crossed with grating), middle component strong.

λ *5161.20*. Series of five lines. Main line determined in fourth-order spectrum of grating. Components at −0.105, −0.186, −0.241, and −0.276 (by echelon alone).

λ *5245.65*. Series of five lines. Main line determined by grating, which did not quite resolve the series. Echelon gave components at −0.053, −0.102, −0.146, and −0.192.

λ *5265.150, 5265.266*. Double line by grating. Separation of components by echelon 0.119.

λ 5338. Close triplet by echelon alone. Main line strongest. Components at −0.041 and −0.083.

λ 5345. Components 1 and 2 faint, 4 and 5 fairly strong. Calling No. 3 the main line, we have for the structure (5 seems to be double) +0.098, +0.029, 0.000, −0.051, −0.10, −0.116.

λ 5356. Echelon crossed with grating shows five components spaced at nearly equal intervals, the width of the whole group being about 0.3 A.

λ 5370. Series of five lines by echelon. Main line 5369.75 (by grating). Components of decreasing intensity at −0.054, −0.099, −0.146, and −0.192.

λ 5405.59. Main line and series of decreasing intensity. Four of the members of the series resolved by grating, namely, .59, .38, .23, .11. The series was a little wider than the distance between the orders of the echelon, the last member (the fifth) falling beyond the main line in the next order. The series would be shown to better advantage by echelon plates of 7 mm thickness. It could be studied only with the echelon crossed with the grating on account of the proximity of 5407 (a four-term series). It is represented thus: 0.00, −0.21, −0.36, −0.48, −0.55. (The distance between echelon orders in this region is 0.473.)

λ 5407. Echelon crossed with grating shows series of four lines of decreasing intensity, 0.000, −0.062, −0.115, −0.160.

λ 5436 is shown to be single, by echelon crossed with grating, and 5438 similar to the 5407 series but with closer spacing, 0.000, −0.05, −0.082, −0.115, and −0.161.

λ 5464.77. Series of five components by echelon alone. Four shown with grating in fourth order. The first and strongest has wave-length given above, and the four others are of decreasing intensity and located at −0.106, −0.190, −0.255, −0.275.

λ 5491.50 is single; 5493.45 and 5494.05 (by grating) appear with echelon crossed with grating as a close doublet of about 0.13 separation—an example of a spurious result due to a confusing of orders.

λ 5497.08. Main line of five member series. Grating measurements gave others as 5496.96, .85, .79, and .73. On account of proximity of other lines, it could be further studied only with echelon crossed with grating, which gave 0.000, −0.134, −0.233, −0.310, and −0.343.

λ 5598.6. A triplet, by echelon crossed with grating. Components 0.00, −0.185, and −0.310 (the latter faint).

λ 5600.2. Close doublet, separation of components about 0.05 A.

λ *5603*. Three components by echelon crossed with grating: 0.000, −0.082, and −0.159.

λ *5612*.8. A doublet with components 0.068 A apart.

λ *5678*. Doublet by echelon alone. Wave-lengths also determined by grating 5678.06 and 5678.15. Echelon gave 0.88 separation.

λ *5691*. Echelon alone gave a bright line with fainter companion 0.078 A toward red, and a very faint one at −0.10 toward violet. The grating alone gave 5690.89 and 5690.96.

λ *5710*. A very complicated line. A strong triplet series with another strong line well separated from it on short wave-length side, and two or three fainter companions:

$$+0.066,\ \overline{0.00,\ -0.045,\ -0.073}^{\text{triplet}},\ -0.110,\ \overline{-0.179}^{\text{strong}}$$

λ *5738.5*. Single.

λ *5739.5*. A triplet.

λ *5774.7*. Five components by echelon crossed with grating. The arrangement of the dots suggests that we may have two superposed lines, as the dots are not quite in line.

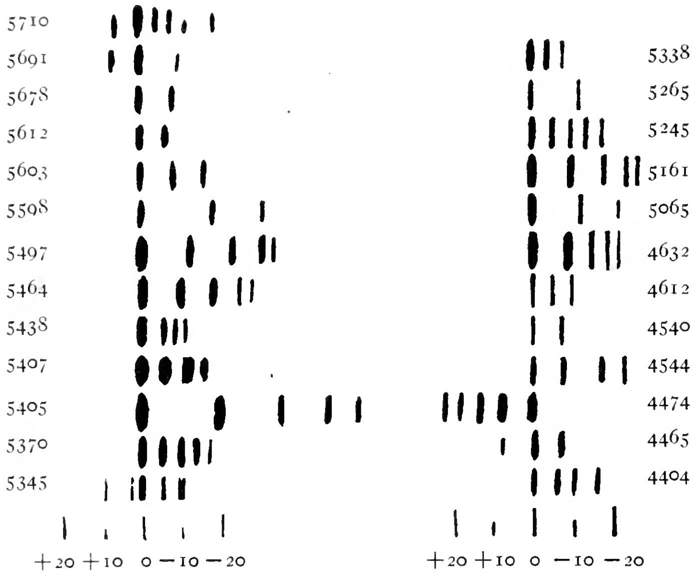

FIG. 3

The structures of these lines are shown by Fig. 3. The reversal of the series in the case of the line 4474 is of especial interest, as it was the only case found. It is also worthy of mention that all of the complex lines belong to the "spark" type. The width of the quadruplet series 5438 is only a little more than one-half of the distance between the first two members of the quintuplet series 5405, in which the separation of the members reaches its maximum value. In the following paper the behavior of these complex lines in a magnetic field will be discussed.

JOHNS HOPKINS UNIVERSITY
July 8, 1917

ZEEMAN-EFFECT FOR COMPLEX LINES OF IODINE

By R. W. WOOD and M. KIMURA

In the previous paper we have given a description of the many complex lines which we have discovered in the spectrum of iodine electrically excited in vacuum tubes.

In the present communication we shall discuss the behavior of some of these lines in the magnetic field. A large number of the lines have a structure resembling that of a short series, thus, . We never observed a series of more than five members, and the width of the group varied from 0.16 to 0.50 A. Paschen and Back[1] have shown that the close doublets of helium and the triplets of oxygen behave in a most interesting manner in the magnetic field, the oxygen triplet, for example, being transformed into a single line for the polarized component vibrating parallel to the field.

Our preliminary observations, which were made without polarizing apparatus, showed that the lines which had the structure figured above gave, in strong fields, a triplet of quite normal appearance. In weak fields the structure became too complicated to follow, and we immediately resorted to a polarizing apparatus by which the components vibrating parallel and perpendicular to the field could be studied separately. The tubes employed were of the same type as that described in the previous paper. The short capillary part of the vacuum tube was mounted between flat pole pieces, the field being essentially homogeneous in the region of the capillary. The tube was observed "end-on," i.e., in a direction perpendicular to the lines of force. A natural crystal of Iceland spar about 1.5 cm thick was used as a double-image polarizing prism. This was placed close to the capillary, and, when properly oriented, gave two polarized images very close together, the one immediately above the other. Real images of

[1] *Annalen der Physik*, **39**, 897, 1912.

these were formed on the first slit of the Hilger constant-deviation spectroscope by means of a photographic objective of high quality. These images were about a millimeter in diameter, as we placed the lens much nearer the tube than the slit, and were separated by a distance of about 3 mm. Both polarized components could thus be photographed with the echelon simultaneously, and by raising the images about 1.5 mm on the slit we could obtain two more records showing the unmagnetized lines in coincidence with the magnetized ones. Various methods of bringing about this vertical shift were tried, but all were found unsatisfactory until the following simple expedient was adopted. A piece of plane-parallel glass (Michelson interferometer flat) was mounted a short distance from the slit of the Hilger spectroscope and arranged to rotate on a horizontal axis, by which it was possible to incline the plate from the vertical at the angle necessary to produce the requisite shift. A graduated paper scale and a light lever attached with sealing-wax, to give the desired rotation, completed the apparatus. The advantage of the plate is that it shifts the two converging beams *without changing their direction.* Acute prisms, and other devices which we tried, changed the direction of the beams, reducing or destroying entirely the illumination of the echelon. In taking our plates we recorded the current flowing in the magnet in each case, and subsequently determined our fields by comparing two deflections of a ballistic galvanometer, produced respectively by the quick removal of a small exploring coil from between the pole pieces and from the center of a standard solenoid giving a known field. This method could not be used for the stronger fields (above 1000 gauss), as the field in the coil was only 860 gauss with a current of 18 amperes. The stronger fields were measured by observing the Zeeman-effect on the green helium line 5016 (which is known to exhibit the normal effect) from the formula

$$H=\frac{\Delta\lambda}{0.94\,\lambda^2}10^4,$$

in which the wave-length is expressed in centimeters, and $\Delta\lambda$ is the separation of the outer components of the Zeeman triplet in a field of H gauss.

The following lines were studied: λλ 5464, 5161, 4632, these having five components each, in the form of a series of decreasing intensity and spacing; λλ 5338 and 5345, each a close triplet; 5691, a doublet having one strong and one weak component; and 5624, a single line.

Photographs of many of the other lines were made, but only those mentioned above were measured. The resolving power of the echelon was not quite sufficient (20 plates of 10 mm each), and as a future investigation with a 40-plate echelon is contemplated by one of us, the results given in the present paper are to be regarded as preliminary in their nature.

λ 5691

We shall begin with a discussion of the action of the magnetic field on this line, as its structure is very simple, a strong line with a fainter companion 0.078 A on the side of longer wave-lengths. The behavior of the perpendicular and parallel components of polarization in fields of increasing strength is shown on Plate XV, *j*. Shorter wave-lengths are to the right in these figures.

In the case of the parallel component the main line remained undisplaced, as in the normal Zeeman-effect up to a field of about 2000 gauss, but its intensity increased and it showed a distinct broadening. The satellite, however, approached it and increased in intensity until, at about 2500 gauss, the intensities appeared about equal and the lines almost fused. At 3220 they had completely united into a single line with a wave-length intermediate between that of the main line and the satellite, with a faint companion on the side of short wave-lengths; with increasing field the bright line suffered a further displacement toward the red, the faint companion moving toward the violet and disappearing in fields above 5000. Over a dozen plates in all were made and measured, and the results are shown in the form of a graph in Fig. 1.

In the case of the perpendicular component, the main line was doubled, the separation being normal up to a field of about 2000 gauss, the satellite remaining unaffected. With an increase of field the satellite was deflected as if pushed along by the positive

branch of the doublet, the two finally fusing as shown in Fig. 1. As will be seen from the diagram, the resulting doublet continues to widen and at 9000 gauss is symmetrical with respect to a point midway between the main line and the satellite. This phenomenon of the fusing of the satellite with the main line in the case of the ∥ component, and with one of the branches of the doublet in the case of the ⊥ component, was observed also in the case of other iodine lines, and is in agreement with observations made by Nagaoka and Takamine on the lines of other elements. In strong

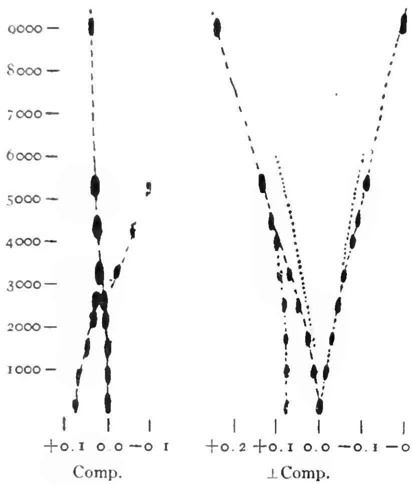

FIG. 1.—λ5691

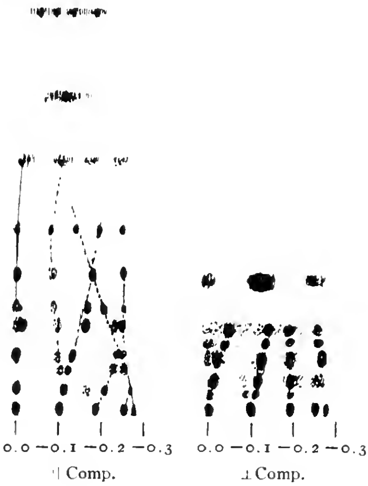

FIG. 2.—λ5464

fields, and in the absence of the polarizing apparatus, we have a triplet, with its central component displaced (from the position originally occupied by the main line for zero field) toward the red. The dotted lines in Fig. 1 (⊥ component) indicate the calculated separation for the normal Zeeman-effect.

λ 5464.77

This line consists of five components, decreasing in intensity and separation toward the side of short wave-length, suggesting a miniature Balmer series. The wave-length of the main line was

determined by the grating, and the components are located at −0.106, −0.190, −0.255, and −0.275, the latter being very faint and not appearing resolved in the reproduction. The appearance of the composite line is shown on Plate XV, *k*. For each field-strength the parallel and perpendicular components of polarization are shown in coincidence with the line in zero field.

We will consider first the behavior of the components of vibration parallel to the field. Notwithstanding the fact that the parallel component is unaffected by the field in the normal Zeeman-effect, in the case of this complex line, the three members of shorter wave-length are so sensitive that even the residual magnetism of the electromagnet, after the current was shut off, was sufficient to alter the appearance of the series in a very marked manner, and this for the *parallel* component, ordinarily uninfluenced. On this account it was necessary to demagnetize the magnet very carefully, by repeatedly reversing and diminishing the current, until no appreciable attractive force was exerted by the poles for a piece of soft iron. It was also important not to pass from a strong to a weak field without demagnetization.

The behavior of the lines, when only the parallel components of vibration are recorded, is shown by Fig. 2. In a field of only 150 gauss a very distinct effect was observed on the two lines of shortest wave-length (−0.255 and −0.275), and at 500 gauss they fused into a single line. The line at −0.190 first widens, and at 600 gauss becomes double. We now have five lines, as in the beginning, though with a different spacing and distribution of intensity. If this were an isolated observation, one might erroneously conclude that the magnetic field had merely pushed the lines closer together. As the field increased in strength the negative branch approached, and finally fused with, the line formed by the fusion of the two referred to above, which appears to move slightly in the positive direction to meet the other line. The other branch (positive) formed by the division of −0.190 could not be followed beyond 600 gauss. The further behavior of these lines with increasing field is well indicated by the figure. The line formed by the fusion of the three lines just referred to divides again, one member increasing its wave-length, the other remaining fixed. The component

at -0.106 divides as indicated, the negative branch, which is strongly displaced, fusing with the positive branch mentioned above. The positive branch, which is much fainter, remains almost in coincidence with the original line. These changes can be followed on Plate XV, *k* (upper figures marked ||). Above 3000 gauss it is difficult to interpret the plates, as the components become hazy. At 3400 gauss we have four lines, and at 4400 gauss five lines again, the probable manner of transition being indicated in Fig. 2. At 6000 gauss we again have but four lines (hazy). At 7500 we found a continuous background, with a hazy line in the center, but we were unable to trace out the transition.

The perpendicular component of polarization is affected in a very different manner. The third line of the series remains undisplaced in fields below 2000 gauss, the first and second lines giving strongly displaced negative branches and very faint positive branches, which show little displacement and are difficult to follow. At 3200 there is a strong central component and two lateral fainter components, but we have not been able to determine from the plates just how the transition takes place, nor have we followed the development of the hazy doublet which appears in very strong fields.

λ 4632

The behavior of this line was studied in fields up to 4000 gauss and is shown graphically in Fig. 3. Normally it is a five-line series similar to λ 5464, but it is affected in a different manner. For the parallel component line No. 1 broadens and shows a suggestion of doubling. No. 2 doubles, the positive branch disappearing in fields above 2000, the negative remaining up to over 3000. No. 3 is unaffected up to 1000 gauss, then deviates rapidly in a negative direction. No. 4 is also displaced in the same direction. At 3900 gauss we have a doublet. For the perpendicular component of polarization, lines Nos. 1 and 3 double and 2 and 4 remain unaffected. The resolving power of the echelon was insufficient to accomplish more than a suggestion of this doubling, however, in the widened lines. At 4900 we have a triplet with a strong middle component. The behavior of the line is shown on Plate XV, *l*.

λ 5161

The parallel component only was studied in the case of this line, which is a five-line series similar to λ 4632 and λ 5464. Its behavior is shown by Fig. 4. The first line of the series exhibited only a slight widening and slight displacement toward the violet as the field-strength increased. The second line behaved in a remarkable manner. It became distinctly double in a field of 1000 gauss, and at 1820 the two components were widely separated. The doubling was not symmetrical, however, for the positive branch attained its maximum displacement at 1820, while the negative

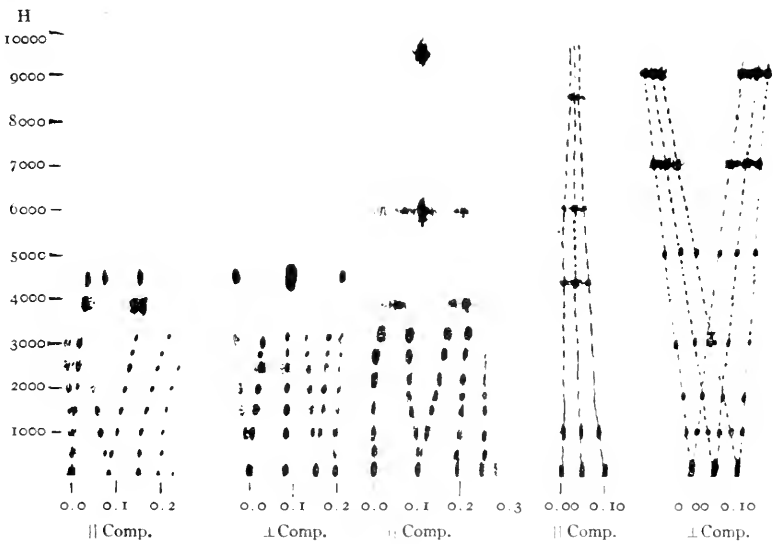

Fig. 3.—λ 4632 Fig. 4.—λ 5161 Fig. 5.—λ 5345

branch continued to move toward the violet with increasing field. The positive branch eventually fuses with line No. 1, which moves over to meet it (4000 gauss). The displacement of the negative branch was proportional to the field-strength up to 3500 gauss. Line No. 3 was very slightly displaced toward the violet with increasing field and fused with the negative branch of No. 2 at 5000 gauss. Lines Nos. 4 and 5 fused at 500 gauss and faded away above 3000 gauss. At 6000 gauss we have a broad hazy line in the position of line No. 2, and at 9700 gauss it is found slightly

displaced from this position toward the violet. These changes are shown on Plate XV, *m* and *n*, the latter figure showing the group in ten different stages, from zero field up to 9700 gauss, with the photographs mounted in coincidence.

λ 5338

In a weak field the main line of this triplet was decomposed into a triplet with normal separation and polarization. The satellite at −0.041 was also decomposed into a triplet, but the component lying on the side toward the main line suffered a greater displacement than the one lying on the other side of the central component. The same was true of the satellite at −0.083 except that the dissymmetry was even greater.

In strong fields we have a diffuse triplet which forms in the manner indicated by Fig. 5, which is, however, the graph for the similar line λ 5345.

λ 5345

The behavior of this line is similar to that of λ 5338, and the measurements made from the plates are recorded in Fig. 5.

λ 5625

This is a single line and gave a symmetrical triplet, with a separation somewhat greater than that of a normal triplet.

$$\text{For normal triplet } \frac{\Delta\lambda}{\lambda^2 H} = 0.94 \times 10^{-4}.$$

$$\text{For } \lambda\ 5625, \qquad = 1.26 \times 10^{-4},$$

that is, in the ratio 3:4.

The chief points of interest which have been brought out in this investigation may be summed up as follows:

The complex lines having the form of a series with decreasing intensity and separation are not all affected in a similar manner by the magnetic field.

In the case of any given complex line the components are affected to very different degrees. Certain components may not be affected at all, while others break up into doublets, the components of which

sometimes fuse with neighboring components and sometimes fade gradually away as the field-strength increases.

In the case of the perpendicular components of polarization we have not traced the development of the widely separated hazy doublet which appears with strong fields from the complex which develops in weak fields. This will require a somewhat higher resolving power than that available in the present work. Obviously the method of the non-homogeneous field would be especially adapted to the study of these complex lines, as the transition could then be traced by very gradual steps. We made some experiments along these lines, with flattened capillary tubes and pointed poles, but the results were not very satisfactory. With the tubes used in the latter part of the work, with internal electrodes, described in the previous paper, it seems probable that excellent results can be obtained in this way.

Johns Hopkins University
July 8, 1917

PHOTOGRAPHIC EFFECTIVE WAVE-LENGTHS OF SOME SPIRAL NEBULAE AND GLOBULAR CLUSTERS

By KNUT LUNDMARK AND BERTIL LINDBLAD

The method of determining color or spectral type based on the measurement of the photographic effective wave-length in grating spectra, especially developed by E. Hertzsprung[1] and by A. Bergstrand,[2] has, as far as we know, until now been applied only to point-shaped objects, fixed stars, and planetary satellites. There is, however, nothing to prevent the use of this method for celestial objects showing a clear surface, provided it is small enough, so that the spectra of the first order and the central image do not begin to overlap. If the time of exposure is sufficiently short, spectra of the first order become almost congruent with the central image, while they elongate when the intensity of image increases. The effective wave-length is calculated by measuring the distance between the two spectra, in doing which we follow the practice of bisecting the blackened surface.

In this investigation we have turned our principal attention to the spiral nebula. As the derivation in the usual way of the color-index of these objects might be beset with considerable difficulties, it seems as if the method of effective wave-length could in this case be of comparatively great importance. As a matter of course, different objects require different instruments. The images should not be very large and diffuse; in this investigation they do not, in some cases, differ very much from images of fixed stars, while some show a considerable surface, as, for instance, M 5 and M 82.

[1] "Sur la dispersion atmosphérique," *Bulletin astronomique*, **25**, 5, 1908; "Über die Verwendung photographischer effectiver Wellenlüngen zur Bestimmung von Farbenäquivalenten," *Potsdam Publications*, **22**, No. 1, 1911; further, several articles in the *Astrophysical Journal*.

[2] "Über die Abhängigkeit der atmosphärischen Dispersionswirkungen von den Sterngrössen," *Astronomische Nachrichten*, **177**, 241, 1908; "Recherches sur les couleurs des étoiles fixes," *Nova Acta R. Soc. Scient. Upsala* (IV), **2**, No. 4, 1909; also in *Comptes rendus*, **148**, 1079, 1909.

The instrument employed was a twin 6-inch astrographic telescope at the Upsala Observatory with Zeiss triplet lenses (aperture 15 cm, focal length 150 cm). A wire grating (the grating constant $c=1.3422$ mm) was placed, with the wires in the direction of right ascension, in front of one objective (No. I); with the other objective a plate was taken without grating. The plates used were Imperial S. S. Emulsion 9220 A, Speed No. (H.O.D.) 275. The measurements were made by both of us with the Repsold measuring machine.[1]

The applicability of this telescope for the determination of effective wave-lengths has been examined by Bergstrand and Lindblad,[2] and the result of the trial was upon the whole encouraging. Some doubt remained as to the fitness of the instrument for very faint images. Since then the objectives have been altered a little, through corrections of the position of their middle lenses, in order to produce the best possible images over as large a part of the fields as possible. It appears that this change has been of advantage also in the applicability of the instrument to the determination of effective wave-lengths. As the matter in question here is very faint images, the so-called photographic *Purkinje-effect*, i.e., the dependence of the effective wave-length on the intensity of image, will be of great importance. For the refractors, the achromatization here plays a large part. By the focusing here employed the wave-lengths are united at 400 $\mu\mu$ and 443 $\mu\mu$, while the mean effective wave-length is at about 424 $\mu\mu$, thus fairly central. The spectrum obtained from a star with short exposure is at first almost rectangular, but with longer exposure it gets more and more elliptical, sometimes more irregularly oval in shape. The effect in question shows itself as a general tendency to an increase of the effective wave-length, with the intensity of image somewhat stronger for yellow stars than for white ones. In order to control the effect for a celestial object with a clear surface, we

[1] Bergstrand, "Recherches sur les couleurs des étoiles fixes," *Nova Acta R. Soc. Scient. Upsala* (IV), 2, 10, 1909.

[2] "Om bestämmingen af de fotografiskt effektiva våglängderna i fixstjärnspectra," *Arkiv för Matematik, Astronomi och Fysik utgifvet af K. Svenska Vetenskapsakademien*, 11, No. 17, 1916.

have photographed Saturn with different exposures and have measured the effective wave-lengths. As appears from Table I, the rate in the λ values is at first quite imperceptible; but at the exposure time of 32^s a tolerably strong increase sets in. The

TABLE I

SATURN: PLATE TAKEN APRIL 25, 1917
SIDEREAL TIME $12^h14^m-12^h24^m$

Exposure Time	Observer		Difference Lk.−Ld.
	Lundmark	Lindblad	
	$\mu\mu$	$\mu\mu$	$\mu\mu$
$\frac{1}{3}^s$	432.5	432.2	+0.3
1	32.7	28.5	+4.2
1	33.0	33.7	−0.7
1	33.0	34.7	−1.7
3	32.7	31.9	+0.8
3	34.3	34.7	−0.4
10	32.5	31.2	+1.3
10	32.3	32.8	−0.5
32	39.6	39.0	+0.6
32	38.5	39.9	−1.4
100	43.8	43.8	0.0
100	443.3	443.8	−0.5

MEANS

Exposure time	λ_{eff}
$\frac{1}{3}^s$	432.3
1	32.6
3	33.4
10	32.2
32	39.2
100	443.7

intensities of images which occur in connection with other objects included in this investigation are all considerably weaker than this critical intensity of image. Otherwise the two series of measurements show no perceptible personal equation for the two observers. $\frac{\Delta\lambda_{eff}}{\Delta_{atm}}$ has been assumed to be 2 $\mu\mu$, in accordance with a determination made by Lindblad, using stars of different types of spectrum.

In his work already referred to, Hertzsprung[1] has assumed $\frac{\Delta\lambda_{\text{eff}}}{\Delta_{\text{atm}}}$ to be $3.5\,\mu\mu$, valid for white stars (the Pleiades). In a footnote he remarks that this correction seems to be smaller for yellow stars than for white ones.

The grating-constant was determined from micrometric measurements as $c = 1.3422$ mm, and from the measured distance of η and b of the Pleiades was obtained with the use of W. Elkin's[2] measurements $f = 1485.5$ mm.

TABLE II

Date	Sidereal time	Exposure	Object	Observers	Remarks
1917 Feb. 16..	7^h53^m3—10^h25^m7	$2^h\ 9^m$	M81, M82, N.G.C. 3077	*Lk., Ld.	Cloudy
22..	6 18.8—10 54.8	4 34	Same object	Lk.	Clouds at the end of exposure
Mar. 15..	11 56.9—13 41.9	1 45	M 94	Lk.	From the beginning of the exposure very fine sky; clouds at the end
22..	10 39.7—12 9.7	1 30	M 3	Lk., Ld.	....
27..	11 15.6—14 15.6	3 0	M 51, W.H.I. 186	Lk., Ld.	Sky very good
April 19..	12 16.8—16 31.0	{1 40, 1 40}	M 64, W.H.I. 84	Lk., Ld.	....
25..	13 22.6—15 0.6	1 30	M 57	Ld.	....
May 10..	14 10.5—16 32.0	2 21	M 53	Lk., Ld.	....
11..	14 36.5—16 36.5	2 0	M 5	Lk.	....
17..	14 55.0—16 55.0	2 0	M 94	Lk., Ld.	Some haze

*Lk. = Lundmark; Ld. = Lindblad.

If s is the distance in mm between two spectra of the first order, we can put in this case with sufficient accuracy:[3]

$$\lambda_{\text{eff}} = \frac{c}{2} \cdot \frac{s}{f}$$

or

$$\lambda_{\text{eff}} = 451.74\,\mu\mu \cdot s$$

Before proceeding to a statement of the measurements and their results, we give (in Table II) a summary of the plates we have had at our disposal for this investigation.

[1] "Über die Verwendung photographischer eff. Wellenlängen, etc.," *Astronomische Nachrichten*, **177**, pp. 24–25, 1908.

[2] "Revision of the First Yale Triangulation of the Principal Stars in the Group of the Pleiades," *Trans. of the Obs. of Yale University*, **1**, 1887–1904.

[3] Bergstrand, "Recherches," etc., *Nova Acta R. Soc. Scient. Upsala* (IV), **2**, 17–18, 1909.

As the aim of the present investigation is in the first place to find out whether the connection between the effective wave-lengths and the spectral type of the stars found by Bergstrand[1] and by Hertzsprung[2] also exists between the corresponding elements for spiral nebulae and globular clusters, the objects examined have with preference been chosen among those whose type of spectrum is known.

From plates taken for a more extensive work, for which he has made preparations and on which he is engaged, Lindblad has found, when only the faintest measurable images are taken into consideration, that the following relation between the effective wave-lengths and the type of spectrum of the stars is obtained for the instrument employed by us:

A	415 $\mu\mu$
F	420 $\mu\mu$
G	427 $\mu\mu$
K	433 $\mu\mu$

These measurements show that for the Upsala 6-inch astrographic the spectral interval A to K (about $1^{M}.0$ in color-index) corresponds to 18 $\mu\mu$ in effective wave-length. It is evident that the instrument is especially appropriate for the determination of effective wave-lengths from the fact that, for the great Meudon reflector[3] and the 60-inch reflector[4] of Mount Wilson Solar Observatory, both of which instruments are especially adapted for the determinations in question, the corresponding interval is, respectively, 21 $\mu\mu$ and 24 $\mu\mu$.

The measurements have been corrected for the effect of the selective extinction in our atmosphere, whereby, owing to the different qualities of the material, we shall here treat separately the objects investigated.

[1] "Recherches sur les couleurs des étoiles fixes," *Nova Acta R. Soc. Scient. Upsala* (IV), **2**, 4, 36, 1909.

[2] "Effective Wave-Lengths of 184 Stars in the Cluster N.G.C. 1647," *Mt. Wilson Contr.*, No. 100, 1915; *Astrophysical Journal*, **42**, 92, 1915.

[3] Bergstrand, "Recherches," *Nova Acta R. Soc. Scient. Upsala* (IV), **2**, 36, 1909.

[4] Hertzsprung, "Effective Wave-Lengths of 184 Stars," etc., *Mt. Wilson Contr.*, No. 100, pp. 3, 19.

M81 = N.G.C. 3031

By accident the images had been doubled on the longer-exposed of the two plates, containing this and the two following nebulae. Thanks to the circumstance that the displacement had taken place nearly at right angles to the direction of the length of grating spectra, the two images, owing to their closeness to each other, could be measured as a whole. With the aid of the pairs of horizontal wires in the measuring microscope it was possible to measure only the stronger of the two images. The measurements, which in the two cases agreed well, gave $\lambda_{\text{eff}} = 435.7\ \mu\mu$. The plate of February 16 showed fairly faint images, which, however, could without any difficulty be measured, and from which was obtained $\lambda_{\text{eff}} = 435.8\ \mu\mu$. E. A. Fath[1] has found that the total spectrum of this nebula exactly resembles the spectrum of a K-type star. *When we assume that the relation between the* λ_{eff} *and the spectral class of the stars, derived by Lindblad, also can be employed for the objects included in this investigation*, we find that the λ_{eff} for M81 determined by us also corresponds to the spectral class K.

M82 = N.G.C. 3034

This was measurable only on the plate having the longest exposure. In this case the two images could not be distinguished, owing to the extension of the object. It was possible to measure only a bright nucleus close to the center. This nucleus occurs to the right of the center of the nebula on Isaac Roberts' photograph.[2]

Our measurements, which nevertheless were rather uncertain, gave $\lambda_{\text{eff}} = 415\ \mu\mu$, which wave-length should correspond to the spectral type A. It is to be hoped that a long exposure will show whether any difference in effective wave-length or spectral type, respectively, exists between the central and the exterior parts of the nebulae. As far as we know, no determinations of the spectral class of this nebula exist.

[1] First Paper: "The Spectra of Some Spiral Nebulae and Globular Star Clusters," *Lick Observatory Bulletin*, No. 149, 1909; Second Paper: "The Spectra of Spiral Nebulae and Globular Star Clusters," *Mt. Wilson Contr.*, No. 49, 1911; *Astrophysical Journal* **33**, 58, 1911; Third Paper: *Mt. Wilson Contr.*, No. 67, 1913; *Astrophysical Journal* **37**, 198, 1913.

[2] *Celestial Photographs*, **1**, 1893, Plate 25.

W.H. I.286=N.G.C. 3077

The λ_{eff} was determined from the plate that was taken on February 22 and is probably very well established, although the images of the grating-spectra were rather faint. A mean value of $\lambda_{eff}=428.6\ \mu\mu$ was obtained. The spectrum of this nebula is unknown. The effective wave-length obtained corresponds to the spectral class G.

M 94=N.G.C. 4736

As this nebula has small extent on our plates (1 mm corresponds to 2'.28), and as its light is strongly concentrated in the more central parts,[1] the effective wave-length could be measured with very great accuracy. If in the measurements only the starlike nucleus (the central condensation) was taken into consideration, a considerably greater value for the effective wave-length was obtained than if regard was paid to the image of the whole nebula, which on the grating-plate measured in diameter about 30″ and consequently contained at least a part of the spiral branches lying very close to the central nucleus. From several sets of measurements, showing a good mutual agreement, there was obtained for the nebula in its entirety $\lambda_{eff}=426.7\ \mu\mu$ and for the nucleus $\lambda_{eff}=432.9\ \mu\mu$. Fath's[2] three spectrograms do not agree with one another, yet it may be gathered from his later investigations (Mount Wilson plates) that the spectrum of the nebula must belong to the G-type. The effective wave-length for the nebula in its entirety obtained by us corresponds exactly to the same spectral type. F. H. Seares[3] has recently shown that the central nucleus contains more yellow light than the spiral branches and their condensations, which are intensely blue; this is in good accordance with the fact that the nucleus has a greater effective wave-length, and belongs to a more advanced spectral class, respectively, than the nebula in its entirety.

[1] J. E. Keeler, "Photographs of Nebulae and Clusters," *Publications of the Lick Observatory*, **8**, 1908.

[2] *Op. cit.*, First, Second, and Third Papers.

[3] "Preliminary Results on the Color of Nebulae," *Communications from the Mount Wilson Solar Observatory to the National Academy of Sciences*, No. 36, 1916; see also *Annual Report of the Director of the Mount Wilson Observatory for the Year 1916*, pp. 248–49.

The plate of this object obtained on May 17 was so strongly blackened and consequently the grating-spectra were so difficult to discern that we have thought we ought not to use it, being so decidedly inferior to the preceding plate, for an accurate determination of the effective wave-length. Measurements made by us on this plate give an effective wave-length, corresponding to the mean of the two given, which confirms Fath's result that the spectrum is of the solar type.

W.H. I.84 = N.G.C. 4725

The plate, including this and the following object, was taken for purposes of stellar statistics; hence the grating-spectra of the two nebulae are not sufficiently exposed. The measurements are rather discordant, yet they deserve to have some weight. We obtained on an average $\lambda_{eff} = 430.4\,\mu\mu$. According to Fath,[1] the spectrum is of solar type (Go). Our λ_{eff} value corresponds to a spectral type intermediate between G and K.

M 64 = N.G.C. 4826

The measurements of the grating-spectra of this nebula show a greater mutual agreement than those of the spectra of the preceding objects, and $\lambda_{eff} = 433.8\,\mu\mu$ was obtained. Fath[2] finds that the color-curve is similar to that of a solar-type star, but his plate contains a very weak spectrogram. Max Wolf[3] has found that the spectrum is of pronounced G-type (identical with that of the Andromeda nebula). To what extent the contradiction between these investigations and our determination of λ_{eff}, corresponding to the K-type, can be explained from the vagueness which impairs our measurements, we hope, later on, when we have had the opportunity of photographing this object with longer exposure than the one used, to be able to decide.

[1] "The Spectra of Spiral Nebulae" (Second Paper), *Mt. Wilson Contr.*, No. 49, 1911; *Astrophysical Journal*, **33**, 58, 1911.

[2] *Ibid.* (Third Paper), *Mt. Wilson Contr.*, No. 67, 1913; *Astrophysical Journal*, **37**, 198, 1913.

[3] "Über die Spektren einiger Spiralnebel," *Sitzungsberichte der Heidelberger Akad. d. Wiss.*, Abth. A. Jahrg. 1912, 15 Abhandlung.

M3 = N.G.C. 5272

A determination of the total effective wave-length was not possible for this object, owing to the fact that its considerable extent prevented the central image and the grating-spectra from being quite clearly separated. Besides, the images dissolved into a number of condensations. One of the strongest of these was near the center of the cluster. As grating-spectra of this condensation could be identified, we measured its effective wave-length and obtained $\lambda_{\text{eff}}=425.1\ \mu\mu$. A number of small condensations and accumulations of stars were also measured in this object and gave effective wave-lengths between 424 and 436 $\mu\mu$. As, however, there was some doubt as to the identification of the grating-spectra of these condensations, we will not attach any importance to these later measurements. Fath[1] has found that the integrated spectrum of M3 lies between A and G, and Hertzsprung[2] has by spectral-photometric measurements shown that the distribution of energy in its integrated spectrum is very nearly like that of a typical F-star. The value found by us for the effective wave-length for the strongest condensation corresponds more to the G than to the F type. Judging from several circumstances, we find that the λ_{eff} should, however, for the cluster in its entirety turn out somewhat lower.

M 51 = N.G.C. 5194

The grating-spectra were fairly well exposed, yet there arose some difficulty at the measuring, because one of them was close to the central image of a condensation in the spiral branches, from which, however, it could be distinguished after a certain amount of practice. Our measurements are probably accurate, giving an average $\lambda_{\text{eff}}=430.7\ \mu\mu$. It was only with the greatest difficulty that Fath[3] determined the spectral type for this nebula. A plate exposed for nearly 30 hours gave a spectrum whose color-curve was similar to that of a K-type star and with the lines G and H present.

[1] "The Spectra of Spiral Nebulae" (Second Paper), *Mt. Wilson Contr.*, No. 49, 1911; *Astrophysical Journal*, **33**, 58, 1911.

[2] "Comparison between the Distribution of Energy in the Spectrum of the Integrated Light of the Globular Cluster Messier 3 and of Neighboring Stars," *Astrophysical Journal*, **41**, 10, 1915.

[3] *Op. cit.*, First and Third Papers.

Wolf,[1] on the contrary, obtained on a plate exposed for 31 hours a spectrum on the whole identical with that of the Andromeda nebula or of the type G. Our value for the λ_{eff} corresponds to a spectral type intermediate between G and K. A star involved in the nebula[2] ($\alpha = 13^h 25^m\!.6$, $\delta = 47°42'$) was measured and gave $\lambda_{\text{eff}} = 418.0\,\mu\mu$ (type A5).

W.H. I.186 = N.G.C. 5195

This object, which forms a secondary nucleus in M 51, situated at the end of one of its spiral branches, showed on our plate a starlike central image with quite distinct grating-spectra, which enabled us to determine the effective wave-length with great accuracy. We obtained $\lambda_{\text{eff}} = 426.3\,\mu\mu$. The spectrum of this object has not been examined. With the aid of a yellow-color filter and isochromatic plates Seares[3] has examined the color of M 51 and N.G.C. 5195 and has found that the central parts of the two objects are much stronger in yellow light than in blue, and that they consequently belong to a more advanced spectral type than the exterior parts which are intensely blue.

M 57 = N.G.C. 6720

The grating-spectra are vague and rather difficult to measure, owing to the considerable extent of the object. The measurements gave $\lambda_{\text{eff}} = 394\,\mu\mu$. This result implies that our grating-spectra are in the main formed by the group of lines λ 397, λ 389, and λ 387, of which especially the first and the third have great intensity.[4] The central star in the ring nebula gave a considerably lower effective wave-length,[5] but, as its grating spectra could not be

[1] *Op. cit.*, p. 6.

[2] See G. W. Ritchey, "On Some Methods and Results in Direct Photography with the 60-inch Reflecting Telescope of the Mount Wilson Solar Observatory," *Mt. Wilson Contr.*, No. 47; *Astrophysical Journal*, **32**, 26, 1910, Plate XV, where the star is at a distance of 22.5 mm from the nucleus.

[3] *Loc. cit.*

[4] See, on this subject, Max Wolf, "Der Ringnebel und der Dumbbell nebel," *Sitzungsber. d. Heidelberger Akad. d. Wissenschaft*, Abth. A, Jahrg. 1915, 1 Abhandlung.

[5] See *Annual Report of the Director of the Mount Wilson Solar Observatory for the Year 1916*, p. 249.

identified with perfect certainty, we will not give here the results of those measurements.

M 53 = N.G.C. 5024

The plate was under-exposed. The extremely vague measurements, for which a slightly magnifying ocular was used on the measuring microscope, gave a mean $\lambda_{\text{eff}} = 420\ \mu\mu$, which, in our opinion, ought not to be included in the summary below. Fath[1] has found that this cluster gives a spectrum lying between A and G.

M 5 = N.G.C. 5904

In the measurements we used the weaker ocular above mentioned. The grating-spectra were distinctly separated from the central image and proved to consist of a smooth, blackened surface, measurable with some difficulty. The value obtained here for the effective wave-length is valid for the bulk of the stars of which grating-spectra occur on our plate. We obtained $\lambda_{\text{eff}} = 419\ \mu\mu$. Fath[2] has found the following absorption lines present in the spectrum of M5: F, H_j, H, K, and a band at λ 419. He is of the opinion that the integrated spectrum of the brighter stars of this cluster lies between F and G.

We have collected the results of our measurements in Table III. Under the heading of "Spectrum Calculated" we have entered the spectral class calculated from the λ_{eff} with the aid of the following formula, derived from Lindblad's measurements mentioned above:

$$\text{Spectral class} = A_0 + \frac{\lambda_{\text{eff}} - 415}{6}.$$

From these preliminary investigations, which we hope later on to have the opportunity of completing and expanding, it appears *that the connection, found for fixed stars, between spectral type and effective wave-length also exists between the (integrated) spectrum and the effective wave-length of the spiral nebulae and globular clusters.* By using the method suggested by us, it consequently ought to be possible, in proportion as the greater instruments are used for the purpose, to determine with tolerable facility the effective wave-length and thereby the spectral type or color-index or temperature

[1] *Op. cit.*, Second Paper. [2] *Op. cit.*, Third Paper.

TABLE III

Object	α_{1900}	δ_{1900}	Galactic Co-ordinates: Long.	Galactic Co-ordinates: Lat.	Magnitude (Holetschek)	Effective Wave-Length: According to Lundmark	Effective Wave-Length: According to Lindblad	Effective Wave-Length: Means	Spectrum Calculated	Spectrum Observed	Remarks
						$\mu\mu$	$\mu\mu$	$\mu\mu$			
N.G.C. 6720 M 57	18h 49m.8	+32° 54′	30°	+14°	8.9	394	394	394	......	......	Ring nebula in Lyra
3034 M 82	9 47.6	70 10	109	42	8.8	414	416	415	A_0	......	Gaseous nebula?
5904 M 5	15 13.5	2 27	333	45	6.7	418.7	419.6	419.1	F	F–G	Globular cluster
5024 M 53	13 8.0	18 42	307	79	7.8	42	......	......	F?	F–G	Globular cluster
5272 M 3	13 37.6	28 53	8	77	6.6	425.3	425.0	425.1	F5	F	Globular cluster
5195 W.H. I.186	13 25.8	47 47	68	71	8.6	426.3	426.3	426.3	G	......	......
4736 M 94	12 46.2	41 40	76	76	7.7	427.3	426.1	426.7	G	G	Spiral nebula. This λ_{eff}-value concerns the nebula in its entirety
						433.0	432.8	432.9	K	......	This λ_{eff}-value concerns the central nucleus
3077 W.H. I.286	9 55.3	69 13	108	42	10.2	427.4	429.7	428.6	G	......	Spiral nebula?
4725 W.H. I. 84	12 45.5	26 3	154	83	8.7	432.0	428.7	430.4	G5	G	Spiral nebula
5194 M 51	13 25.7	47 43	68	71	8.4	431.7	429.7	430.7	G5	G, K	Spiral nebula
4826 M 64	12 51.8	22 13	295	84	8.6	433.8	433.9	433.8	K	G	Spiral nebula
N.G.C. 3031 M 81	9 47.3	+69 32	109	+42	8.0	435.7	435.6	435.7	K	K	Spiral nebula. λ_{eff} from plate taken Feb. 22, 1917
						436.3	435.4	435.8			Spiral nebula. λ_{eff} from plate taken Feb. 16, 1917

also for a considerable number of small, faint nebulae and globular clusters, which are not accessible for investigations of their spectrum by our present spectral apparatus.

If we assume that the parallax of the spiral nebulae and globular clusters examined lies between the same limits as those derived by H. Shapley[1] for M 13:

$$0''00010 > \pi_{M13} > 0''00001,$$

we find that, provided the value of c derived by P. J. van Rhijn[2] is right, the change in color-index T, caused by the selective absorption in space, must lie between the limits

$$2^{M} < \Delta T < 19^{M}$$

Hence it follows that the spectral type calculated by us should on an average differ from those determined in the usual way, where the spectral lines have been observed, by an interval at least twice as large as A–K. This not being the case, it seems to us that our investigation can be considered as a confirmation of the result found by Shapley[3], Hertzsprung[4] and others, *that no sensible absorption exists in space.*

ASTRONOMICAL OBSERVATORY,
UPSALA
June 1917

[1] "Studies Based on the Colors and Magnitudes in Stellar Clusters," *Mt. Wilson Contr.*, Nos. 115–117, 1915–1916.

[2] "Derivation of the Change of Color with Distance and Apparent Magnitude," Dissertation, Groningen, 1915.

[3] *Loc. cit.*

[4] *Loc. cit.*

A NEW FORM OF SPECTRO-COMPARATOR

By RALPH E. DE LURY

By employing a half-silvered surface and two microscopes, it is possible for one to construct a comparator possessing special advantages in the measurements and comparisons of photographs of spectra. The device may be arranged in several ways:

I. The microscopes may be set up with their axes intersecting on either side of the objectives and with the half-silvered surface at the intersection.

II. The microscopes may be set up with the axes of the objectives parallel. One beam of light, after passing through its objective, is turned by reflection through one or two right angles to meet, at the half-silvered surface, the direct or similarly reflected beam from the other objective.

The half-silvered surface transmits and reflects in either case about half of each beam; and the images, side by side or overlapping, may be observed in two directions at right angles to one another.

By employing the arrangement I in such form as shown in the diagram, it is possible to produce, with a minimum of optical surfaces, two sets of images in convenient positions for alternate measurement, the configurations appearing rotated 180 degrees with respect to one another. In the case of spectra, one eyepiece would show "violet right" and the other "violet left." Furthermore, in order to compare personal errors of measurement, two observers could conveniently measure together, one at each eyepiece, viewing each other's settings or making alternate settings. (Such a method of comparison between two observers should prove valuable in such observations as the transits of stars, occultations, etc.) To facilitate such comparisons in the measurements of spectra, the comparator should be made to rotate and should be set up on a narrow table with a seat on each side. To avoid the interference of reflections from the outside surfaces, the silvered surface should lie between two wedges or rectangular prisms of glass.

For arrangement II the Hartmann spectro-comparator[1] may be easily adapted by removing the usual silver mask between the components of the double-prism placed below the eyepiece, half-silvering the face of one of the prisms, and cementing the prisms together again with Canada balsam. With this device the Hartmann comparator possesses all the advantages it ordinarily has, and in addition the following:

1. One double-prism suffices for all comparisons. In the Hartmann instrument various kinds of masks are made between

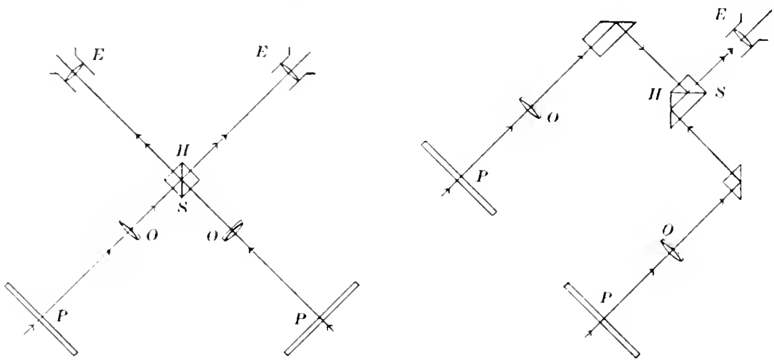

FIG. 1

I. New Form of Spectro-comparator II. Adapted Hartmann Spectro-comparator

E, eyepiece; *HS*, half-silvered surface; *O*, objectives; *P*, photographic plates

the components of the double-prisms, to make possible the comparison of spectra of different configurations; and it is also necessary to focus both images at this mask in order to have the edges of the latter sharp. If the center of the mask be in focus, the outer edges are out of focus (as Hartmann pointed out), and this has a disturbing effect on the observer. Furthermore, a vertical line at this surface is in focus only at its middle and can therefore not be used for setting on spectrum lines in the ordinary way. These difficulties are all overcome by using the half-silvered surface. In this case the images may be brought to focus beyond the double-prism where masks of any desired configuration may be employed,

[1] *Zeitschrift für Instrumentenkunde*, **21**, 205–207, 1906; also *Astrophysical Journal*, **24**, 285, 1906.

and where adjustable spider lines may be used when desired. Thus comparisons and line bisections may be made at the same time.

2. When the half-silvered surface is used, it is possible to place the images in coincidence in order to detect differences.

3. The new device makes possible the employment of the comparator for measuring photographs of spectra by the promising method suggested by Evershed, namely, that of using, with the negative to be measured, a positive plate made from it, and placed reversed to negative end for end.[1] I have adapted the Hartmann comparator for this purpose and have found it very satisfactory. It overcomes the disadvantages of the method Evershed has employed of sliding positive above negative, in that by the new method the films of positive and negative may be focused in the same plane. (The difficulty of having the films not in the same focus has been practically overcome by Evershed by employing an objective of long focus.) The new method possesses an additional advantage in that the intensity or color of the beams of light from positive and negative may be altered independently, thus making it possible, by matching the intensities of the positive and negative or by increasing the contrast, to measure the displacements of spectral lines of almost any character, even the broad lines in some stellar spectra.

4. By overlapping the spectra no parts of them need be cut out, as is the case in using the arbitrary masks of the Hartmann instrument.

In Table I are given the means of the measurements of displacements (produced mechanically) of 15 lines (λλ 4196.599–4291.630, intensities 1–5), of six exposures of the spectrum of the solar limb by: (1) the ordinary method of bisecting with spider line, taking the differences between means of four settings on the middle strip and means of two settings on each of two outside strips of spectrum, each way (violet right and violet left)[2]; (2) by measuring negative (violet left) with positive of itself (violet right) on the adapted Hartmann comparator, taking differences

[1] *Kodaikanal Observatory Bulletin*, No. 32, 1913.

[2] *Journal Royal Astronomical Society of Canada*, 5, 308, 1911.

between means of two settings on the middle strip and means of two settings on each of the two outside strips of spectrum, each way (violet left and violet right). A comparison of the results of the two methods shows the practicability of the new device and the lessening (nearly halving) of the probable error of setting obtained by employing the positive-with-negative method.

TABLE I

Measurements of Plate L701 by Two Methods

	a	b	c	d	e	f	Means
I. Bisection with Spider Line, 1911							
Mean displacement in mm.	0.0690	0.0699	0.0702	0.0699	0.0690	0.0686	0.0694
Probable error, single line.	0.0029	0.0030	0.0027	0.0027	0.0015	0.0029	0.0026
Probable error of mean.	0.0008	0.0008	0.0007	0.0007	0.0004	0.0008	0.0007
II. Positive-with-Negative, 1917							
Mean displacement in mm.	0.0692	0.0700	0.0708	0.0704	0.0690	0.0692	0.0698
Probable error, single line.	0.0020	0.0014	0.0015	0.0012	0.0015	0.0014	0.0015
Probable error of mean.	0.0005	0.0004	0.0004	0.0003	0.0004	0.0004	0.0004

When the adapted Hartmann comparator is used, remeasurements of solar rotation plates show good agreement with my earlier measures by the method of bisection of lines; and the comparisons promise to throw light on the systematic differences in the measurements of the same lines on the same plates by different observers. For example, the measurements of 19 lines, λ 5506.095–5688.436, Plate L845(1), are:

Bisection of lines by spider thread:

De Lury, mean of 9 measures, 1911–1916, is... 1.618 km/sec.
Plaskett, mean of 2 measures, 1911, is .... 1.695 km/sec.

Positive-with-negative:

Evershed, 1 measure, 1916, is .... 1.636 km/sec.
De Lury, mean of 3 measures, 1917, is.......... 1.619 km/sec.

To test the applicability of the new form of comparator to the determination of stellar radial velocities, I have measured plates of β Geminorum with positives of solar standard plates (or vice versa),

selecting plates which had been previously measured[1] (by J. S. Plaskett) in testing the Hartmann spectro-comparator. Instead of groups of comparison lines and groups of stellar and solar lines being aligned in turn, as in the Hartmann method, these groups are brought into coincidence in turn, positive fitting into negative in a most satisfying manner. In all cases two settings were made on solar and stellar coincidences; and one on each of the two strips of comparison spectrum at each region. The plates were then reversed and measured in the same way. Less than a minute at each region each way was the average time expended. The measurements are given in Table II, and the degree of reliability of the measures may be judged by the following measurements of a zero displacement (22 regions) of solar standard 1462 with positive plates A and B from it:

1462 with positive A, −0.0010 mm
1462 with positive B, −0.0007 mm; 0.0000 mm; 0.0016 mm; 0.0004 mm
Mean, 0.00006 mm; or 0.04 km/sec.
Probable error of single measure, ±0.24 km/sec.; probable error of mean, ±0.14 km/sec.

(It is to be expected that the errors can be lessened considerably by adopting Evershed's method of using negative violet left and positive violet right, or vice versa, even though the measurement is in this case confined to a single comparison line and a single solar and stellar line—a method which has the obvious advantage of yielding information concerning the behavior of the individual lines.)

From Table II it will be seen that repeated measurements are close in the case of Plates 1527 positive A and 1520 negative, and much closer for 1527 negative and 1520 positive A. For the former the probable error of a single measure is ±0.43 km/sec.; and for the mean, ±0.22 km/sec.; while for the latter the probable error of a single measure is 0.07 km/sec.; and for the mean, 0.03 km/sec. This latter is a degree of accuracy seldom attained, I believe, for stellar plates of the low dispersion used, by either the bisection method or the Hartmann method for so few settings and

[1] *Report of the Chief Astronomer*, 1909, pp. 183 f.

regions. Accuracy of setting is dependent to a great degree on the careful balancing of the intensities of positive and negative when making the settings or when printing out the positive. The agreement between the overlapping-positive-with-negative measurements (De Lury) and the aligning-negative-with-negative measurements (Plaskett) is only fair; and both measures seem greatly superior to the agreement of the solar standards or of the stellar plates.

TABLE II

MEASUREMENTS OF β GEMINORUM BY TWO METHODS

(De Lury using positive-with-negative; Plaskett using the Hartmann method, negative-with-negative.)

β GEM. PLATE	SUN PLATE	NO. OF REGIONS	D (DE LURY, 1917) Measured Velocity km/sec.	D Reduced Velocity km/sec.	D Mean Velocity km/sec.	P (PLASKETT, 1909) Measured Velocity km/sec.	P Reduced Velocity km/sec.	P Mean Velocity km/sec.	DIFFERENCE P−D km/sec.
1373−....	1360+A	20	24.80	2.88	2.88	24.78	2.86	2.86	−0.02
1373−....	1462+A	22	24.88	3.08	......	24.23	2.43	......	....
1373−....	1462+B	22	25.01	3.21	......	24.31	2.51	......	....
1373−....	1462+C	22	24.84	3.04	3.11	....	......	2.47	−0.64
1373−....	1520+A	11	23.74	1.84	......	25.01	3.11	......	....
1373−....	1520+A	11	23.30	1.40	......	....	......	......	....
1373−....	1520+A	11	23.94	2.04	1.76	....	......	3.11	+1.35
1472−....	1520+A	11	31.12	1.88	1.88	31.27	2.03	2.03	+0.15
1527+A...	1520−	11	25.56	0.85	......	27.03	2.32	......	....
1527+A...	1520−	11	26.15	1.44	......	....	......	......	....
1527+A...	1520−	11	27.12	2.41	......	....	......	......	....
1527+A...	1520−	11	25.76	1.05	1.44	....	......	......	....
1527−....	1520+A	11	26.31	1.60	......	....	......	......	....
1527−....	1520+A	11	26.13	1.42	......	....	......	......	....
1527−....	1520+A	11	26.09	1.38	......	....	......	......	....
1527−....	1520+A	11	26.12	1.41	1.45	....	......	2.32	+0.87
					2.22			2.56	+0.34

The main advantages of the new form of comparator are:

1. A minimum of optical surfaces are used.

2. Images may be brought into coincidence; and thus the Evershed positive-with-negative method of measuring spectra may be employed without the disadvantages found in using the positive over the negative and with the added advantage that the intensities of positive and negative may be altered independently and contrast gained by using colored filters.

3. Two sets of images are produced and may be observed in two different eyepieces, the instrument being in fact a *double* comparator.

4. In measuring spectra, the instrument may be used in the ordinary way of bisecting the lines with a spider thread; or it may be used as a comparator for overlapping or aligning, for which masks for various configuration may be readily interchanged.

SOLAR PHYSICS DIVISION
DOMINION OBSERVATORY, OTTAWA
April 1917

ON THE SPIRAL OF OBSCURATION IN THE STELLAR UNIVERSE

A REPLY TO MR. F. H. SEARES

By H. H. TURNER

Mr. Seares has made a valuable determination of the galactic condensation[1] from part of the material collected by me from counts of the zones of the *Astrographic Catalogue*. There are several points in his paper on which I offer comments, but it is natural to take first the most important one in which I find myself directly at variance with him. He states that "the deviations of the observed densities [Table VI of his paper] . . . . are not in agreement with the spiral of obscuration derived by Turner from the same data." This statement I challenge directly; and I submit the following figures taken straight from his Table VI. The equation suggested for the spiral is

$$\alpha+3.66\,\delta=247^{\circ}.$$

The values of α calculated from this equation for the declinations of Table VI are shown in the second line below:

Dec. . . .	$+62^{\circ}$	$+28^{\circ}$	$+17^{\circ}$	$+9^{\circ}$	-1°	-17°	-32°	-41°	-42°
α.	$+20^{\circ}$	$+145^{\circ}$	$+185^{\circ}$	$+214^{\circ}$	$+257^{\circ}$	$+309^{\circ}$	$+4^{\circ}$	$+37^{\circ}$	$+41^{\circ}$
Used. . .	1^{h}	9^{h}	13^{h}	15^{h}	17^{h}	21^{h}	1^{h}	3^{h}	3^{h}
Error. . .	-5°	-10°	$+10^{\circ}$	$+11^{\circ}$	$+4^{\circ}$	$+6^{\circ}$	$+11^{\circ}$	$+8^{\circ}$	$+4^{\circ}$

Since Table VI only gives results for the odd hours 1^{h}, 3^{h}, 5^{h}, etc., we cannot use these exact values of α but must take the nearest odd hours, as shown in the third line of the table. We now simply rearrange Table VI by commencing the circuits of R.A. with the hours above indicated. Further, we do not want the numbers of Table VI themselves, but merely the differences between the first and last column of each declination. (See table of differences on p. 227.) Simple sums are given at the foot. It will be seen that those from (11) to (19) are all positive but one, the mean of them being $+31$; those from (21) to (9) are, with one excep-

[1] *Mt. Wilson Contr.* No. 135; *Astrophysical Journal*, 46, 117, 1917.

tion, negative, the mean being -35. Analyzing the twelve terms harmonically we get

$$-13 \sin\theta - 33 \cos\theta = -35 \cos(\theta - 20^\circ),$$

so that the maximum *obscuration* occurs when $\theta = +20^\circ$, about an hour later than is given by the formula

$$\alpha + 3.66\,\delta = 247^\circ.$$

[In the foregoing work the Melbourne zone has been omitted for reasons indicated in *Monthly Notices*, **77**, 224, 1917; the spiral no longer follows the foregoing equation south of $\delta = -55^\circ$.]

DIFFERENCES OF TABLE VI RE-ARRANGED IN SPIRAL FORM

(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)	(19)	(21)	(23)
−3	−15	−5	+7	−1	−6	+9	+2	+2	+8	−16	−11
+1	−5	+7	−7	−16	−15	+1	+1	−14	−34	−24	+23
+8	−9	−12	−9	+8	+27	+6	+5	−5	−6	−23	−19
+4	−7	−13	−10	−5	+21	+22	+11	−13	+22	−3	+11
−2	−4	+2	−2	+8	+12	−2	+11	+6	−3	−5	+5
+3	+3	+5	+10	−28	−9	−1	+13	+1	+9	−16	+14
−3	−1	+3	−8	+1	−11	−4	+2	−1	+5	+9	0
−19	−2	−12	−26	+7	−1	+14	+12	+3	+6	−8	−12
0	−9	−4	−8	+5	+7	+13	+10	+14	+6	−2	−5
−11	−49	−29	−53	−21	+25	+58	+67	−7	+13	−88	+6

Adopting the crude supposition that the differences are directly proportional to differences of magnitude, since the latter are

$+62^\circ$	$+28^\circ$	$+17^\circ$	$+9^\circ$	-1°	-17°	-32°	-41°	-42°
2.6	3.3	2.3	2.0	1.2	3.4	2.5	2.5	0.9,

the sum of which is 20.7, then the coefficient 35 of the foregoing harmonic represents $0.35/20.7 = 0.017$ per magnitude. This would be denoted 1.7 in such a series as that for R_0' in Table V of *Monthly Notices*, **77**, 225, 1917, and is distinctly small; but it is of the right order of magnitude and its smallness is, at any rate, partly due to the crude manner of deducing it by throwing together material from widely different parts of the sky.

It is accordingly submitted that Mr. Seares's Table VI, so far from being "not in agreement with the spiral of obscuration," gives very good evidence in support of it; the small discordances

being what we might expect when a portion of the whole material is separately discussed.

I will now add one or two remarks on other points. In his opening sentences Mr. Seares represents it as my "purpose to show that important conclusions may be derived simply by counting the stars within each interval of brightness, even when the scale of luminosity is arbitrary." This is a fair statement of what the purpose has become in the course of time; but I am anxious that the *original* purpose of these papers should not be forgotten. I urged that counts of the kind mentioned would be the simplest way of co-ordinating the various scales of the *Astrographic Catalogue* one with another, and I hope that the advantages and facility of such comparisons have been amply demonstrated. The method was rejected at the last meeting of the Permanent Committee in favor of a series of special investigations on sample plates. There can be no question of the value of such a piece of work, which Professor E. C. Pickering has done much to facilitate, but it will be costly in time and labor; and it is not free from pitfalls, as those who have tried it know from their experience. The pitfalls are being discovered and avoided and the labor is being faced, so that we shall ultimately have a really satisfactory scale to which all the Astrographic magnitudes may be referred. But meanwhile we can by these counts co-ordinate them very closely with a single, though arbitrary, scale, which will never be work thrown away.

The physical results, for which the counts have lately become useful, were not part of the first purpose, except that *incidentally* the first few counts seemed to throw doubt on the progression of the galactic condensation and it was interesting to follow up this clue. It led directly to the recognition of some other phenomenon (now regarded as the spiral of obscuration) which seemed to be more marked than the galactic phenomenon; but ultimately (as noted by Mr. Seares) the progression of the galactic condensation was recognized and even crudely determined. That a better determination has not hitherto been given is due simply to the fact that more material is being accumulated—there is a goodly stock already in hand—and it was thought better to defer the definitive discussion. Mr. Seares's paper is however most welcome; it seems to show that

Kapteyn's determination is so good that it may profitably be adopted (as at any rate a first approximation) to clear the results from the galactic condensation before presentation. Up to now they have not been so cleared, at first because the progression of the condensation seemed to be discredited, and later because it had not been sufficiently well determined for use.

Finally, I demur to the following statement of Mr. Seares: "The question under consideration is really that of the uniformity of the stellar distribution in galactic longitude." I have stated several times that the spiral has no relation to the Galaxy. Hence galactic longitude *alone* is inappropriate for the discussion; we must include at the same time galactic latitude, just as we should have to use *both* ecliptic longitude and latitude, or *both* of any series of co-ordinates except those which are definitely related to the spiral itself. Only when we recognize the spiral in our co-ordinates is it possible to conduct the discussion in one co-ordinate; it is because we recognize the Galaxy in our co-ordinates that we can discuss the galactic condensation in terms of galactic latitude only. How much importance Mr. Seares attaches to the foregoing statement I scarcely know; but he makes it as though to clarify the discussion, whereas I am concerned to point out that it obscures it.

University Observatory
Oxford
July 1917

COMMENTS ON MR. SEARES'S REJOINDER

(Added September 13, 1917)

This discussion was opened by Mr. Seares in the September number of this *Journal*. He courteously sent me his paper in manuscript, and on receipt of it (July 14) I replied at once in the hope that my reply might appear with his paper. But apparently this was not possible. The reply was returned to me (received September 12) with a rejoinder, and with the intimation that copies of both were in the hands of the editor of this *Journal* and would appear in October or (more probably) November if I cabled to that effect. After reading the rejoinder I sent the cable. But

as there is a certain lack of symmetry in the conduct of a discussion which leaves the last word always on one side, I send these few lines in the hope that they too may be added.

Mr. Seares changes his ground in the rejoinder. His former statement was that certain deviations "were *not in agreement* with the spiral of obscuration." For simplicity of reference let us substitute the statement that certain quantities, -3, -2, -1, $+1$, $+2$, $+3$, $+5$, $+6$, $+7$, are "not in agreement" with a positive mean value, the responsibility for italics and for substitution being entirely mine. My reply was to add them up and show that the sum is $+18$. Mr. Seares replies by asking, "Is not something more than a positive sum required *for the substantiation of a positive mean value?*" The italics and translation are again mine. I use them to make clear the change of ground and to simplify the reply. To the former charge of want of agreement the reply is already made. To this quite different question my reply is: Certainly something more is required. With these particular numbers we can only *suggest* a positive value, we cannot *substantiate* it. For that we require more material, which I am doing my best to accumulate. Already there is a great deal in print, of which, for excellent reasons, Mr. Seares has so far been unable to take account. But the existence or non-existence of the spiral (represented above by the positive mean value $+2$) will ultimately be settled by the whole material, not by my views or Mr. Seares's views of the significance of a limited sample.

It is true that the sample examined contains a very large number of stars and cannot be disregarded. Had it been in contradiction with the main thesis, the objection would have been serious. But there is a distinction between "not in agreement with" and "being sufficient to substantiate." The former phrase suggests a tendency to substantiate the contrary, and against it I have made my protest; whereas, that we have as yet sufficient evidence to substantiate the spiral I have never claimed deliberately. If there be some careless phrase which seemed to claim it, may I not rather be judged by my acts than by my words—my diligence in collecting more material and in pleading that others will do the same?

One word more—in considering some of the points raised by Mr. Seares it is important to remember that he is not raising them for the first time. He suggests, for instance, a possible seasonal effect on the observations. This point was considered in the second paper of the series (dealing with the Bordeaux observations);[1] and more recently the evidence for and against a seasonal effect has been given in detail. Again, as regards the accordance of different zones, details have been collected more than once. Had such important points been overlooked the criticisms would have had more point. But Mr. Seares has, as a matter of fact, adduced very little that is really new. He has given a welcome precision to the progression in galactic condensation; but the crude estimates already given in the papers were of the right order of magnitude, and a definitive discussion would naturally have followed in due course when additional materials now available had been put in shape. Perhaps we may profitably postpone the remainder of this discussion till then?

H. H. TURNER

[1] *Monthly Notices*, **72**, 464, 1912.

THE SPIRAL OF OBSCURATION[1]

COMMENTS ON PROFESSOR TURNER'S REPLY

By FREDERICK H. SEARES

Professor Turner discusses[2] numerically the data in Table VI of *Mount Wilson Contribution* No. 135[3] and concludes that they give very good evidence in support of the spiral of obscuration. He demonstrates the substantial agreement of the progressive change in the sums of quantities derived from this table, when appropriately rearranged,[4] with the varying ratio of faint stars to bright stars defined by the spiral. But is not something more than accordance in the sums required for the substantiation of the spiral? Such an agreement might appear when the differences in each line but one of Professor Turner's table were all zero, although the evidence obviously would not then support the spiral, but indicate instead a local phenomenon affecting only a single zone of declination. Evidently the number of zones agreeing with the spiral must also be considered, and unless these are numerous the support of individual favorable cases will be accepted with caution.

Examining Professor Turner's table, we find that the clustering of signs is to be traced mainly to the similar variations affecting the zones at $+17°$, $-41°$, and $-42°$, although part of it is attributable to the circumstance that the algebraic sums of the quantities in each line are not zero. Adding a constant to each line to make the sums zero and transferring the results for $+17°$ to the bottom, we have Table I, whose sums display the same progressive change as was found by Professor Turner.

For the first six zones the clustering is no longer conspicuous. It is pronounced, however, for the last three, whose minima agree satisfactorily with the obscured region of the spiral. Two zones $(+62°, -1°)$ among the first six are also in agreement, although the amplitude is not much larger than the accidental errors. The

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 139.

[2] *Astrophysical Journal*, **46**, 226, 1917. [3] *Ibid.*, **46**, 117, 1917.

[4] See last line of his table on p. 227 and the related discussion.

sequence of signs for $+9^\circ$ suggests an agreement, but the beginning of the series corresponds to a nodal point rather than a minimum. The remaining three cases ($+28^\circ$, -17°, -32°) are unfavorable. The zone at -65° is not included, as it has already been rejected by Professor Turner as discordant. Noting that the two Cape zones have essentially the same declination and count, therefore, as one, we find no excess of favorable cases, even when the last three zones are included.

TABLE I

(Unit = 0.01 in Logarithm)

Zone	First Hour	(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)	(19)	(21)	(23)
+62°	1	− 1	−13	− 3	+ 9	+ 1	− 4	+11	+ 4	+ 4	+10	−14	− 9
+28	9	+ 8	+ 2	+14	0	− 9	− 8	+ 8	+ 8	− 7	−27	−17	+30
+ 9	15	+ 1	−10	−16	−13	− 8	+18	+19	+ 8	−16	+19	− 6	+ 8
− 1	17	− 4	− 6	0	− 4	+ 6	+10	− 4	+ 9	+ 4	− 5	− 7	+ 3
−17	21	+ 3	+ 3	+ 5	+10	−28	− 9	− 1	+13	+ 1	+ 9	−16	+14
−32	1	− 2	0	+ 4	− 7	+ 2	−10	− 3	+ 3	0	+ 6	+10	+ 1
−41	3	−16	+ 1	− 9	−23	+10	+ 2	+17	+15	+ 6	+ 9	− 5	− 9
−42	3	− 2	−11	− 6	−10	+ 3	+ 5	+11	+ 8	+12	+ 4	− 4	− 7
+17	13	+10	− 7	−10	− 7	+10	+29	+ 8	+ 7	− 3	− 4	−21	−17
Sum first six		+ 5	−24	+ 4	− 5	−36	− 3	+30	+45	−14	+12	−50	+47
Sum last three		− 8	−17	−25	−40	+23	+36	+36	+30	+15	+ 9	−30	−33
Sum all zones		− 3	−41	−21	−45	−13	+33	+66	+75	+ 1	+21	−80	+14
Mean lat., first six		39°	38°	39°	39°	37°	32°	30°	26°	26°	31°	32°	35°
Mean lat., last three		64	43	20	4	18	27	24	12	18	33	56	71

A certain amount of progressive fluctuation is to be expected, however. Seasonal factors, for example, are almost certain to influence the results; and even when their phases are distributed at random from zone to zone, the number of cases agreeing with an assigned distribution closely enough to be accepted as favorable to that distribution will be considerable. In the present instance it does not appear that this number is large enough to establish a relation between the observed distribution and the spiral of obscuration. These details may be verified by an inspection of the curves in the right half of Fig. 2 of *Mount Wilson Contribution* No. 135.

Examining now the question quantitatively, we form the sums for each column of the first six zones and find an oscillation of sign

that appears to be wholly fortuitous, whence we conclude that the fluctuation noted by Professor Turner is caused by the pronounced variation affecting the zones at -41°, -42°, and $+17^\circ$. In other words, the irregular distribution in these two regions affects appreciably the mean for all the data, and it is this dominating influence of the Cape and Bordeaux results that appears in Professor Turner's figures; but, since the total number of cases of agreement is not greater than might have been expected because of inherent uncertainties, I see no reason for modifying the judgment previously expressed.

Irregularities in the stellar distribution undoubtedly exist, and eventually it may appear that those remaining after eliminating the effect of the galactic condensation possess a systematic character; but I should hesitate to believe that at present we have any reliable indication of such a phenomenon. As far as the data under discussion are concerned, the spiral

$$\alpha - 2.24\,\delta = 206^\circ$$

seems to represent them much better than the one proposed by Professor Turner. The matter is immediately tested by drawing a line across the right half of Fig. 2 of my original paper in such a way as to intersect the $+62^\circ$ axis at $\alpha = 23^h$, and the -65° axis at $\alpha = 4^h$. This will then represent the line of greatest obscuration, and inspection shows that in no case will the agreement with the minima of the individual curves be less satisfactory than before, while for $+28^\circ$, -32°, and -65° there will be an appreciable improvement. But at the present moment, in view of the undoubted presence of seasonal influence and other systematic errors in the Astrographic counts, I should attach no significance to this circumstance.

Finally, it is important to note that an appropriate modification of the galactic condensation adopted for the faint stars will remove entirely the progressive variation which arises from the preponderating influence of the zones at -41°, -42°, and $+17^\circ$. Let us examine the mean latitudes in the last two lines of the table. For the first six zones these are sensibly the same for all the columns,

but for the last three zones they show a marked variation. Now the quantities in the body of the table are differences of the form[1]

$$\delta = \log \frac{D_F}{D_B} - \log \frac{N_F}{N_B},$$

the first term involving the observed value of the ratio of faint stars to bright, while the second includes the value of this ratio calculated from the mean densities in Table III, *Mount Wilson Contribution* No. 135. A change in the adopted galactic condensation of faint stars relative to bright stars will modify the second term, but for a specified galactic latitude the change will be constant and the *relative* values of δ for a series of equal latitudes will remain unchanged. Since the mean latitudes for the first six zones are nearly equal, the sequence of the sums of the tabular quantities for these zones will not be appreciably affected by a change in the condensation.

On the other hand, the three zones which show a marked fluctuation in δ also show a wide range in the mean latitudes. Positive values of δ in these zones correspond in general to small galactic latitudes, and negative values to large latitudes. Now suppose the tabulated mean density for the faint stars to be slightly increased in the lower latitudes. This will increase N_F and reduce the positive values of δ to zero. A similar but opposite change in N_F for the higher latitudes will neutralize the negative values of δ.

I have previously mentioned[2] the uncertainty affecting the determination of the galactic condensation from the Astrographic data, and a reference to the lower curve in Fig. 1 of *Mount Wilson Contribution* No. 135, which illustrates the difficulty experienced in drawing some of the curves, will indicate whence this uncertainty arises. The modification necessary to eliminate entirely the progressive variation in the sums of δ for all the zones is well within this uncertainty, and for this reason the mean densities derived in *Mount Wilson Contribution* No. 135 may be accepted for the present.

These conclusions are also in harmony with other circumstances. For example, it is a contention of this discussion that the spiral of

[1] See equation (1) of *Mt. Wilson Contr.*, No. 135.

[2] *Mt. Wilson Contr.*, No. 135, p. 13; *Astrophysical Journal*, 46, 129, 1917.

obscuration, for the most part, is only the increasing galactic condensation which appears with increasing limiting magnitude exhibited under another form, and that the spiral disappears when the data are freed from the effects of the condensation. Treated by themselves, the Cape results for $-41°$ indicate a very high relative value of the condensation, much higher than the mean for all the data. When corrected with the aid of tables of mean distribution, traces of the galactic concentration (or of the spiral) will remain, and this is just what we find for the zone in question.

Professor Turner has commented on the magnitude of the condensation for this zone,[1] and it is of interest to note that, with the exception of the zone at $+62°$, the other values of the condensation coefficients given by him[2] are all similarly related to the systematic variations shown in the right half of Fig. 2, *Mount Wilson Contribution* No. 135. Thus the Bordeaux and Algiers coefficients ($+17°$, $-1°$) are also large, and the corresponding curves, like that for the Cape, have been counted as cases favorable to the spiral, while the small values for Oxford and Perth ($+28°$, $-32°$) correspond to unfavorable cases.

The remaining comments to be made on Professor Turner's reply are altogether minor in character. He refers to my use of only a part of the material collected by him. To avoid misunderstanding it may be well to repeat that some of his published data are not in a form suitable for the investigation undertaken in *Mount Wilson Contribution* No. 135. All that could be used were included in the discussion.

I think that we are entirely in agreement as to the importance of the counts he has collected. They undoubtedly admit of a close co-ordination of the scales of the various Astrographic zones, as Professor Turner himself has shown by the comparisons that he has made with the scale of Chapman and Melotte. The regularity of increase in stellar density with decreasing galactic latitude, which appears even when the counts are restricted to comparatively small areas, is an impressive phenomenon and an excellent indication of the value of density as a measure of stellar brightness.

[1] *Monthly Notices*, **75**, 608, 1915. [2] *Ibid.*

The meaning of the sentence from my article quoted by Professor Turner in the last paragraph of his reply would have been clearer had the wording been different. The implication was that we wish information as to the uniformity of distribution after the galactic condensation has been removed. By looking for irregularities along parallels of latitude, the effect of the condensation will not enter, and we may feel certain that any irregularities thus discovered will not be the consequence of residual errors in the adopted values of the condensation. But to trace the course of any irregularity, galactic latitude, the second co-ordinate, naturally requires consideration, as Professor Turner suggests.

MOUNT WILSON SOLAR OBSERVATORY
August 22, 1917

Reviews

Collected Papers on Spectroscopy. By G. D. Liveing and Sir J. Dewar. Cambridge University Press, 1915. Pp. xv+556. With 35 plates, 18 maps, and numerous diagrams. 30*s*.

This collection of "Papers on Spectroscopy" has a value entirely its own, given to it partly by the high standing of its joint authors, partly by the inherent importance of the results set forth. The work of Liveing and Dewar, covering the last quarter of the nineteenth century, is well known. It has long illustrated the effectiveness of co-operation when the talents combined are really complementary. In this particular case we see two eminent scholars, one mainly a chemist, the other mainly a physicist, joining forces upon a problem which is really physico-chemical.

The principal issue in spectroscopy has shifted greatly from the time when Newton employed the solar spectrum for a study of color, or when Young and Fresnel measured diffraction spectra in order to test certain theories of light, or when Bunsen used the prism chiefly for the discovery of new chemical elements, or when the constitution of various celestial objects commanded the attention of Sir William Huggins.

Current literature indicates that spectroscopic endeavor is now largely directed along electro-optical lines, including spectral series and the criteria which they furnish for theories of atomic structure. To say that all the beautiful advances which have recently been made in the manufacture of fine dispersion-pieces are merely ancillary to this more fundamental problem does not in any way detract from their importance.

Between these earlier and later purposes fall the labors of these two Cambridge scholars, who early set themselves the task of finding what they could concerning the physical constitution of laboratory sources—the mechanics of the arc, spark, and flame. It was well recognized even at the commencement of their work that a single substance may present several different spectra. Accordingly their search resolved itself into finding a one-to-one correspondence between a series of spectra and a series of physical states in the source. The difficulty of the quest lay partly in the fact that many of these spectra had never been described, but more largely in the fact that it is tremendously difficult—practically

impossible—to depict the physical conditions which are encountered in almost any of the sources.

To be sure, one can give the voltage across an arc, either on open or closed circuit; the mean amperage also; the temperature of the solid electrode; the end-products of the chemical action. And it must be confessed that the profound knowledge of chemistry brought to bear by these two authors fitted them in an eminent degree for "guessing" at the chemical processes going on inside these electrical sources and flames. But how far all this is from knowing the instantaneous electrical and magnetic fields, the mechanical motions, the pressure, the density, the chemical constitution, the actual structure of the tiny mechanism which emits light!

Bohr's theory offers us a most interesting atomic skeleton; but, as Professor Millikan has pointed out (*Science*, **45**, 330), it does not explain the mechanics of radiation. No one recognizes how far we are from the goal more clearly than these two authors themselves, who in a late paper frankly confess that we are still "wholly ignorant of the mechanism by which the gas is lighted up."

Nevertheless, one must admire the vigor of their pursuit; for these papers are filled with keen observations and cleverly devised experiments. Witness their early attempts to discover series-relations, their observations on lines which are now designated as "enhanced," their early distinction between the nitrocarbon and hydrocarbon bands, their resolution of the magnesium fluting (λ 5007) into lines, their suspicion that the edge of this fluting did not coincide with the green nebular line, their recognition of asymmetric reversals, the effect of nitrogen and hydrogen atmospheres in altering the relative intensity of certain lines, and a host of other phenomena.

Their paper on the spectrum of magnesium was, at the time of its appearance, almost a treatise on spectroscopy. Paper No. 57, dealing with the absorption spectrum of oxygen, led to the important conclusion that there was no resemblance between the absorption in the compounds of oxygen and the absorption in the element itself—a marked extension of the work of Janssen and his colleagues in the Royal Stables at Meudon.

Among the important contributions in the collection must be mentioned those papers which describe the recently discovered gases of the earth's atmosphere, those which detail the isolation of these elements, and those which give the preparation of spectrum-tubes by use of liquid air and liquid hydrogen.

The methods employed traverse the entire history of the science from liquid hydrogen back to the days of Rutherfurd gratings and arcs operated by 25 Grove cells. On page 283, where the Rowland grating is compared with that of Rutherfurd, one's curiosity is aroused by the remark that "the Rutherfurd grating is in some respects the better of the two." Again, in Paper 43, "On the Use of the Collimating Eyepiece in Spectroscopy," one wonders how the so-called Littrow form of spectroscope (invented, as Littrow himself says, by Duboscq) ever came to be described under the title of an "eyepiece."

It goes without saying that in all their pages there is a spirit of careful scholarship which no camouflage can ever imitate. There are those who think that the appreciation of such work dates from August 1914; but it is really very old. Sir David Brewster well understood the value of such science more than half a century ago. It was during the Crimean War that he wrote his *Memoirs of Sir Isaac Newton*, in the dedication of which he says: "It is from the trenches of science alone that war can be successfully waged; and it is in its patronage and liberal endowment that nations will find their best and cheapest defence."

H. C.

THE

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THE VARIATION WITH TIME OF THE CHARACTERISTICS OF A POTASSIUM PHOTO-ELECTRIC CELL AS TO SENSIBILITY ACCORDING TO WAVE-LENGTH

By HERBERT E. IVES

The measurements presented here were obtained during a study which was started with a twofold object: first, to try to work out a method of making photo-electric cells which would have the same characteristics of sensibility according to wave-length from cell to cell; secondly, to determine whether the spectral sensibility-curve remained constant with time and use.

A previous study,[1] part of a series planned to learn the possibilities of the photo-electric cell in photometry, established the fact that the distribution of sensibility through the spectrum varied greatly among cells made up in the manner usual at that time. If the proposition so to screen a cell as to make it equivalent to the eye is to have any practicability, it is essential that the cell's characteristics remain fixed; and if the manufacture of any considerable number is to be undertaken, it would be practically prohibitive to have to work out a separate correction for each cell. Hence the desirability of working out, if possible, a standardized cell of permanent characteristics.

[1] Ives, "Wave-Length Sensibility-Curves of Potassium Photo-Electric Cells," *Astrophysical Journal*, **40**, 182, 1914.

The test for permanence giving disappointing results, as will be seen, the other portion of the study was necessarily abandoned,

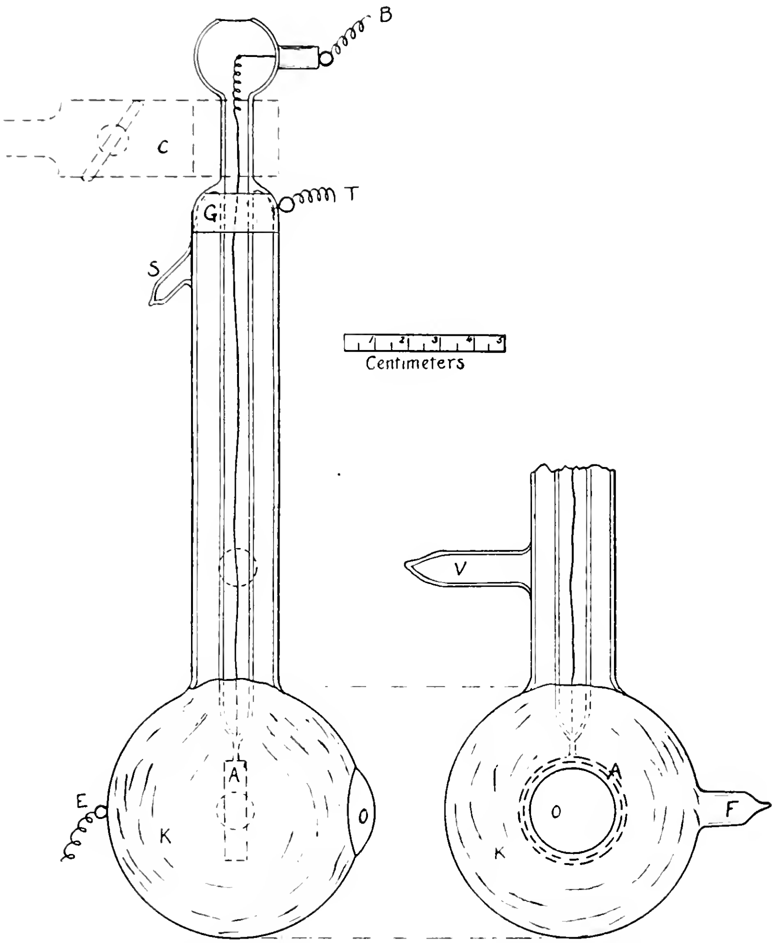

FIG. 1.—Photo-electric cell studied. *K*, Bulb whose inner surface coated with potassium forms the kathode; *A*, Anode; *E*, Electrometer connection; *O*, Opening for the admission of light; *F*, Filling tube; *V*, Evacuating tube; *S*, Tube used for silvering inner guard-ring; *G*, Guard-ring formed by interior and exterior silvering; *T*, Earth connection for guard-ring; *B*, Battery connection; *C*, Supporting clamp.

and work was limited to a single cell. Its design is shown in Fig. 1, with its accompanying legend. In general, it is made to conform

to the requirements for securing a rectilinear relationship between illumination and current as established by an earlier research,[1] namely, the reduction to a minimum of all surfaces on which charges can accumulate. The form adopted was the one described in the earlier paper as the Hughes design, that is, one in which the walls are covered with alkali metal, so that the whole cell approximates a black body. The choice of this form rather than of the Richardson design with the alkali metal at the center of the inclosure was dictated by the idea that the approximation to a black body might give the desired uniformity of characteristics from cell to cell. In order to secure sensibility all the way down to the red end of the spectrum, which is essential for photometric purposes, the cell was colored by a hydrogen glow discharge. This process was carried to the point where a maximum of sensibility was just passed, and all the conditions, such as voltage, pressure, etc., were carefully measured, so that they could be copied on subsequent cells. The resultant surface was dark blue in color. After the operation of coloring, the hydrogen was carefully pumped out, and helium was introduced to a pressure which gave the maximum sensibility as tested with the cell still on the pump. The curve of voltage-current of this cell was that characteristic of the gas-filled type—convex toward the voltage axis. The illumination-current relation was strictly rectilinear.

In testing the wave-length sensibility the light from a monochromatic illuminator was projected on to a piece of ground glass directly before the window of the cell, the idea being so to scatter the light that the average effect of all elements of the rather large area of potassium surface would be obtained.

The cell was mounted for test in a galvanized iron box which communicated with the electrometer inclosure, both being sealed air-tight with paraffin seals and well provided with drying material. During the whole period of the test neither cell, electrometer, nor monochromatic illuminator was disturbed in position. The source of light used was a standard 4.85 wpsc carbon lamp illuminating a ground glass before the slit of the monochromatic

[1] Ives, Dushman, and Karrer, "Factors Affecting the Relation between Photo-Electric Current and Illumination," *Astrophysical Journal*, **43**, 9, 1916.

illuminator. This lamp was used for no other purpose and provided a source of radiation of practically perfect uniformity of spectral distribution throughout the period of the test.

Measurements through the spectrum were made at first every day, then weekly, and then irregularly at longer intervals. After the first two weeks the cell was continuously illuminated, by an auxiliary lamp, with current running for a period of two weeks, to determine whether use accelerated the rate of change already detected. No effect directly chargeable to use was found. Five of the curves of wave-length sensibility for 2, 36, 63, 93, and 220 days (after which the arrangement of apparatus was disturbed) are shown in Fig. 2. These are just as obtained, not corrected either for the distribution of energy in the source or for the dispersion of the spectrometer. They are plotted to equal values at their maxima.

It is shown by these curves that the wave-length sensibility suffers a continuous change with time. The exact nature of the change cannot be inferred from these data, as it may be either an increase in the sensibility for the long waves or a decrease for the short waves. Thus, if the curves of Fig. 2 are plotted to equality in the yellow or yellow-green region of the spectrum, they all appear to be practically coincident until the blue-green is reached, when the variation shows as a progressive decrease of sensibility with time. This point could only be settled by preserving the constancy of the current-sensibility of the apparatus throughout the test and working with an applied voltage accurately the same on each trial, which was not attempted. For the main purpose of the test—establishing permanency or lack of it—it is of small importance whether the variation is due to an increase or a decrease of sensibility in any given region.

While the measurements give no information as to the cause of this variation, the suggestion may be hazarded that what is taking place is a gradual sintering of the highly sensitive and presumably unstable colloidal or hydride colored surface. This, as is well known, loses its characteristics if the hydrogen atmosphere, used to form it, is left in the cell. According to Elster and Geitel, an atmosphere of inert gas (helium, argon, neon) prevents this deterio-

ration. The present work indicates that some change still takes place with helium as the gas of the cell. From the standpoint of permanence alone, probably a very different result would be obtained if the coloring of the surface were dispensed with and

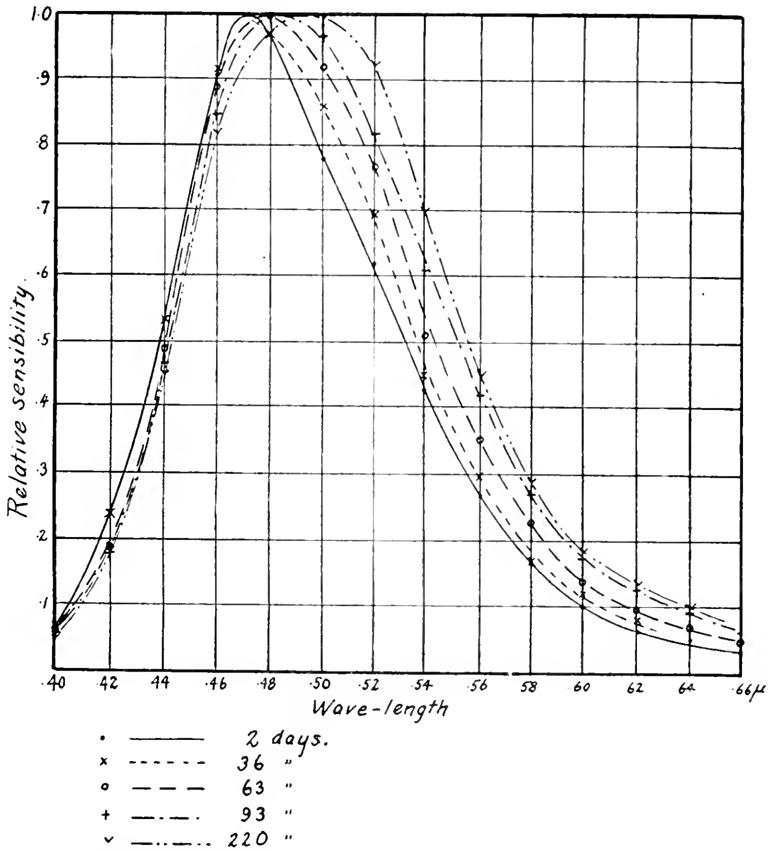

FIG. 2.—Variation with time of curve of sensibility according to wave-length

the potassium were simply made as pure and clean as possible by multiple distillation in vacuo. Pure potassium, however, is insensitive beyond $0.6\,\mu$ and so would be useless for the photometric purpose in view. Rubidium is sensitive farther into the red, but not far enough, while caesium, sensitive still farther, is too rare and of too low melting-point to be considered seriously.

It appears from this work that the prospect is not bright of making permanent cells of standardized characteristics. Consequently the practical fabrication of cells screened to imitate the human eye is not as yet feasible.

This investigation forms the conclusion of the series begun by the writer some years ago, aimed specifically at applying the photo-electric cell to photometry. For this purpose the cell should have certain characteristics. It should first of all exhibit a simple uniform relation between illumination and current. Next, it should have a curve of the relation between wave-length and sensibility which could be modified by screening so as to duplicate that of the average eye. The cell and its auxiliaries should, in addition, be sensitive, simple to use, and permanent. With regard to the first point, the writer's work[1] showed that in cells as ordinarily constructed, the relation of illumination and current was very erratic from cell to cell. Later[2] the explanation for this was found in the fact that whenever opportunity was offered for electrical charges to accumulate on free glass surfaces in the cell the current would be affected. Cells in which the relation of illumination and current could be altered at will were constructed. This led to the statement of the conditions necessary for a rectilinear relation of illumination and current, namely, the elimination, as far as possible, of such free glass surfaces, and to the production of cells possessing the desired relationship. The second point is covered by the measurements of curves of sensibility with wave-length of a series of cells, published some time ago,[3] and by the experimental work of the present paper.

The net result of this whole research is to show that the photo-electric cell is inapplicable at present to photometry, where by *photometry* is meant the measurement of luminous flux, that is, radiant flux evaluated according to its capacity to produce the

[1] Ives, "The Illumination-Current Relationship in Potassium Photo-Electric Cells," *Astrophysical Journal*, **39**, 428, 1914.

[2] Ives, Dushman, and Karrer, "Factors Affecting the Relation between Photo-Electric Current and Illumination," *Astrophysical Journal*, **43**, 9, 1916.

[3] Ives, "Wave-Length Sensibility-Curves of Potassium Photo-Electric Cells," *Astrophysical Journal*, **40**, 182, 1914.

sensation of light. The answer is quite different, however, in so far as *radiometry* is concerned, where under radiometry we include those measurements often treated under photometry, in which the characteristics of the radiometer as to sensibility according to wave-length do not enter. Such, for instance, are measurements of distribution of radiant flux, etc., for which the only requirement is the rectilinear relationship of illumination and current and adequate sensibility.

Among radiometric applications for which the properly constructed photo-electric cell is well suited may be noted, in particular, variations in intensity of certain types of stars, distribution curves of illuminants, densities of photographic plates, and finally spectro-radiometry in the visible and ultra-violet regions where the non-selective radiometers and the eye are too insensitive. For this latter purpose it is necessary, in order to get sufficient sensibility to make such measurements as those of diffuse reflecting power, to use an electrometer, or possibly a highly sensitive Thomson galvanometer. Some means of amplifying the current similar to those used in wireless telegraphy, to bring it into the range of the simple d'Arsonval galvanometer, would be a great boon. Some experiments made for the writer some time ago by Dr. A. W. Hull, of the General Electric Research Laboratory, showed the entire feasibility of amplifying photo-electric currents by the electron tube amplifiers developed in that laboratory. But while amplification of a thousand fold or more presented no difficulty, these amplifications were of currents already large enough to be measured by a sensitive d'Arsonval galvanometer. Small currents, of the order of magnitude measured by the electrometer, presented great difficulty, owing to leakage and other troubles in the amplifiers. Further work along this line is desirable.

Another improvement in the cell to make it applicable to spectro-radiometry would be to master the technique of producing a flat, extended curve of sensibility with wave-length, such as one of those accidentally found in one of the cells measured in the work quoted.[1] The very sharp maximum of the more usual type, such as that exhibited by the cell reported on in this paper, is

[1] *Ibid.*

undesirable, necessitating very narrow slit-widths to avoid corrections.

These points have been discussed in part to emphasize the necessary distinction between the very valuable characteristics of the photo-electric cell in *radiometry* and its failure in *photometry*. For the latter the only physical instrument so far developed is the screened non-selective radiometer, e.g., thermopile.[1]

Physical Laboratory
The United Gas Improvement Company
Philadelphia, Pa.
September 25, 1917

[1] Ives and Kingsbury, "Physical Photometry with a Thermopile Artificial Eye." *Physical Review*, **6**, 319, 1915.

THE PRINCIPLE OF GENERALIZED RELATIVITY AND THE DISPLACEMENT OF FRAUNHOFER LINES TOWARD THE RED[1]

By CHARLES E. ST. JOHN

According to Einstein's equivalence principle of generalized relativity, the lines in solar and stellar spectra should be displaced to the red when referred to the corresponding terrestrial lines. For any given frequency the magnitude of the effect depends upon the difference in gravitational potential between the gravitational field in which the emitting center is located and the terrestrial field where the radiation is received and measured. Einstein[2] deduces an approximate relation between the frequencies in the solar and terrestrial fields, namely,

$$(n_0 - n)/n_0 = 2 \times 10^{-6},$$

or, postulating the constant space-velocity of light,

$$\lambda - \lambda_0 = 2 \times 10^{-6} \lambda.$$

The calculated displacement of two parts in a million is well within the possibilities of observation, being 0.010 A for λ 5000. A less approximate and perhaps more objective statement is, "at the surface of the sun the displacement is equivalent to the Doppler displacement produced by a radial velocity of 0.634 km per sec."[3]

As the occurrence or non-occurence of such displacements is of fundamental importance in the theory of relativity and in the interpretation of observations of solar and stellar spectra, definite results from an investigation carried on with powerful instruments at command would have a double significance. The present contribution presents the details of such an investigation in which the primary object has been to determine what consideration must

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 138.

[2] *Annalen der Physik*, **35**, 905, 1911.

[3] A. S. Eddington, *Monthly Notices*, **77**, 380, 1917.

be given to this deduction from the equivalence principle of generalized relativity in the discussion of observations of solar spectra.

INSTRUMENTS AND METHODS

The 30-foot Littrow spectrograph and the 60-foot tower telescope on Mount Wilson were used in obtaining the main portion of the data. A grating of high resolving power and excellent definition was at our disposal through the kindness of the Physical Laboratory of Johns Hopkins University, the ruled surface being 114×160 mm and the number of lines 95,000. The third and fourth orders with scales of 0.56 A and 0.36 A per mm were used.

A common defect in narrow comparison spectra obtained by the usual occulting arrangement at the slit is the unsymmetrical ends of the arc lines and the irregular edges of the solar spectra. This disturbing element was eliminated, and the ease and accuracy of the measurements were increased by stretching a violin string on each side of the central strip of spectrum and close to the photographic film. This also removed the effect of a slight astigmatism without involving the movement of an occulting shutter. Other possible instrumental displacements were avoided by making all comparison exposures simultaneous, the relative intensities being controlled by a rotating sector.[1] The data in particularly important cases are the means of the closely agreeing measures made by three observers, and in all instances depend upon at least two.

SELECTION OF OBSERVATIONAL MATERIAL

Besides the principle of relativity, three other causes have been suggested which might account for a displacement of the Fraunhofer lines in the direction of longer wave-lengths—namely, differences of pressure, motion in the line of sight, and anomalous refraction. While anomalous refraction may produce sporadic effects under occasionally favorable density-gradients in the solar atmosphere, the conclusion from investigations and observations at this observatory is that, within the present limit of precision of measurement, the positions of the Fraunhofer lines in the spectrum

[1] St. John and Babcock, *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

of the solar disk are not systematically affected by anomalous refraction.[1] Any effect due to the difference in gravitational potential between the solar and terrestrial fields may be freed from that due to pressure by using lines whose pressure-shifts are negligible, such as those in the bands of the carbon arc at λ 3883 variously assigned to carbon, cyanogen, and nitrogen;[2] it may be freed from the influence of motion in the line of sight by observations upon the same lines at the sun's polar limb.

The richness of the ultra-violet Fraunhofer spectrum and the multiplicity of lines in the carbon-arc spectrum make the selection of suitable lines a matter of grave consideration from the point of view both of precision in measurement and of contamination by blends with lines sensitive to pressure changes. From an examination of high-dispersion spectrograms of excellent definition lines were selected whose separation from neighboring lines was sufficient to assure that the measurements would be free from systematic errors. A previous investigation had revealed such errors and shown that their magnitude depends upon the degree of proximity and the character and intensity of the adjacent lines; it also furnished data for determining for spectrographs of the dispersion and resolving power used in this investigation the limiting separation from neighboring lines within which measurements become unreliable.[3]

The lines finally selected were those of the nitrogen (cyanogen) bands listed in Tables I and II. Since they show wide variations in character and surroundings, an estimate was made of the weight to be assigned to results for individual lines. This estimate, based upon the appearance of the lines in both solar and arc spectra, is the observer's a priori judgment whether the measurements would have high, medium, or low weight. These estimated weights are entered in the appropriate tables.

[1] St. John, *Mt. Wilson Contr.*, Nos. 93 and 123; *Astrophysical Journal*, **41**, 28, 1915, and **44**, 311, 1916.

[2] The observations of Grotrian and Runge show that these bands are produced under experimental conditions that apparently preclude the presence of carbon, and that the essential condition is the presence of nitrogen in the absence of oxygen (*Physikalische Zeitschrift*, **15**, 545, 1914).

[3] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

THE OBSERVATIONS

A. *The arc wave-lengths.*—The wave-lengths of the lines under investigation were determined in the arc in terms of the same international iron standards and under the same instrumental conditions as were later used for their solar wave-lengths. The agreement with the wave-lengths of Uhler and Patterson was so close that the mean was taken as the probable value. For a few difficult cases, in which evidently neighboring lines might have affected the measures, appeal was made to the difference, Rowland *minus* International, to decide which was the more probable wave-length of the arc-line. Uhler and Patterson[1] consider their wave-lengths for sharp lines on a clear background to be correct in absolute value to ±0.005 A. This appears to be an underestimate of the accuracy, as the mean difference, Uhler and Patterson *minus* Mount Wilson, taken without regard to sign, is only 0.003 A even when difficult lines are included in the comparison. The decimals of the adopted values of the wave-lengths are in the fourth column of Table I.

B. Sun—arc displacements.—The critical questions are the behavior of the investigated lines at the center of the solar disk and the relation between center and limb displacements. Owing to the importance of the possible results and the limited observational material available, the questions have been approached from several directions:

1. At the center: (*a*) The center—arc displacements were directly determined from simultaneous exposures on the sun and the equatorial sections of a carbon arc 6–7 mm long, fed by a current of 6–7 amperes. These and all other comparisons between solar and terrestrial sources have been corrected for the earth's motion. The results are in the first column of Table I.

(*b*) The absolute wave-lengths of the solar lines were obtained from simultaneous exposures on the sun and on the iron arc, in terms of the same iron standards as were used for the arc wave-lengths. The differences between the solar and arc wave-lengths in the third and fourth columns of Table I furnish an independent determination of the center-arc displacements given in the fifth column.

[1] *Astrophysical Journal*, **42**, 438, 1915.

(*c*) The stable iron lines are displaced to the red in the solar spectrum. The positive differences, Rowland wave-length *minus* International wave-length, for such lines when decreased by the corresponding displacements center *minus* arc give the difference R−I that would be shown by lines whose solar and arc wave-lengths are equal. For the spectral region under consideration the mean value of this difference is 0.137A, which will be referred to as the "standard" R−I. Lines for which R−I is greater than 0.137A are displaced to the red in the sun; those for which it is less are displaced to the violet, the excess and deficit being the measures of the corresponding center−arc displacements. Hence a

TABLE I

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF THE NITROGEN (CYANOGEN) BANDS AT CENTER OF SUN

SECTION A. LINES OF SOLAR INTENSITY 00–1

(*a*) Sun−Arc	λ Center of Sun		λ Arc I.U.	(*b*) λ Sun *minus* λ Arc	(*c*) Rowland *minus* λ Arc	Intensity Rowland	Series	Weight
	R-System	I.U.						
+0.002..	3819.200	.061	.062	−0.001	0.138	1 Nd	B_1	High
+ .002..	3825.260	.126	.125	+ .001	.135	00	C_1	High
− .002..	3833.744	.610	.610	.000	.134	0	C_1	Medium
.000..	3842.780	.644	.648	− .004	.132	0	......	High
+ .001..	3845.152	.021	.019	+ .002	.133	1	B_1	High
.000..	3846.667	.534	.536	− .002	.131	00	......	High
.000..	3848.981	.848	.848	.000	.133	1 N	B_1	High
+ .002..	3850.784	.652	.648	+ .004	.136	0	B_1	Low
.000..	3854.192	.062	.063	− .001	.129	0	B_1	High
+ .002..	3854.988	.855	.854	+ .001	.134	1	B_1	Low
.000..	3857.058	.926	.922	+ .004	.136	0	......	High
+ .002..	3857.288	.156	.158	− .002	.130	1	B_1	High
+ .002..	3858.723	.591	.592	− .001	.131	0	B_1	Medium
− .002..	3867.116	.980	.983	− .003	.133	0	A_2	High
+ .002..	3867.200	.064	.061	+ .003	.139	0	A_2	High
− .003..	3867.908	.776	.779	− .003	.129	1	A_2	High
.000..	3868.542	.410	.410	.000	.132	1	A_2	Low
+ .004..	3868.624	.489	.486	+ .003	.138	00	B_2	Low
.000..	3868.702	.570	.572	− .002	.130	0	A_2	High
+ .003..	3868.782	.648	.644	+ .004	.138	00	A_2	Low
− .002..	3872.312	.178	.181	− .003	.131	0	A_2	High
− .001..	3876.448	.312	.315	− .003	.133	0	A_1	High
− .001..	3877.482	.346	.351	− .005	.131	1	A_1	High
+ .001..	3879.716	.580	.579	+ .001	.137	1	A_1	Medium
+0.003..	3880.102	.966	.964	+0.002	0.138	1	A_1	Low
+0.0006........	....	....	....	−0.0002	0.134	....	....	....

Average number of measures per line, 13. Average mean deviation per line, 0.002 A.

TABLE I—*Continued*

SECTION B. LINES OF SOLAR INTENSITY 2–4 AND PARTIALLY RESOLVED PAIRS

(a) Sun−Arc	λ Center of Sun R-System	λ Center of Sun I.U.	λ Arc I.U.	(b) λ Sun *minus* λ Arc	(c) Rowland *minus* λ Arc	Intensity Rowland	Series	Weight
	R.346					1	A_1	
+0.001..	3819.384	.241	.244	−0.003	0.140			Low
	R.412					1	A_1	
	R.400					0	A_1	
+ .001..	3822.435	.290	.290	.000	.145			Low
	R.470					0	A_1	
	R.745					0	B_1	
.000..	3830.774	.630	.637	− .007	.137			Low
	R.801					0	B_1	
+ .004..	3831.174	.038	.034	+ .004	.140	3	A_1d	Low
	R.639					1	A_1	
+ .005..	3836.660	.520	.516	+ .004	.144			Low
	R.689					1	A_1	
+ .005..	3844.377	.238	.235	+ .003	.142	4	A_1d	Low
− .002..	3846.131	.993	.999	− .006	.132	2	B_1	High
	R.777					1	A_1	
.000..	3846.796	.657	.656	+ .001	.140			Low
	R.814					1	A_1	
+ .004..	3851.426	.292	.286	+ .006	.140	2 N	A_1d	Medium
+ .006..	3852.541	.408	.402	+ .006	.139	2 N	A_1d	Low
− .003..	3853.621	.487	.491	− .004	.130	2 N	A_1d	Low
+ .006..	3856.800	.667	.665	+ .002	.135	2 N	A_1d	Low
+ .004..	3858.822	.692	.684	+ .008	.138	2 N	A_1d	Medium
+ .002..	3862.626	.493	.489	+ .004	.137	2	A_1b	Low
+ .004..	3863.534	.402	.399	+ .003	.135	3 N	A_1b	Low
+ .004..	3864.438	.308	.303	+ .005	.135	3	A_1	High
+ .003..	3865.284	.150	.150	.000	.134	3	A_1sB_1	Low
0.000..	3866.125	.989	.993	−0.004	0.132	3 N	A_1sB_2	Low
+0.0024	....	....	....	+0.0012	0.138	....	....	....

Average number of measures per line, 22. Average mean deviation per line, 0.003 A.

comparison of this "standard" value with R−I for the carbon-arc or nitrogen bands furnishes a method independent of the direct center−arc determinations.

A preliminary examination indicated that as a class the carbon-arc lines were not appreciably displaced to the red in the solar spectrum. A possible explanation of this systematic difference in the behavior of the iron and the band lines was that the Rowland wave-lengths for the two classes of lines did not form a homogeneous system. It is known that Rowland made an intensive investigation of these band lines, but it is not clear how their wave-lengths were connected with his standards. The solar wave-lengths

of the lines under investigation have therefore been redetermined in the Rowland system, the Rowland values for the neighboring iron lines being used as standards. There are no systematic differences between the Rowland and Mount Wilson measurements, the sums of the positive and negative differences for 57 lines being +0.086 and −0.094 A. The means of the two measurements have been taken as the wave-lengths in the Rowland system and are entered in the second column of Table I. In the sixth column are given the differences, Rowland *minus* International. For the lines of solar intensity 00–1 a mean displacement of 0.003 A to the violet is indicated, and for those of intensity 2–4 a displacement of 0.001 A to the red.

(*d*) A fourth independent method of determining the center−arc shifts is based upon the observations detailed below, which refer to the limb and its relation to the center, as the displacements, limb−arc, decreased by the corresponding limb−center values, are another measure of the center−arc shifts.

2. At the sun's limb: (*a*) The absolute wave-lengths of the investigated solar lines were measured at the limb in terms of the iron standards used for the arc wave-lengths. The results are in the second column of Table II. Simultaneous exposures were made on the center of the iron arc and the sun's limb in latitude 90° at a point distant from the edge one-eightieth of the diameter of the solar image. The displacements at the limb are then λ limb−λ arc as given in the third column.

(*b*) A check is furnished by increasing the center−arc displacements by the limb−center shifts independently obtained. The latter were found by the method used by Adams,[1] and also from simultaneous exposures on the center and on a point on the limb in latitude 90°. The means of the closely agreeing results given by these two methods are entered in the fifth column of Table II under the heading "Limb *minus* Center."

It is to be remarked that the Rowland intensities given in Tables I and II should in many cases be modified. It is especially difficult to estimate the intensities of the components of close pairs. In general, the values assigned are too low. For example, the

[1] *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

double lines $\left\{\begin{matrix}3836.639,\ 1\\ 3836.689,\ 1\end{matrix}\right\}$ and $\left\{\begin{matrix}3846.777,\ 1\\ 3846.814,\ 1\end{matrix}\right\}$ are like 3844.378, 4*d*, in appearance and intensity in both arc and solar spectra. Though quite differently estimated by Rowland, they are placed in the same section of these tables, and, similarly, all partially resolved doublets measured as single lines are classed with nominally stronger lines, which in reality are more or less close pairs of similar lines.

The lines under investigation are within a region of 60 A, in which Rowland catalogues about 250 lines belonging to the bands. In equal regions to the red and violet of the bands there are 400 and 450 metallic lines, respectively. If the region λ3819–λ3880 is equally rich in such lines, the probability of blends among the 250 band lines is high. It is highest for broad lines formed by the coalescing of the components of doublets, higher even than for narrower lines of greater inherent intensity; moreover, the errors of measurement may be systematic, as frequently the compound lines are unsymmetrical. As the probable occurrence of blends is least for the

TABLE II

Behavior of the Lines of the Nitrogen (Cyanogen) Bands at the Sun's Limb

SECTION A. LINES OF INTENSITY 00–1

Lines	(*a*) λ Limb	λ Limb *minus* λ Arc	λ Limb *minus* λ Center	λ Limb *minus* λ Center	Intensity	Series	Weight
3819.197 ...	.064	+0.002	+0.003	+0.003	1 N	B_1	High
3825.256 ...	.126	+ .001	.000	+ .003	00	C_1	Low
3833.744 ...	.611	+ .001	+ .001	+ .004	0	C_1	Medium
3842.779 ...	.646	− .002	+ .002	+ .005	0	......	High
3845.149 ...	.023	+ .004	+ .002	+ .002	1	B_1	Medium
3846.666 ...	.536	.000	+ .002	.000	00	......	Medium
3848.979 ...	.846	− .002	− .002	− .002	1 N	B_1	High
3854.191 ...	.065	+ .002	+ .003	+ .004	0	B_1	High
3854.989 ...	.855	+ .001	.000	+ .001	1	B_1	Low
3857.063 ...	.927	+ .005	+ .001	+ .002	0	......	Medium
3857.288 ...	.156	− .002	.000	+ .008	1	B_1	Medium
3867.118 ...	.982	− .001	+ .002	+ .001	0	A_2	Low
3867.906 ...	.779	.000	+ .003	+ .002	1	A_2	High
3876.448 ...	.310	− .005	− .002	.000	0	A_1	High
3877.481 ...	.346	− .005	.000	− .002	1	A_1	High
3879.716 ...	.579	.000	− .001	− .002	1	A_1	Medium
3880.105 ...	.964	0.000	−0.002	−0.004	1	A_1	Medium
....	....	0.0000	+0.0007	+0.0015	....	......	....

TABLE II—*Continued*

SECTION B. LINES OF INTENSITY 2–4 AND PARTIALLY RESOLVED PAIRS

Lines	(a) λ Limb	λ Limb *minus* λ Arc	λ Limb *minus* λ Center	λ Limb *minus* λ Center	Intensity	Series	Weight
3819.346 } 3819.412 } ..	.247	+0.003	+0.006	+0.004	{ 1 { 1	A_1 } A_1 }	Low
3822.406 } 3822.470 } ..	.291	+ .001	+ .001	+ .002	{ 0 { 0	A_1 } A_1 }	Low
3830.745 } 3830.801 } ..	.636	− .001	+ .006	− .002	{ 0 { 0	B_1 } B_1 }	Low
3831.174 ...	.039	+ .005	+ .001	+ .002	3 d	A_1d	Low
3836.639 } 3836.689 } ..	.523	+ .007	+ .003	+ .002	{ 1 { 1	A_1 } A_1 }	Low
3844.378 ...	.239	+ .004	+ .001	+ .004	4 d	A_1d	Low
3846.131 ...	.997	− .002	+ .004	.000	2	B_1	High
3846.777 } 3846.814 } ..	.660	+ .004	+ .003	.000	{ 1 { 1	A_1 } A_1 }	Low
3851.427 ...	.295	+ .009	+ .003	+ .005	2 N	A_1d	Low
3852.541 ...	.406	+ .004	− .002	+ .002	2 N	A_1d	Low
3853.620 ...	.492	+ .001	+ .005	+ .004	2 N	A_1d	Low
3856.802 ...	.671	+ .006	+ .004	+ .007	2 N	A_1d	Low
3858.822 ...	.692	+ .008	.000	+ .002	2 N	A_1d	Low
3862.626 ...	.494	+ .005	+ .001	+ .001	2	A_1b	Low
3863.533 ...	.401	+ .002	− .001	.000	3 N	A_1b	Low
3864.438 ...	.308	+ .005	.000	+ .003	3	A_1	High
3865.282 ...	.154	+ .004	+ .004	+ .002	3	A_1sB_1	Low
3866.122 ...	.995	+0.002	+0.006	+0.004	3 N	A_1sB_2	Low
....	....	+0.0037	+0.0025	+0.0023	....	....	....

narrowest lines, the division between Sections A and B of Tables I and II has been based upon line-intensity. For this reason, and because of the relatively high precision of measurement, greater weight is to be attributed to the data of Sections A.

The series classification in the column next to the last in Tables I and II is from the work of Uhler and Patterson.[1] A_1 is the nominally "singlet" series from the first head, but in all probability it is a series of doublets. Its ultimate doublet character, according to Uhler and Patterson, appears at λ 3860.626, line 47 of the series,

[1] *Astrophysical Journal*, **42**, 434, 1915.

though preliminary widening is evident in line 44 at λ 3863.390. They say:

> On leaving the head and going toward shorter wave-lengths one acquires the impression that the A_2 series (doublets from the first head) gradually grows out of and away from A_1. The first component of A_2 appears as a close companion of the eighth line of A_1. The second component of A_2 appears as a close component of line 13 of A_1 [λ 3881.587].

The number of lines suitable for measurement in the series A_1 is limited by the conditions that they be free from the A_2 lines and the disturbing influences of their own doublet character. In the shorter B_1 and C_1 series—singlets from the second and third heads—duplicity is not a disturbing factor, and the components of the A_2 and B_2 doublets are probably by nature single. The conditions that the corresponding solar lines, inherently more difficult to measure, should be measurable with fair accuracy, further limits the available material. Sections A of Tables I and II contain nearly all the lines under investigation that fulfil the conditions necessary for measurements of high weight.

DISCUSSION

The mean results from the data in Tables I and II are exhibited in Table III and may be briefly summarized as follows: The magnitude of the displacement to the red required by Einstein's deduction is approximately 0.008 A for the spectral region considered. Compared with this the observations at the center of the sun show a mean displacement of approximately 0.001 A to the violet for the 25 lines of greater weight, and of 0.001 A to the red for the 18 lines of lesser weight, or a zero displacement for the 43 lines. As a displacement due to relativity would be independent of the character of the radiating centers, this discrepancy between the calculated and observed values for the center of the solar disk requires some influence producing displacements of solar lines to the violet of just the magnitude to balance the predicted effect. Anomalous refraction, if active, would tend to displace the lines to the red; the wave-lengths of the lines employed are not appreciably affected by pressure; there is no evidence that wave-length depends upon the temperature of the source; of known causes there is left only the

possibility of an outward radial movement of the solar vapors with a velocity at this level of 0.634 km per second, which masks the assumed gravitational effect. At the sun's limb the radial movement would be across the line of sight, the Doppler effect would vanish, and the gravitational displacement of 0.008 A to the red should appear. For the lines whose measurement at the limb is most dependable, the 17 lines of Section A in Table II, the mean shift between the sun's limb and the arc is zero. There are three reasons for assigning a high weight to this mean, namely: under the

TABLE III

Comparative Solar and Terrestrial Wave-Lengths of Lines in the Nitrogen (Cyanogen) Bands

	Section A	Section B
At Center of Sun	25 Lines	18 Lines
Intensity	00–1	2–4
a) Direct comparison of solar and arc spectra	+0.0006	+0.0024
b) λ in center spectrum *minus* λ in arc spectrum	− .0002	+ .0012
c) (R−I) for band-lines *minus* "Standard" (R−I)	− .0030	+ .0010
d) (λ limb−λ arc) *minus* (λ limb−λ center)	− .0011	+ .0008
Mean (center−arc)	−0 0009	+0.0013
At Limb of Sun	17 Lines	18 Lines
Intensity	00–1	2–4
a) λ in limb spectrum *minus* λ in arc spectrum	0.0000	+0.0037
b) (λ center−λ arc)+(λ limb−λ center)	+0.0001	+0.0035
Mean (limb−arc)	0.0000	+0.0036

highest resolving power used the lines show no sign of duplicity; the narrowness of the lines reduces the chances for blends and in so far increases the probability that we are working with uncontaminated lines; and their isolation from adjacent lines is sufficient for measures of precision.

If the results of lower weight for the 18 compound and broader lines in Section B are included, the displacement to the red at the limb is 0.0018 A, or one-fourth of the calculated amount. The displacement shown by the broader lines may, however, be due to blends with lines of other elements. Adams found that, at the

limb, displacement of the metallic lines to the red is the general rule. In passing from the center to the limb the mean for 120 lines is 0.007 A. Undetected blends with such lines tend to introduce systematic displacement to the red at the limb for lines not sensitive to edge-effect and normally undisplaced.

Adams found for 14 lines of the ultra-violet fluting a shift, relative to the center, of 0.002 A to the red, with which the mean for the 35 lines in Table II is in agreement. He attributes this displacement between center and limb to rising convection currents. The 25 lines in Section A, Table I, lines of lowest level, indicate a rise of the vapors from the sun's interior of 0.08 km per second. This compares well with 0.12 km per second suggested by Adams. The displacement to the red in passing from the center to the limb for these low-level lines appears to be due to the disappearance at the limb of the Doppler-Fizeau effect in the ascending currents at the level where these weak lines originate. For the strong lines in Section B, the small displacement to the red at the sun's center may be accounted for by a downward drift at the higher level of origin, in which all high-level lines appear to be involved. For both the weak and the strong lines the displacement is, however, of the order of the errors in measurement, and great weight cannot be given to their distinctive behavior.

OTHER OBSERVATIONS

In an investigation bearing upon the behavior of the high-level calcium vapor in the solar atmosphere[1] the wave-lengths of the H and K lines were determined in the arc and at the solar limb, with results as follows:

	H_3	K_3
1 mm from the limb....	3968.478	3933.667
At the limb...........	3968.476	3933.665
In the arc............	3968.476	3933.667

Though the close agreement between the solar and arc wave-lengths may in part be fortuitous and the evidence for equality

[1] St. John, *Mt. Wilson Contr.*, No. 48; *Astrophysical Journal*, **32**, 36, 1910.

should not be unduly stressed, the observations indicate with a high degree of probability that the difference is not of the order of 0.008 A, as required by the principle of relativity.

As the gravitational effect is proportional to wave-length, the displacement due to it should be more evident for lines of great wave-length. The Mount Wilson data on the comparative wave-lengths of the iron lines in the sun and arc give for 25 lines of the stable *b* group, mean λ 6300, a mean displacement to the red at the sun's center of 0.004 A instead of 0.013 A, required by the relativity theory, a discrepancy between calculated and observed values too large to be attributed to errors of observation.

A complete report of an investigation by Schwarzschild, "Über die Verschiebungen der Bande bei 3883 Å.E. im Sonnenspektrum,"[1] has not yet reached America. According to the abstract in the *Beiblätter*, he finds that at the center of the sun there is a displacement indicating a downward velocity of 0.2 km per second and that, if this downward movement is taken into account, there remain only slight displacements, which cannot be considered favorable to the Einstein theory.

For lack of fuller data a satisfactory comparison cannot be made between Schwarzschild's results and those in this paper. If the lines used by him are similar in character and intensity to those in Section B of Table I, as seems probable, there is good agreement between his displacement to the red interpreted as a downward movement of 0.2 km per second and the mean of 0.0014 A for the lines in Section B corresponding to a downward velocity of 0.1 km per second.

In *Bulletin* No. 39 of the Kodaikanal Observatory Evershed and Royds give the results of their measurement of 12 lines in the band at λ 3883 and their rather startling deduction of the receding of the solar atmosphere at the sun's center and all around the circumference where it is supposed to attain a velocity of recession of about 1 km per second. This suggested flow of the solar vapors from the hemisphere turned earthward at all points of the sun's limb is assumed to be due to a repulsion of the solar gases by the earth. They base this hypothesis upon a displacement to the red.

[1] *Beiblätter*, **39**, 480, 1915.

indicated by their measurement of lines in the band at λ 3883. They appreciate the difficulties involved in the idea, but consider that there is no alternative. The crucial point is the behavior of such lines at the sun's limb. As is evident from the summation in Table III, the Mount Wilson observations, based upon a larger number of lines, fail to confirm the results found at Kodaikanal. For the lines of highest weight there is no displacement to the red either at the center or at the limb. The measurements are inherently difficult, and results may be more or less influenced by the choice of lines and by the resolving power, definition, and dispersion of the spectrographs used.

The displacements between center and arc observed at Kodaikanal and at Mount Wilson, the appearance upon the Mount Wilson plates, and the estimated weights of the Mount Wilson measures are given in Table IV. The unresolved triplet groups (A_1, A_2, A_2), near the head of the band, where the A_2 doublets begin to appear and to separate from A_1, are unsymmetrical in the sun and arc. After the triplet character becomes discernible, the components vary progressively in intensity and spacing so that settings upon the complexes are difficult and liable to systematic errors until the separation is sufficient to free a line completely. The weights of the Mount Wilson measures are based largely upon the degree of isolation of the lines.

Evershed and Royds are of the opinion that the negative shifts for λ 3876 and λ 3877 are due to arc conditions and that the general shift to the red of 0.005 A observed by them is too small for the same reason. They state that the general effect of arc conditions is to reduce sun—arc shifts. When light is taken from near the poles of an iron arc or when a short arc with strong current is used, the sun—arc displacements for a large group of iron lines are negative, but this has not been observed for the lines of all elements. An examination of the carbon arc for pole-effect shows that the wave-lengths of these band lines are free from this disturbing influence. Simultaneous exposures were made upon the pole and center of the carbon arc with the 30-foot spectrograph in the fifth order of a Michelson grating of 66,000 lines. The scale of the spectrograms is 3 mm to an angstrom. The result for lines of the series A_1, B_1,

C_1, A_2, and B_2 are given in Table V. There is no evidence of such instability as the iron lines of groups *d* and *e* show.

TABLE IV

THE APPEARANCE ON THE MOUNT WILSON PLATES OF THE LINES USED AT KODAIKANAL

Lines	Intensity	Center *minus* Arc: Kodaikanal	Center *minus* Arc: Mt. Wilson	In the Arc	In the Sun
3863.533....	3 N	+0.010	+0.002	A_1, broad, confused with B_2	Trace of line on red edge: weight very low in sun and arc
3864.438....	3	+ .008	+ .003	A_1	Free: weight high
3876.448....	0	− .002	− .003	A_1	Free: weight high
3876.556.... 3876.622....	0 0	+ .005	+ .003	A_2 A_2	Incomplete resolution: weight low
3877.481....	1	− .001	− .003	A_1	Free: weight high
3877.587.... 3877.646....	0 0	+ .006	+ .002	A_2 A_2	Red component > violet, weight low
3879.331.... 3879.394.... 3879.458....	1 0 0	+ .005	....	A_1 A_2 A_2	Not measurable as single line. A_2 blended with another line
3879.796.... 3879.851....	0 0	+ .003	+ .003	A_2 A_2	Partial resolution: weight low
3880.465.... 3880.532.... 3880.596....	1 2 1	+ .007	+ .003	A_1 A_2 A_2	Resolved triplet, middle component a blend, weight very low
3880.815.... 3880.931....	1 1	+ .004	+ .003	A_1 A_2 A_2	Resolved triplet, middle component weak, weight low
3881.729.... 3881.825....	1 1	+ .008	+ .002	A_1 first appearance of 2d component of A_2 series A_2 A_2	Apparent doublet, violet component of triplet blends with middle component, weight low
3882.828.... 3882.893....	1 0	+0.008	+0.002	A_1 first appearance of 1st component of A_2 series A_2	Unsymmetrical in arc and sun, weight low
Mean..........		+0.005	+0.0015	....	....

In the case of λ 3876 and λ 3877 an observed displacement to the violet might arise from the proximity of the lines on the red, if dispersion and resolution are insufficient, as there is then a tendency

on the part of the measurer to displace the violet and red components of close pairs to the violet and red, respectively.[1] On the Mount Wilson spectrograms the resolution is complete, and the separation from the adjacent lines is sufficient to show an equally intense background on both sides of the lines—conditions that experience has shown are necessary and sufficient for measures of precision. Two similar lines, λ 3879 and λ 3880, nearer the head and less separated from the adjacent lines on the red, whose measurement is presumably more liable to errors of this type, give 0.000

TABLE V

NEGATIVE POLE *minus* CENTER OF THE CARBON ARC

λ	Series	Pole-Center	λ	Series	Pole-Center
3819.197	B_1	0.000	3867.906	A_2	0.000
3825.256	C_1	−.001	3868.539	A_1	.000
3833.744	C_1	.000	3868.625	B_2	.000
3845.149	B_1	.000	3868.700	A_2	.000
3846.131	B_1	−.001	3868.785	A_2	.000
3864.438	A_1	.000	3872.312	A_2	+.001
3867.118	A_2	.000	3876.448	A_1	.000
3867.205	A_2	−0.001	3877.481	A_1	0.000
			Mean		0.000

and +0.002 A, the mean of the four being −0.001 A. As a score of other lines of like intensity yield a similar result, it seems probable that small displacements are characteristic of these low-level lines and that within the limits of error displacements to the violet are not, on that ground alone, to be discarded. These observations remove the necessity felt by Evershed and Royds of assuming an effect of the earth driving the solar vapors from the sun's earthward-turned hemisphere and leave the question of limb-effect for further investigation, with pressure, level, and line-intensity as probable contributing factors.

I wish to express my appreciation of the assistance given in this difficult investigation by Miss Miller, who has had a large share in the measurements of each type, and to Miss Ware, who has checked the results of Miss Miller and myself in the most critical cases.

[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

SUMMARY AND CONCLUSIONS

1. The wave-lengths of a group of lines in the band spectrum of nitrogen (cyanogen) have been measured at the center and at the limb of the sun and in the carbon arc, in terms of identical iron standards and their solar wave-lengths have been redetermined in the Rowland system.

2. Emphasis was put upon the selection of lines, with a view to precision of measurement and freedom from blends, which is particularly important for observations of the limb, where measurements are inherently difficult and blends with metallic lines introduce systematic errors of the sign required by the relativity theory.

3. Sun—arc displacements have been determined at the center directly and by three indirect methods.

4. Limb—center and limb—arc displacements have each been determined by two methods.

5. The sun—arc displacements for iron lines in the region were measured, and from these a standard value for Rowland *minus* International was found for lines showing no displacement at the sun's center. With this the corresponding R—I for the selected band lines was compared.

6. The mean sun—arc displacement at the sun's center for 25 band lines of intensity 00–1 is −0.001 A, and for 18 lines of intensity 2–4 it is +0.0014 A, with a mean of zero for the 43 lines.

7. The mean sun—arc displacement at the limb for 17 band lines of intensity 00–1 is 0.000 A, and for 18 lines of intensity 2–4 it is +0.0036 A, with a mean of +0.0018 A for 35 lines.

8. The conclusion is that within the limits of error there is no evidence in these observations of a displacement to longer wave-length, either at the center or at the limb of the sun, of the order of 0.008 A, as required by the principle of relativity.

9. There is a limb-effect not due to motion, in which pressure, level, and line-intensity appear to be involved in varying degrees for different elements.

MOUNT WILSON SOLAR OBSERVATORY
June 1917

RELATION TO PROPER MOTION OF PREFERENTIAL MOTION AND OF THE PROGRESSIONS OF SPECTRAL CLASS AND MAGNITUDE-VELOCITY

By C. D. PERRINE

The following investigation concerns the relation between the preferential motion or streaming of the stars as interpreted by the ellipsoidal theory, and the size of proper motion, and is based upon radial velocities. It was suggested by the contrary behavior of 110 stars of magnitude 2.9 and brighter in the matter of the positions of the solar apex and by the consistently different apices derived for the A, F, and G stars from those given by the other spectral classes. These peculiarities correspond in general to differences in the sizes of proper motions.

It was desired also to see what effect there might be on the ellipsoidal motion of the A, F, and G stars by the use of the apex and velocity derived from these classes instead of the uniform apex and velocity used by Eddington and Hartley in their determination.[1] Advantage was taken, therefore, to combine both problems. The stars were separated according to brightness and also according to size of proper motion. The limit of annual proper motion was taken at 0."10. Those above this limit are called large and those below, small. The stars in Campbell's L. O. catalogues[2] were divided into two classes, those of 2.9 and brighter and those of 3.0 and fainter, for each spectral class. The mean magnitude of these latter is about $4\frac{1}{2}$, and they contain very few fainter than magnitude $5\frac{1}{2}$. Adams' Mount Wilson list[3] of 500 radial velocities was also available, the stars ranging almost entirely from 5.5 to 6.5 magnitude with a mean at about 6.0. This list contains no stars south of $-25\frac{1}{2}°$ declination and was selected especially with the view of extremes in the size of proper motion.

[1] *Monthly Notices*, **75**, 521, 1915.

[2] *Lick Observatory Bulletins*, **6**, 108, 1911; **7**, 20, 51, 113, 1913; *Publications of Lick Observatory*, **9**, 329, 1911.

[3] *Mt. Wilson Contr.*, No. 105; *Astrophysical Journal*, **42**, 172, 1915.

The limits adopted for the regions around the major and minor axes of the spheroid are practically the same as those used by Kapteyn and Adams[1] in similar investigations within 50° of either vertex for the major axes and from 60° to 120° from a vertex for the minor axes. The southern vertex used in the present investigation is that derived by Eddington[2] from the Boss stars, $\alpha = 274°$, $\delta = -12°$.

The results are given in the following tables, in which ρ_2 and ρ_1 are the major and minor axes respectively. The stars of Campbell's catalogues were cleared of solar motion as follows:

B	−20.0 km	toward	270°, +30°
A, F, G	−18.	toward	260°, +15°
K, M	−19.5	toward	270°, +30°

The solar motion used by Adams in obtaining the values of the velocities of the stars (v') given in his catalogue was −20 km toward 17^h59^m, $+30.^\circ8$.

There is a consideration which at least ought to be mentioned in this connection. In the classification by size of observed proper motion the effect of the portion which is due to the sun's motion is neglected. It is not easy properly to eliminate this solar effect because of the lack of knowledge of distances. There are reasons for thinking that most of the conclusions resting upon such classifications are really free from any serious results of the unconscious selection which has thus been practiced. But until this matter has been put definitely beyond suspicion it seems well at least to bear in mind the possibility of some such effect.

The first value obtained for the ratio of the axes for the small μ of the F stars (Table I) was 1.91. An examination showed that the average μ of these stars was higher than the others. The experiment was then tried of rejecting the values over $0.''05$, with the result that the ratio was reduced to 1.17. These larger values were then included with the large μ. Two stars of 98 and 92 km were omitted in ρ_1 for large μ. None of the other classes were so treated with respect to restriction to smaller proper motions.

[1] *Proceedings Nat. Acad. Sci.*, 1, 14, 1915.

[2] *Stellar Movements*, p. 102.

These results in general show a smaller prolateness for the small than for the large proper motions. The fainter stars of K and M (of Adams' list, Table II), as well as the bright stars of the

TABLE I

PROLATENESS FROM RADIAL VELOCITIES

STARS OF 3.0 AND FAINTER FROM CAMPBELL'S L. O. CATALOGUES

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(60) 7.9	(53) 8.2	0.96	....	....	....
A......	(64) 12.8	(63) 10.8	1.18	(8) 12.9	(23) 7.2	1.79
F......	(18) 12.2	(20) 10.4	1.17	(46) 19.4	(80) 11.9	1.63
G......	(47) 14.0	(34) 10.7	1.31	(13) 39.0	(28) 25.4	1.54
K......	(90) 15.3	(136) 14.1	1.09	(43) 29.1	(81) 20.5	1.42
M......	(18) 19.7	(34) 16.5	1.19	(4) 30.7	(9) 15.7	1.96

TABLE II

PROLATENESS FROM RADIAL VELOCITIES

ADAMS' MOUNT WILSON CATALOGUE OF 500 STARS

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(52) 8.6	(29) 10.5	0.82	....	....	....
A......	(53) 11.3	(57) 9.2	1.23	(2) 19.4	(7) 10.3	1.88
	(49) 9.7	(55) 8.1	1.20	....	....	....
F......	(7) 12.8	(11) 9.6	1.33	(7) 51.4	(4) 25.7	2.00
	(6) 10.8	(10) 7.3	1.48	....	....	....
G......	(26) 12.8	(29) 7.8	1.64	(8) 79.4	(21) 27.6	2.88
	(23) 10.6	....	1.36	(7) 48.0	....	1.74
K+M...	(41) 14.3	(43) 12.2	1.17	(14) 30.4	(17) 25.5	1.19

TABLE III

PROLATENESS FROM RADIAL VELOCITIES

STARS 2.9 AND BRIGHTER FROM CAMPBELL'S L. O. CATALOGUES

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(16) 8.9	(14) 4.9	1.81	....	....	....
A, F, G..	(9) 20.5	(9) 4.5	4.56	(8) 16.6	(17) 11.1	1.50
K, M...	(9) 11.2	(11) 9.1	1.23	(7) 16.7	(11) 19.1	0.87

same spectral classes, show no such difference. This may be only a coincidence, particularly in the case of the brighter stars, as the amount of data is very small, but requires further investigation from a larger number of stars. With the exception of the K and M stars already noted, the bright stars, including those of type B, show considerable values for the prolateness.

The evidence of the 30 bright B stars is quite consistent as far as it goes, as will be seen from the individual velocities which are given below.

Vertices	Minor Axis
+10.6	+8.5
24.3	+3.7
10.6	+6.3
9.3	+7.6
13.4	−9.0
4.9	−0.9
2.6	+5.3
1.0	+6.3
6.4	+5.3
7.6	+2.3
0.4	+7.6
2.8	+1.0
14.7	+8.0
10.1	+1.7
21.7	
+ 1.3	
Mean 8.9	Mean 4.9

Half of the velocities about the vertices are larger than any in the regions of the minor axis.

The increase of prolateness in the stars of large μ can be traced directly to the greater increase of velocity along the ellipsoidal axis. If we take the difference large μ minus small μ for ρ_2 and ρ_1, and then difference these in the sense ρ_2 minus ρ_1, we get the following:

LARGE μ *minus* SMALL μ, ρ_2 *minus* ρ_1

Class	Table I	Table II
A	+ 3.7 km	+ 7.0 km
F	+ 5.7	+22.5
G	+ 0.3	+15.4
K	+ 7.4	....
M	+11.8	....
K+M	....	+ 2.8
Mean	+ 5.8	+11.9

There is a dissymmetry or skewness in the average velocities with respect to the northern and southern groups about the minor axis which calls for remark in connection with the effect on the prolateness of the spheroid. It has been observed that in the classes A, F, and G the northern groups of the stars of small μ at right angles to the axis give smaller average velocities than the southern groups. In such cases there is a nearer equality of the southern group to the velocities from the vertices, the northern group standing out as a deficiency. Such a condition causes the minor axis to be reduced, and, when compared with the major axis, to show a prolateness. Indeed, if we take the average of the values of ρ_1 for the Campbell stars of $3^{M}.0$ and fainter in the spectral classes A, F, and G (B not showing any prolateness), we find 7.8 for the northern groups and 11.6 for the southern. If we assume 11.6 for the major axis, we obtain a prolateness of 1.20 due entirely to such a deficiency in the velocity of the northern stars. If such stars had been unsymmetrically distributed and we had had no southern stars, we should have found a prolateness of 1.49 for these three classes, a value which differs but little from that obtained from the ellipsoidal theory. The small proper motions resulting for the A, F, and G stars of Adams' list give a mean value of 12.3 for ρ_2 and 8.9 for ρ_1 and are consistent in both cases. These values yield a prolateness of 1.38. Not yet having corresponding southern stars, it is not possible to judge with certainty, but this appears to be a case in point.

It is suggestive that we find such an effect only in these three classes where ellipsoidal motion is most pronounced. There is no marked effect of this sort in the B, K, and M stars ($3^{M}.0$ and fainter, Campbell), where there is no prolateness for B, and values of only 1.09 for K and 1.19 for M stars. There are indications that this effect is in reality related to a dissymmetry on opposite sides (north and south) of the axis of solar motion for the stars of small μ. A well-marked effect of this kind which is believed to be an indication of rotation has been found in the B and K stars. The other classes have not yet been directly tested for this effect. This and other peculiarities of stars of both small and large μ will be made the subject of a separate investigation.

It is believed that these preferential motions are in the form of more or less definite streams, which it is hoped to isolate by means of their velocities.

The conclusion arrived at in this investigation from radial velocities, that the preference for motion along the ellipsoidal axis is almost entirely confined to the stars of larger proper motion (and presumably nearer), receives strong support from Dyson's[1] investigations of the larger proper motions, which showed (I infer from references, as I have not access to his original papers) a more pronounced streaming for these stars than for the groups used by other investigators where small proper motions were included. In their investigation of the relation between radial velocity and proper motion of the stars of classes F, G, K, and M, Kapteyn and Adams came to the conclusion[2] from much the same series of observations which I used that "the values of ρ_2/ρ_1 are no smaller for the stars having the very smallest proper motions than for the other stars in the list." Their table of results bears out this conclusion quite well. A minute examination, however, shows some slight signs of an increase of the prolateness with increasing size of proper motion. Combining the groups of the stars of smaller proper motion in the classes F and G, and separating the K and M stars at about 0."10, the following simple means of the ratios ρ_2/ρ_1 are obtained:

ρ_2/ρ_1

Class	Small μ	Large μ
F.	1.51	1.93
G.	1.52	1.99
K.	1.17	1.33
M.	1.40	1.61

This somewhat arbitrary and biased reclassification would have little weight by itself. The great differences between my results and those of Kapteyn and Adams are probably to be explained by the general deficiency in the radial velocities of the stars of small μ to the north of the ellipsoidal axis, combined with the fact that the

[1] *Proceedings Royal Society Edinburgh,* **28,** Part III, No. 13; **29,** Part IV, No. 21.

[2] *Proceedings Nat. Acad. Sci.,* **1,** 17, 1915.

Mount Wilson stars (which formed a not inconsiderable part of the investigation) are almost entirely northern stars, and would therefore have a tendency to increase the prolateness for the groups of smaller μ.

RELATION OF PROPER MOTION AND RADIAL VELOCITY TO DISTANCE

Kapteyn and Adams[1] showed that there is an increase of radial velocity with increasing proper motion in the spectral classes F, G, K, and M, and they were able to satisfy this condition by a non-Maxwellian distribution, with indications also of a relation to magnitude, without the aid of a third explanation of the decrease of velocity with increasing distance. The latter is, perhaps, the most direct interpretation of such an effect as that observed, and deserves consideration in view of the conclusion indicated by the present investigation that the difference in the behavior of the stars of large and small proper motion is primarily due to distance.

It is inconceivable with even the little absolute knowledge which we have of the distances of the stars that, in general, the brighter stars and those with the largest proper motions should not be nearest. The direct determinations of the parallactic displacements of a number of the stars of large proper motion confirm this explanation in a way which seems final. There are as yet only a small number of well-determined parallaxes, but these, as far as they go, indicate greater radial velocity for the nearer than for the more distant stars. Forty-one stars have been found whose parallaxes are 0."06 and over, which appear to be reliable—the probable error being well below the foregoing value. These were arranged in three groups, as given in Table IV. The second group contains a considerable number of large velocities. If we reject four with velocities of from 73 to 98 km, we obtain the results given in the second line of the second group in Table IV.

If these rejections are justifiable, the results show a definite decrease of velocity with increasing distance. This is shown also in another and probably more reliable way by this same group of 41 near stars. The mean velocity of the group is 26.9 km (or

[1] *Proceedings Nat. Acad. Sci.*, 1, 14, 1915.

20.7 km, if we omit the four stars with velocities of from 73 km to 98 km), whereas the average velocity of a large number of stars of similar magnitude and spectral class (including many stars of small μ), which are undoubtedly at a much greater distance than the 41 stars, may be taken at 15 km or 10 km, if limited to small proper motions. The 41 stars are very well distributed with respect to the different parts of the ellipsoid, and appear to be comparable on the whole with those giving the smaller velocities in all ways except distance.

TABLE IV

RELATION OF RADIAL VELOCITY TO PARALLAX

π	Mean Mag.	Mean π	Mean v	No. Stars	Mean μ
			km		
0".06 to 0".09	3.1	0".07	17.9	13	0".63
0.10 to 0.19	4.5	0.13	34.1	18	1.73
	4.3	0.13	19.8	14	1.17
0.20 to 0.76	2.9	0.34	25.5	10	2.22

Such conclusions presuppose that the Doppler effect is not sensibly modified by distance.

The number of stars is small and the weight of the evidence, therefore, not great. As far as it goes, however, it is confirmatory of the simple explanation that the radial velocities (and perhaps the total velocities) of the nearer stars are, on the average, greater than those of the more distant stars. The conclusion that the difference in velocity is due to distance appears to be strengthened by the investigations of Kapteyn and Adams, already referred to, in which they find (p. 18) that "a peculiar feature of this result is that if all the stars are used, and not alone those on which the stream motion has little influence, the exponential constants remain essentially unchanged." In other words, I understand this to mean that the observed non-Maxwellian distribution has no relation to the preference for motion along the ellipsoidal axis, that the distribution observed is more general than that. The factor of distance would seem to supply such a general requirement. It would be of interest to know how these constants would be affected by limitations in the matter of proper motions.

THE PROGRESSION OF RADIAL VELOCITY WITH SPECTRAL CLASS

Evidence has been encountered in this work that the spectral class velocity-progression may also depend upon the proper motions (or total motions), that the radial velocities of the stars of small proper motion and presumably great distance are on the whole nearly free from this effect. Of Adams' list 326 of the stars fail to show any progression whatever with spectral class when classified with respect to the ellipsoidal axis, and a number of groups of small proper motion of Campbell's catalogues also fail to show such progression when similarly classified. Table V shows the results

TABLE V

STARS OF SMALL μ

ADAMS' MOUNT WILSON CATALOGUE

Spectral Class	All Stars		Rejecting R.V. of 30 km or Over		
	ρ_2	ρ_1	ρ_2	ρ_1	Mean ρ
	km	km	km	km	
B..........	(52) 8.6	(29) 10.5	....	....	9.6
A..........	(53) 11.3	(57) 9.2	(49) 9.7	(55) 8.1	8.9
F..........	(7) 12.8	(11) 9.6	(6) 10.8	(10) 7.3	9.0
G..........	(26) 12.8	(29) 7.8	(23) 10.6	(29) 7.8	9.2
K..........	(28) 14.0	(26) 10.1	(25) 11.6	(24) 8.1	9.6
M..........	(13) 14.8	(17) 15.4	(11) 10.0	(13) 11.9	10.9

from all the stars of small proper motion in Adams' catalogue as originally selected and after the rejection of 22 of these stars having velocities of 30 km or over. All of the 348 stars show a slight increase of velocity with spectral class. After the rejection of the 22 stars there is no increase whatever, which indicates, in these stars at least, that the progression with spectral class has been due to the 22 stars with large velocities. The proportions of the stars of very small μ but relatively large radial velocity of these stars of Adams' list are found to be progressive, the later spectral classes containing larger proportions than the earlier classes. The distribution of such stars in the sky appears to be somewhat systematic. Interpreted literally, such progression would seem to indicate a tendency among the later types for the motions to become radial.

It should be recalled in this connection that a small proportion of stars will be moving nearly in the line of sight, and therefore, if selected on a basis of proper motion alone, that a few will show abnormally large radial velocities. It would seem to be justifiable to classify such stars of large radial motion upon the general ground of total motion.

It is of interest in this connection to classify with respect to magnitude, and northern and southern regions of sky, the values of ρ_1 for the stars of small μ in the catalogues of both Campbell and Adams. These are given in Table VI. As previously used, the F stars were restricted to those having proper motions of 0.″05 or less.

TABLE VI

ρ_1 FOR STARS OF SMALL μ

CAMPBELL'S L. O. CATALOGUES. NORTHERN AND SOUTHERN SEPARATELY

Spectral Class	$2^{\mathrm{M}}.9$ and Brighter	$3^{\mathrm{M}}.0$ and Fainter	Spectral Class	$2^{\mathrm{M}}.9$ and Brighter	$3^{\mathrm{M}}.0$ and Fainter
	km	km		km	km
Northern			Southern		
B.........	(3) 6.2	(16) 7.8	B.......	(11) 5.0	(37) 8.4
A.........	(3) 4.2	(33) 8.1	A	0 ...	(30) 13.8 (27)† 11.7
F.........	(1) 7.3	(4)* 5.5	F........	(2) 6.0	(14) 10.2
G.........	(1) 1.8	(13) 8.2	G.......	(2) 3.2	(21)‡ 12.3 (19)† 8.2
K.........	(5) 10.9 (4) 6.2	(54) 14.4 (47)† 10.9	K.......	(5) 8.8 (4) 6.3	(82) 13.9 (73) 10.8
M.........	0	(12) 16.7 (9) 8.3	M	(1) 1.5	(22) 16.4 (19) 12.2

* One star of 39 km omitted.
† Omitting velocities of 30 km and over.
‡ One star of 227 km omitted.

The 34 stars of magnitude 2.9 and brighter according to this classification show no certain increase of velocity with spectral class. The slightly larger values of the K stars indicate at first sight a small increase, but if we reject one star out of each series (29.7 and 19.0) the remainder show no real increase.

The northern stars of $3^{\mathrm{M}}.0$ and fainter show a sudden break between the G and K types, whereas in the southern stars a similar break occurs between B and A. If in these 338 stars we reject the 27 stars with velocities of 30 km and over (8 per cent), there is

little or no evidence in the remaining 311 of an increase with spectral type. Further restriction in the size of proper motion would probably have some effect also. The number of stars is small, and it seems better to await more material before making a further discussion of the matter. Table VII contains the values

TABLE VII

ρ_2 FOR STARS OF SMALL μ

CAMPBELL'S L. O. CATALOGUES

Type	$2^{\rm M}9$ and Brighter	$3^{\rm M}0$ and Fainter	Type	$2^{\rm M}9$ and Brighter	$2^{\rm M}0$ and Fainter
	km	km		km	km
B..........	(16) 8.9	(60) 7.9	G..........	0	(47) 14.0 (44) 11.4
A	(4) 25.6	(64) 12.8 (56) 10.5	K..........	(6) 8.8	(90) 15.3 (80) 12.5
F..........	(5) 16.4 (4) 12.3	(18) 12.2	M..........	(3) 16.0 (2) 6.1	(18) 19.7 (13) 12.4

for ρ_2 from Campbell's catalogues in the two classes of magnitudes previously adopted. If large radial velocities are rejected from a number of these groups on the hypothesis previously adopted, there is left no real progression with spectral class. No rejections were made in the stars of class A of $2^{\rm M}9$ and brighter because at least three of the four belong to the class of larger motion.

If we accept the evidence, which seems direct and strong, that the nearer stars are on the whole moving with greater velocities, in the line of sight at least, than the distant ones, then the greater part of the observed increase of velocity with spectral type appears to be very well explained upon the simple assumption of a greater proportion of nearer stars in the later spectral types. Adams came to the conclusion[1] that a direct interpretation of his results indicated that among the very distant stars the change of velocity with spectral type is slight. It seems to me probable that the slight effect still remaining in his results is entirely similar to that which I found to result from a very few comparatively large velocities in the later types among the stars of small proper motion, the bearing of which has been considered to some extent above.

[1] *Mt. Wilson Contr.*, No. 105, p. 21; *Astrophysical Journal*, **42**, 172, 1915.

It may be noted in this connection that if there is a loss of light in the stars as they progress in spectral type from B to M (as seems very probable) there would be a tendency, in any classification according to a given brightness, for the later types to be nearer on the average. If, then, the conclusion arrived at earlier, that the nearer stars have larger radial velocities and proper motions than the more distant ones, is correct, there will be an apparent increase of radial velocity with spectral class in stars so classified. Such a suggestion was indeed made by Eddington, but later was considered to be entirely disproved.[1]

The fact that there appears to be a close relation between proper motions and radial velocities would seem to indicate that the increase of motion with decreasing distance is in three dimensions.

THE MAGNITUDE-VELOCITY EQUATION

It is of some importance to see what effect this classification with respect to proper motion (and the ellipsoidal axis) will have upon the observed increase of radial velocity with decrease in brightness. The values of ρ_2 and ρ_1 from the stars of small and large μ for all types for the three classes of brightness are given in Tables VIII and IX. ρ_2 and ρ_1 have the same meaning as before. μ is the angular motion on a great circle and ρ_2' and ρ_1' are the axes reduced to unity (ρ_2, μ and ρ_1, μ). The numbers of stars in each group are given in parentheses. The small numbers of stars of 2.9 and brighter of large μ are greatly affected by a few very large motions, as will be seen by the effect of omitting the two largest in each group.

The stars of small μ show no change of velocity with change of brightness. The stars of large μ show a well-defined increase of velocity with decreasing brightness, greatest in the direction of the major axis. When, however, the velocities for the different groups are reduced to a common proper motion, this progression disappears almost completely. The slightly smaller values for the stars of 2.9 and brighter which remain can scarcely be held to furnish any real evidence of an effect due to magnitude. However, on account

[1] *Stellar Movements*, p. 161.

of the very consistent progressions which have been shown in so many other classifications, and until the condition that the nearer bright stars also approximate more closely to the sun in spectral type is satisfactorily accounted for, the conclusion that such progressions have alone resulted from varying mixtures of stars of different distances in relation to ellipsoidal motion will require further careful study.

TABLE VIII

CLASSIFICATION ACCORDING TO BRIGHTNESS

SMALL μ

	ρ_2	ρ_1	ρ_2/ρ_1
	km	km	
$2^{M}9$ and brighter.	(34) 13.5	(34) 6.2	2.18
3.0 to $5\frac{1}{2}$.......	(297) 13.6	(317) 11.8	1.15
$5\frac{1}{2}$ to $6\frac{1}{2}$.......	(179) 12.4	(169) 10.4	1.19

TABLE IX

CLASSIFICATION ACCORDING TO BRIGHTNESS

LARGE μ

	ρ_2	μ	ρ_2'	ρ_1	μ	ρ_1'	ρ_2/ρ_1	ρ_2'/ρ_1'
	km		km	km		km		
$2^{M}9$ and brighter	(15) 16.6	0.″42	39.5	(28) 13.8	0.″62	23.1	1.28	1.71
	(13) 15.9	0.29	54.8	(26) 13.6	0.38	35.8	1.17	1.53
3.0 to $5\frac{1}{2}$......	(96) 26.4	0.38	69.3	(212) 17.6	0.44	40.2	1.57	1.72
$5\frac{1}{2}$ to $6\frac{1}{2}$.......	(31) 45.2	0.64	70.6	(49) 22.3	0.56	39.8	2.03	1.77

The tentative suggestion of B. Boss,[1] "it would appear that within this region (a space around the sun having a radius equal to the distance represented by $M=6''$ [$\pi=0.''015\pm$]) the evolution of stars has been going on so long that almost all the B stars that may have once existed there had either moved out or had developed into older types," may have a bearing on this point. A difficulty for the first of these explanations would arise, however, if in reality the velocities depend simply upon distance or, what seems more probable, on their position in a system which has different velocities in different parts. *Such a condition would not provide any mechan-*

[1] *Astronomical Journal*, 26, 189, 1911.

ism by which a sorting-out or mixing process could take place, such as would result from increases of velocity with age or decreasing brightness. Under such circumstances, in order to account for the tendency to concentration of the intermediate spectral types (with larger motions) nearer the sun, it would seem necessary to suppose that these nearer stars form a group which has had a separate origin or different development from the rest of the system, as indicated by Boss in the second of his suggestions above.

Such an assumption destroys at once the idea of a common origin in time and space for all the stars of our system. Whether such a hypothesis is tenable remains to be seen. If the stars have in reality evolved from the great irregular nebulae (of which we see some signs), such a condition would appear to favor such a theory of separate origins for groups of stars in the same system, in place of a single common history.

So long as the increase of velocity with proper motion was supposed to have no relation to distribution in space, but to be purely accidental in that respect, some mechanism of acceleration which would account for such differences of velocity as have been so abundantly observed was necessary. If, however, the relation of radial velocity to proper motion is in fact a relation to distance, it is almost certainly connected with some general motion of the system, and the necessity for accelerations is much less or perhaps non-existent. Motions which would very well satisfy the conditions as this investigation shows them might be conceived to exist in the spiral nebulae, for example, where the stream velocities near the center would be in very different directions from those in the outer portions of the arms, and where the actual velocities may differ widely also.

The suspicion exists that behind this apparently simple dependence of preferential motion upon proper motion or distance lurks another cause which is related to, or is, the real general motion (or motions) of the system. I have encountered in practically every considerable group of stars which I have examined and discussed in other ways, particularly in the radial velocities, a tendency among the residuals *not* to follow the law of accidental errors even approximately. That this is but another manifestation of the effects of

Kapteyn's two streams seems very probable. These peculiarities are being made the subject of a separate investigation.

Discussion of many questions and relations which suggest themselves, including the bearing upon the general motions of the stellar system, is reserved, awaiting more complete confirmation and the conclusion of other investigations, particularly of the stars of small μ. Although these latter appear to be devoid of preference for the ellipsoidal axis, they give some indications of a different form of preferential motion.

CONCLUSIONS

I. The preference for motion in the direction of the ellipsoidal axis appears to be confined to the stars of larger proper motion and brighter than 3.0 magnitude in all of the spectral classes, with the possible exception of K and M. The limit of proper motion below which little or no preference for the ellipsoidal axis is shown appears to be about $0''.05$.

II. The increase in prolateness in the stars of large proper motion can be traced chiefly to the direction of the ellipsoidal axis.

III. This relation to proper motion furnishes a satisfactory explanation of the peculiar behavior of the stars of class B.

IV. The radial velocities of the nearer stars are larger than those of the more distant stars.

V. When classified according to size of proper motion, and when a few large motions are excluded, there is no certain increase in the radial velocities of the different spectral classes.

VI. When the radial velocities are classified according to size of proper motion, there is no certain change with magnitude.

Observatorio Nacional Argentino, Córdoba
March 31, 1917

NOTES ON THE COLOR-CURVE AND LIGHT-ELEMENTS OF W URSAE MAJORIS[1]

By HARLOW SHAPLEY AND J. VAN DER BILT

The variable W Ursae Majoris[2] is of particular interest because of its exceptionally short period (four hours between minima) and because when interpreted as an eclipsing binary its mean density is more than twice that of the sun. Discussions of the orbit have been published by H. N. Russell[3] and H. Shapley.[4] A further study of the light-variations and orbit, on the basis of simultaneous photographic and photo-visual observations, was begun with the 60-inch reflector in 1916. The work was discontinued, however, upon learning that a similar investigation, based on Potsdam visual observations and unpublished photographic measures from Harvard, was already under way at Princeton.[5] The following pages contain a brief discussion of the color-curve obtained from the preliminary Mount Wilson observations, and an investigation of the light-elements based upon a considerable number of published and unpublished observations of minima during the last thirteen years.

OBSERVATIONS AT MOUNT WILSON AND UTRECHT

The magnitudes of the comparison stars used on the photographs with the 60-inch reflector were determined in the usual manner from ten polar-comparison plates (Table I). Magnitudes of the variable, derived from a series of Seed 27 and Cramer isochromatic plates made on April 26, 1916, are given in Table II. Six, seven, or eight exposures were made on each plate with exposure-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 140.

[2] W Ursae Majoris = B.D. $+56^\circ 1400$; $\alpha = 9^h 36^m 43^s$, $\delta = +56^\circ 24'.5$ (1900.0); spectrum G.

[3] *Astrophysical Journal*, **36**, 139, 1912.

[4] *Contributions from the Princeton University Observatory*, No. 3, p. 82, 1915.

[5] Since this paper was written the Princeton results have appeared in *Astrophysical Journal*, **45**, 306, 1917.

times of one-half, one, or two minutes. The phases were computed with the formula given below. A plot of these results shows that the color-index on the Mount Wilson system is about +0.85, with little evidence of change during the cycle of light-variation.[1]

TABLE I

MAGNITUDES OF COMPARISON STARS

Star	B.D.	*H.A.*, 63 Visual	Mount Wilson		Color-Index
			Photo-visual	Photographic	
a	+56°1397	. . . .	6.64	7.65	+1.01
b	+56 1399	8.74	8.76	9.90	+1.14
c	+56 1398	8.73	8.80	10.08	+1.28

TABLE II

MOUNT WILSON OBSERVATIONS OF W URSAE MAJORIS
April 26, 1916; J.D. 2420980

Photographic					Photo-visual				
Plate	Gr. H.M.T.	Apertures	Phase	Mag.	Plate	Gr. H.M.T.	Apertures	Phase	Mag.
3034P .	^{d}672	14, 9, 6	−0^{d}142	8.76	3035P	^{d}676	14, 9, 6	−0^{d}138	7.91
3036. . .	.702	14, 9	− .112	8.70	3037. .	.706	14	− .108	. . . . . .
3038. . .	.720	14, 9	− .094	8.64	3039. .	.724	14, 9	− .090	7.88
3040. . .	.737	14, 9	− .077	8.81	3041. .	.740	14, 9	− .074	7.96
3042. . .	.754	14, 6	− .060	8.84	3043. .	.758	14, 9	− .056	8.07
3044. . .	.771	14, 9	− .043	8.96	3045. .	.774	14, 9	− .040	8.03
3046. . .	.789	14	− .025	9.06	3047. .	.792	14, 9	− .022	8.18
3048. . .	.802	14, 9, 6	− .012	9.20	3049. .	.805	14, 9	− .009	8.41
3050. . .	.814	14, 9	.000	9.27	3051. .	.817	14, 9	+ .003	8.29
3052. . .	.829	14, 9, 6	+ .015	8.79	3053. .	.832	14, 9	+ .018	8.15
3054. . .	.841	14, 9	+ .027	8.67	3055. .	.844	14, 9	+ .030	7.98
3056. . .	.856	14, 9, 6	+0.042	8.52	3057. .	.860	14, 9, 6	+0.046	7.81

Though the light-curves of this variable often show an asymmetry which, if real, could not be accounted for completely by the simple eclipse theory, the star evidently is not a Cepheid variable, as is sometimes supposed, since no conspicuous change of color is found. Moreover, Russell has no difficulty in interpreting the light-curve, or in explaining the differences between visual and photographic curves through differences in darkening at the limb.[2]

[1] The last two plates are uncertain because of the low altitude of the field.

[2] *Popular Astronomy*, **25**, 30, 1917.

At the Utrecht Observatory 105 observations of W Ursae Majoris were obtained with the polarizing photometer attached to the $4\frac{1}{2}$-inch refractor.[1] At maximum the variable is a little too bright for this instrument, but the minima can be determined very sharply. The following comparison stars were used:

a = B.D. $+54^\circ 1329$ = Potsdam Gen. Cat. 5731;	Mag. =	7.73
b = $+56^\circ 1412 = a + 0^{\mathrm{M}}.33$		8.06
c = $+56^\circ 1399 = a + 1\ \ 38$		9.11

The brightness of both b and c was derived from the measured differences with a.

The method of observation differed somewhat from the usual one inasmuch as more settings were made on the rapidly changing variable than on the comparison stars. In the interval covered by the observations on each evening three to five measures of each of the comparison stars were secured, and these were plotted against the time on squared paper. After allowance for extinction, the settings were usually remarkably constant and could be read accurately for the times corresponding to the measures on the variable. In this way the latter could be closely followed through the steep ascents and descents of its light-curve.

The Utrecht observations are given in detail in Table III, the fifth column containing the differences in hundredths of a magnitude between the adopted value and that given by each of the comparison stars.

THE LIGHT-ELEMENTS

Since about 6000 epochs have elapsed since the latest discussion, a revision of the light-elements can now be made. In view of the possibility that the light-curve may be changeable in form, as many individual minima are included in the computation as can be derived with sufficient accuracy from the published series of observations.

The list in Table IV contains 67 minima, extending over an interval of 29,273 epochs. That numbered 61 has not been used in the derivation of elements. Nos. 59 and 60 were received from Professor Jordan after the completion of the solution and have been

[1] The instrument has been described in *Recherches astronomique de l'Observatoire d'Utrecht*, **6**, 10, 1916.

TABLE III

UTRECHT OBSERVATIONS OF W URSAE MAJORIS

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals* *a*	*b*	*c*
1916	0082.461	0^d071	8.42	+5,	....	−5
April 28	.505	116	8.16	−2,	....,	+3
	.548	158	7.72	+1,	....	0
	0083.408	.018	8.07	−9,	....,	+9
	.420	.029	8.11	−7,	....,	+7
	.429	.039	8.06	−5,	....,	+5
	.440	.049	8.14	−1,	....,	+2
April 29	.452	.062	8.24	0,	....,	0
	.503	.113	8.30	+1,	....	0
	.515	.125	8.23	+2,	....	−2
	.524	.134	7.97	+3,	....	−2
	.535	.144	7.90	+4,	....,	−3
	.544	.153	7.87	+5,	....,	−5
	0995.408	.008	7.80	−11,	....,	+12
	.421	.021	7.94	−9,	....,	+10
	.428	.028	7.87	−8,	....,	+8
	.433	.033	7.92	−7,	....,	+7
	.438	.038	8.02	−6,	....,	+7
	.444	.044	8.06	−5,	....,	+6
	.450	.050	8.15	−5,	....,	+5
	.459	.059	8.24	−4,	....,	+5
	.468	.068	8.48	−4,	....,	+5
May 11	.473	.073	8.43	−4,	....,	+4
	.480	.080	8.65	−4,	....,	+5
	.489	.089	9.09	−3,	....,	+3
	.495	.095	8.71	−1,	....,	+2
	.501	.101	8.76	0,	....,	+1
	.508	.108	8.54	+1,	....,	0
	.514	.114	8.30	+1,	....,	0
	.525	.125	8.16	+1,	....,	0
	.547	.147	7.99	+2,	....,	−1
	.556	.156	7.99	+1,	....,	0
	1001.410	.005	7.92	....,	+1,	−1
	.423	.018	7.92	....,	+1,	0
	.428	.023	7.88	....,	0,	0
	434	.029	7.97	....,	0,	0
	.438	.033	7.99	....,	0,	0
	.448	.043	7.97	....,	−2,	+3
	.461	.056	8.14	....,	−3,	+3
	.465	.060	8.19	....,	−3,	+3
	.469	.065	8.28	....,	−3,	+4
May 17	.474	.069	8.39	....,	−4,	+5
	.483	.078	8.48	....,	−5,	+5
	.486	.082	8.60	....,	−5,	+5
	.491	.086	8.69	....,	−5,	+5
	.495	.090	8.70	....,	−5,	+5
	.500	.095	8.55	....,	−5,	+6
	.505	.101	8.44	....,	−5,	+5
	.517	.112	8.32	....,	−5,	+6
	.527	.122	8.02	....,	−6,	+6
	.536	0.131	7.89	....,	−5,	+6

* A positive residual means that the adopted magnitude is brighter than that derived from the omparison star.

TABLE III—*Continued*

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals *a* *b* *c*
1916				
May 19..........	1003.408	0d001	7.95	+8, −4, −3
	.415	.009	7.88	+5, −3, −3
	.424	.019	7.98	+3, −2, −2
	.429	.024	7.96	+1, −1, −1
	.434	.027	8.00	0, +1, −1
	.439	.034	7.99	−2, +2, 0
	.448	.043	8.01	−4, +3, 0
	.456	.051	7.99	−4, +4, +1
	.465	.060	8.18	−4, +3, +2
	.469	.064	8.26	−4, +3, +2
	.473	.068	8.50	−4, +2, +3
	.478	.072	8.53	−4, +2, +3
	.482	.076	8.41	−5, +1, +3
	.487	.081	8.47	−5, 0, +4
	.496	.090	8.70	−3, −1, +4
	.500	.094	8.71	−3, −1, +3
	.505	.098	8.65	−2, −1, +3
	.510	.104	8.41	−2, −1, +2
	.520	.113	8.26	0, 0, +1
	.530	.123	8.05	−1, +2, −2
	.541	.135	7.93	−3, +4, 0
	.554	.148	7.89	−6, +5, +1
May 21..........	1005.415	.007	8.18	...., +2, −2
	.421	.013	8.03	...., +3, −3
	.433	.025	8.10	...., +6, −6
	.443	.035	7.90	+9, +1, −10
	.451	.043	8.01	+7, +3, −9
	.459	.051	7.97	+6, +2, −7
	.466	.058	8.14	+5, 0, −6
	.472	.064	8.30	+4, 0, −5
	.475	.067	8.35	+5, 0, −5
	.479	.071	8.44	+5, −1, −4
	.483	.075	8.60	+5, −1, −4
	.487	.079	8.63	+5, −2, −3
	.490	.082	8.54	+5, −2, −2
	.496	.088	8.84	+5, −2, −4
	.500	.092	8.86	+6, −1, −5
	.505	.097	8.70	+7, 0, −6
	.509	.101	8.62	+8, 0, −8
	.516	.108	8.51	+10, +1, −11
	.526	.118	8.28	+10, +2, −13
	.533	.125	8.09	+11, +2, −14
May 28..........	1012.420	.007	8.08	...., +3, −2
	.432	.018	8.01	...., 0, 0
	.437	.023	7.96	...., −1, +2
	.442	.028	7.93	...., −3, +3
	.450	.036	8.07	...., −6, +6
	.459	.045	8.26	...., −6, +6
	.480	.067	8.52	...., −2, +2
	.486	.073	8.44	...., 0, 0
	.499	.085	8.74	...., 0, +1
	.500	.095	8.57	...., −2, +2
	.517	.104	8.54	...., −1, +2
	.525	0.112	8.34	...., +1, 0

TABLE IV

List of Minima

No.	Observed Minima Date			Gr. M.T.	Red. to Sun	Epoch	Obsd. J.D. (Helioc.)	O.−C. Days	O.−C. Min.	Observer	Remarks	Reference
1	1903	Jan.	14	4h 34m	+6m	0	6129.194	−0.006	8.0	MK	....	Ap. J., **17**, 205
2			17	4 40	+6	18	6132.199	− 4	5.2	"	....	
3			17	8 31	+6	19	6132.359	− 11	15.3	"	....	
4			18	8 34	+6	25	6133.361	− 10	13.8	"	....	
5		Feb.	20	5 25	+6	222	6166.230	− 4	5.2	"	....	A.N., 4005
6		Apr.	24	10 51	0	601	6229.452	− 7	9.5	"	....	
7	1904	Jan.	10	8 50	+6	2165	6490.372	+ 7	10.7	F	....	A.N., 3963
8			17	8 59	+6	2207	6497.378	+ 7	10.7	"	....	
9			27	5 17	+6	2266	6507.224	+ 11	16.4	"	Uncertain	
10			27	9 12	+6	2267	6507.387	+ 7	10.7	"	Good	
11		Feb.	13	5 14	+6	2368	6524.222	− 7	9.5	MK	....	A.N., 4005
12			17	9 40	+6	2393	6528.407	+ 8	12.1	F	Fair	A.N., 3963
13			19	5 43	+6	2404	6530.242	+ 8	12.1	"	Fair	
14		Mar.	14	6 30	+4	2548	6554.274:	+ 18:	26.5	"	Uncertain	
15	1905	Jan.	17	8 43	+6	4401	6863.367	− 6	8.0	MK	....	A.N., 4005
16		Nov.	9	19 15	+1	6178	7159.803	− 8	11.8	PJ	Fair	Ap. J., **23**, 84
17		Dec.	7	20 7	+4	6346	7187.841	+ 4	5.9	"	Poor	
18	1906	July	26	9 7	−6	7728	7418.376	− 6	8.0	MK	Good	A.N., 4128
19	1907	Mar.	4	5 59	+5	9052	7639.253	+ 2	3.3	T	Orange filter*	*Pulkowa Mitt.*, 2, No. 21
20			5	10 0	+5	9059	7640.419	+ 1	0.9	"	Green-blue†	
21			7	9 55	+5	9071	7642.417	− 3	4.9	"	Orange‡	
22			24	6 24	+3	9172	7659.269	− 0	0.1	"	Orange*	
23			29	6 36	+3	9202	7664.277	+ 3	4.9	"	Ultra-violet*	
24			30	6 40	+3	9208	7665.280	+ 3	4.9	"	Ultra-violet*	
25		Apr.	1	6 42	+2	9220	7667.281	+ 5	6.6	"	Ultra-violet‡	
26			10	6 54	+2	9274	7676.289	+ 4	6.2	"	Ultra-violet*	
27			12	6 54	+1	9286	7678.288	+ 2	2.2	"	Orange§	
28			13	6 59	+1	9292	7679.292	+ 5	6.6	"	Green-blue‡	
29			15	7 1	+1	9304	7681.293	+ 4	5.5	"	Green-blue†	
30			29	11 11	0	9389	7695.466	− 3	4.2	"	Orange*	
31		May	1	11 23	0	9401	7697.474	+0.003	4.8	"	Ultra-violet*	

32	1907	June 8	8h 9m	−4m	9628	7735.337	−0.002	2.3	Ba	Fair	
33		9	8 14	−4	9634	7736.340	+ 0	0.6	"	Good	
34		19	8 24	−5	9694	7746.347	− 2	2.3	"	Fair	
35		July 18	9 7	−6	9868	7775.376	+ 0	0.6	"	Good	
36		22	9 7	−6	9892	7779.376	− 4	5.2	"	Good	
37		25	9 10	−6	9910	7782.378	− 4	5.2	"	Good	
38		Aug. 6	9 23	−6	9982	7794.387	− 6	8.0	"	Fair	
39		11	9 38	−6	10012	7799.397	− 1	0.9	"	Uncertain	*M.N.*, **69**, 86
40	1908	Jan. 2	8 45	+6	10875	7943.369	+ 6	9.2	"	Good	
41		14	8 56	+6	10947	7955.376	+ 2	3.5	"	Fair	
42		Mar. 25	10 28	+3	11373	8026.438	− 1	0.9	"	Fair	
43		Apr. 15	10 56	+2	11499	8047.457	− 2	2.3	"	Fair	
44		26	11 18	0	11565	8058.471	+ 2	3.5	"	Good	
45		May 3	11 28	−1	11607	8065.477	+ 2	3.5	"	Good	
46		14	11 44	−2	11673	8076.487	+ 2	3.5	"	Good	
47	1910	Apr. 4	6 44	+2	15808	8760.282	− 2	2.3	H	Good	
48		6	6 43	+2	15820	8768.281	− 5	6.6	"	Good	*Riv. di Astr.*, **4**, 310
49		7	6 58	+2	15826	8769.292	+ 5	7.8	"	Fair	
50	1911	July 9	9 2	−6	18572	9227.372	− 2	2.3	L	Good	*A.N.*, 4542
51	1913	June 25	9 11	−5	22870	9944.379	+ 15	21.6	P	Uncertain	
52		July 1	9 21	−6	22906	9950.385	+ 15	22.4	"	Fair	
53		3	8 53	−6	22918	9952.366	− 6	8.0	"	Fair	
54		4	9 2	−6	22924	9953.372	− 1	0.9	"	Good	*Mem. Sp. It.*, **2**, 193
55		5	9 2	−6	22930	9954.372	− 1	0.9	"	Fair	
56		6	8 40	−6	22936	9955.357	− 17	22.5	"	Uncertain	
57		7	8 54	−6	22942	9956.366	− 9	12.4	"	Uncertain	
58		8	9 15	−6	22948	9957.381	+ 5	7.8	"	Uncertain	
59	1915	Jan. 28	17 19	+6	26361	0526.726	− 5	7.5	J	....	Letter
60		29	17 22	+6	26367	0527.728	− 4	5.9	"	....	
61	1916	Apr. 26	19 25	0	29083	0980.809	− 5	7.3	Sh	Good	
62		29	11 28	0	29099	0983.478	− 6	8.0	Bi	Fair	
63		May 11	11 48	−2	29171	0995.490	− 5	6.6	"	Good	
64		17	11 53	−2	29207	1001.494	− 6	8.0	"	Good	This paper
65		19	11 54	−2	29219	1003.494	− 8	11.0	"	Good	
66		21	11 59	−3	29231	1005.497	− 7	9.5	"	Good	
67		28	12 14	−3	29273	1012.508	−0.002	2.3	"	Fair	

* Good. † Uncertain; minimum phase very flat. ‡ Uncertain. § Uncertain; light-curve discordant.

used only to indicate the representation in 1915 by the adopted elements.[1]

For the designation of the observers the following abbreviations are used, the method of observation being indicated in parentheses: MK = Müller and Kempf (photometric); F = Fauth (estimates); PJ = Parkhurst and Jordan (photographic); T = Tikhoff (photographic); Ba = Baldwin (photometric); H = Horn (photographic); L = Lazzarino (photometric); P = Padova (photometric); J = Jordan (photographic); Sh = Shapley (photographic); Bi = Van der Bilt (photometric).

The following points are to be noted:

1. The minima published by Müller and Kempf and those by Parkhurst and Jordan have been taken as they stand. There are two misprints in the latter's comparison with Müller and Kempf's data;[2] the calculated time of the minimum of December 7 should be 19^h55^m, and the residual minus instead of plus.

2. The detailed observations of Fauth, made by Argelander's method, are not accessible. Though the results are probably less accurate than those obtained by photometric methods, we have not thought ourselves justified in rejecting them on account of this a priori assumption.

3. All the observations made by Tikhoff have been plotted and several minima derived which had not been discussed previously. Though the various filters seem to give systematic differences in the time of minimum, the accompanying tabulation, based on data

Color Screen	Residuals (Unit is 0^d0001)	Mean Residual
Orange	+23, −33, − 1, +15, −29	− 5
Green-blue	+6, +46, +38	+30
Ultra-violet	+34, +34, +46, +43, +33	+38

[1] For the sake of completeness, times of minima have also been computed for the new Harvard photographic observations from the data given by Russell, Fowler, and Borton in Table IX, *Astrophysical Journal*, **45**, 321, 1917. The representation of these times by our adopted light-elements is as follows:

Mean Epoch	1,120	7,380	16,000	24,600
O.−C.	$+0^d0015$	$+0^d0008$	$+0^d0016$	-0^d0004

[2] *Astrophysical Journal*, **23**, 84, 1906.

in Table IV, shows that these differences hardly exceed the observational uncertainty.[1] The individual measures have a considerable accidental error, and as only one of the color-filters was used at each minimum systematic errors may be involved if the period is not strictly uniform.

4. All the observations by Baldwin have also been plotted, and with the aid of the mean light-curve given by him several new minima were derived from incomplete light-curves and are available for the discussion.

5. Horn, working with photographic trails, gives the times of three maxima. Since it is uncertain whether the minima occur exactly half-way between the maxima, and since the latter can usually not be determined with the same accuracy as the minima, we have made a new grouping of Horn's observations for an independent derivation of the minima. A striking feature of his mean light-curve, entirely differing from the results of other observers, is that the horizontal line through the points of median magnitude is divided by the light-curve into approximately equal parts. The curves obtained by other observers all indicate that the part of this line below the maximum is nearly three times as long as that above the minimum. The actual figures are:

Müller and Kempf	$0^d.125$	and	$0^d.042$
Baldwin	0.121	"	0.046
Van der Bilt	0.125	"	0.042
Horn	0.097	"	0.070

Further, one (and probably two) of Horn's observed curves shows a constant minimum of more than 25 minutes. The explanation of both phenomena may be that the photographic trails at minimum were so faint that they could not be measured accurately.

6. The mean light-curve published by Padova was derived from a very non-homogeneous grouping. A new mean curve has been constructed and the separate minima determined as accurately as the observations allow.

[1] For a note on the interpretation of Tikhoff's results see Kron, *Potsdam Publication*, **22**, Pt. 3, p. 56, 1912.

The 64 minima used in the computation yield the following new light-elements:

$$\text{Heliocentric Minimum} = \text{J.D. } 2418075.8176 + 0^{\text{d}}.1668196E, \text{ Gr.M.T.},$$

the period of revolution being double the interval between minima.

The residuals to which the foregoing formula leads are given, both in fractions of a day and in minutes of time, in the sixth and seventh columns. The mean residual is $0^{\text{d}}.0050$. The algebraic sum of the residuals is $-0^{\text{d}}.0014$. There are 30 positive and 34 negative residuals, 37 recurrences of sign, and only 26 changes, which points to some systematic change in the period. The evidence for this is strengthened by the fact that the first and the last six residuals are all negative, with practically the same mean value, viz.:

Mean epoch =	148,	Mean residual =	$-0^{\text{d}}.0066$
" " =	29,200,	" " =	-0.0053.

Were the time of normal minimum displaced by this amount, all but four of the intermediate residuals would become positive.

There is, however, no indication of a systematic difference between the odd and even minima. We derive:

Mean residual of 39 even minima $= +0^{\text{d}}.0005$
Mean residual of 25 odd minima $= -0.0009$

a difference wholly covered by the observational errors.

From the present study a perturbation of some character appears certain, but the data are as yet insufficient to determine the law of variation.

SUMMARY

1. Mount Wilson photographic observations of W Ursae Majoris, the star of highest known density, give a color-index in agreement with its spectral type (G), and show no conspicuous change of color between minimum and maximum light.

2. From a series of photometric measures made at the Utrecht Observatory (Table III) the times of six minima have been derived.

3. A study of 64 minima, distributed throughout an interval that includes 29,000 recurrences of minimum light, reveals a small but distinct perturbation of the period (Table IV).

MOUNT WILSON SOLAR OBSERVATORY
June 1917

BAXANDALL'S EXPLANATION OF AN "ABNORMAL" SOLAR SPECTRUM[1]

By GEORGE E. HALE

The following letter from Mr. Baxandall gives a simple and adequate explanation of two "abnormal" photographs of the solar spectrum in the neighborhood of a sun-spot, taken at the Kenwood Observatory in 1894, and described in this *Journal* in 1902.[2] The photographs in question were among several made in parallel strips on a single photographic plate, for the purpose of ascertaining the exposure times required for spectra of different orders. The slit of the spectrograph happened to lie across a sun-spot in the 2-inch solar image given by the Kenwood refractor, and most of the photographs on the plate show the usual bright reversals of H and K over the spot region. In two of them, however, these reversals were absent, and the spectra were so changed in appearance as to be unrecognizable. The idea that their peculiarities might be due to the chance superposition of two spectra of different orders was entertained at the time, but the absence of the bright H and K reversals where they should have appeared, together with other causes, unfortunately led us to dismiss this explanation. Finally, after much hesitation because of the improbable character of the spectrum, and after copies of the photographs had been sent to a number of spectroscopists in the hope that an explanation could be found, I published the photographs, together with measures of the lines and estimate of their intensities by Dr. Adams. These showed that most of the lines of the "abnormal" spectra were solar lines in Rowland's table, greatly changed in relative intensity. One of the "abnormal" spectra was intermediate in character between the solar spectrum and the other "abnormal" spectrum, and both showed two unknown bright lines, which, with other features mentioned by Mr. Baxandall, should

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 143.

[2] "Solar Research at the Yerkes Observatory," *Astrophysical Journal*, **16**, 220, 1902.

have furnished us the clue so effectively recognized and applied by him. The fact that the apparent wave-lengths of these bright lines changed appreciably between the two exposures helped to mislead me, but I shall not attempt to offer any excuse for being so easily deceived.

Mr. Baxandall's letter is as follows:

SOLAR PHYSICS OBSERVATORY
Cambridge, June 4, 1916

DEAR PROFESSOR HALE:

It is only after considerable hesitation that I have decided to send you the present communication. It relates to your paper of 1902, containing an account of an apparently anomalous solar spectrum which has been considered to be due to a fleeting upheaval in the solar atmosphere. Doubtless, in common with many others engaged in spectroscopic investigations, I have always felt somewhat sceptical as to the reality of the phenomenon to which the abnormality of the spectrum was supposed to be due, although I have never been able to explain the strange spectra from any other point of view. My unbelief was based mainly on the following heads:

1. The isolated nature of the type of abnormality shown.
2. The restricted part of the spectrum in which it revealed itself.
3. The fact that there was no kind of lead given by the type of line or chemical elements involved.

I have on many occasions referred to your plates in the hope of getting some light on the real origin of the peculiar spectra, but with no success, until on Friday last, while examining the spectra again, I became conscious of the fact that the grouping of many of the most outstandingly strong lines in the abnormal spectrum had a familiar aspect, and the aspect was that of some of the chief landmarks in the solar spectrum itself. Thus the three strong lines 3921.87, 3927.77, and 3930.45 of the abnormal spectrum and the weaker bunch of lines 3916.84–3918.50 to the left, struck me as being uncommonly like the Fe triplet 4045, 4063, 4072, and the manganese bunch 4030–4036 of the normal solar spectrum. Then it dawned on me that probably by some chance circumstance, a small-dispersion solar spectrum had been superposed on your third-order spectra. Measuring on your reproduction the range of the three strong lines mentioned, I found it was exactly one-third the space occupied by the three Fe lines 4046–4072 on your third-order spectra. This suggests that the superposed spectrum was one of the first order of the grating used in your research. Employing this measured interval between your abnormal lines 3921.87, 3930.45 as a unit, and working backward on your spectra, I found that the two strange narrow bright lines 3884.64, 3896.21 agree in position with the H and K lines of the superposed spectrum. As these bright lines apparently extend equal distances on each side of the spot band, they are probably reversals of H and K in the added spectrum.

Working in the same way to the right, and taking the most outstanding line λ 3981 of your list, this revealed itself as the 4226.9 (Ca) line of the superposed spectrum. As the same proportion of one to three held in every case where intervals between solar landmarks were considered, it seems to me that the evidence is very conclusive that the strengthened lines in the abnormal spectrum are really due to the strong and outstanding lines of the small-scale superposed spectrum. The fact that it is only over a restricted region of your large-scale spectra where the abnormalities occur is also in favor of the contention that a small-scale superposed spectrum is involved.

As the intermediate spectrum shows a far closer resemblance to the normal solar spectrum, I have not thought it worth while to investigate in a similar manner to the above the irregularities in that spectrum, especially as, if it is satisfactorily established that any superposition of spectra has taken place, the whole argument for an upheaval in the solar photosphere must of necessity fall to the ground.

I feel sure that if a solar spectrum is prepared of such a scale that the lines 4071.9 Fe, 4226.9 Ca, on it are the same interval apart as the abnormal lines 3930.45, 3981.98 of your reproduction, and the strips so adjusted that these two pairs fit, the counterparts of the outstanding groups and lines of the normal spectrum will be apparent in your reproduced abnormal spectra along the whole range of the superposed spectrum. A few of the normal groups and lines will doubtless be modified in intensity where an outstandingly strong line of your "real" third-order spectrum blends with a line or group in the superposed spectrum.

I append a short list of lines which I consider, from rough measurements on the reproductions, to be identical: that is, the apparently abnormal lines of the reproduction, which I think are the real lines of the superimposed spectrum.

After I had come to the conclusion expressed above, I noticed that you mention in your paper having a suspicion at the time of the investigation that superposed spectra might be the cause of the apparent anomaly, but that after consideration you had come to the conclusion that that view was untenable. There is now little or no doubt in my mind that this is the real explanation. At the same time, I concede that, if you still have the original plates, you have the best opportunity of testing the points I raise.

I may mention just one other point. If the two bright narrow lines are not H and K reversals of the added spectrum, what are the chances against your measured wave-length interval between the two (when assumed to be real lines in the third-order spectra) being exactly one-third of the real wave-length interval between H and K? Thus your mean interval between the lines is 11.635 $\mu\mu$. One-third the interval between H and K is 11.000 $\mu\mu$. The difference between these two is probably within the errors of measurement.

Yours very truly,

F. E. BAXANDALL

All the lines in the first column are well-strengthened lines in the abnormal spectrum, and all those in the second column are conspicuous lines in the

Strengthened Lines in "Abnormal Spectrum"	Corresponding Lines in Superposed Spectrum
3884.67 (bright)	3933.83 K (Ca) reversal
3896.21 "	3968.63 H (Ca) "
3908.41	4005.41 Fe
3908.90	4007. Narrow solar group
3916.84 to 3918.50	4030.9 to 4035.9 Mn group
3921.87	4045.98 Fe (Well-known triplet)
3927.77	4063.76 Fe (Well-known triplet)
3930.45	4071.91 Fe (Well-known triplet)
3972.61	4198.40 to 4199.27 Narrow solar group
3973.74	4202.20 Fe
3981.98	4226.90 Ca
3989.91	4250.29 Fe, 4250.95
3993.05	4260.15 Fe, 4260.64
3996.80	4271.33 Fe, 4271.93
4005.41 to 4011.03 wide hazy group	4299. to 4315. Fraunhofer's "g" group
4013.95	4324. Narrow solar group
4014.70	4325.94 Fe
4033.81	4383.72 Fe (Well-known triplet)
4040.80	4404.93 Fe (Well-known triplet)
4044.19	4415.29 Fe (Well-known triplet)

normal solar spectrum. If any two of the former be taken it will be found that the interval between them is approximately one-third the interval between the corresponding two of the latter. The chance against this being a fortuitous result seems to me to be "beyond arithmetic."

As a further check we may compute the approximate wave-lengths of the solar lines corresponding to lines of the "abnormal" spectrum, on the assumption that a first-order spectrum is super-

posed on the third order, λ 3921.87 of the third order being coincident with λ 4045.98 of the first order.

Abnormal	Calculated First Order	O.−C.
3884.5 bright......	3933.3 (K)	+0.5
3896.1 "	3968.3 (H)	+0.3
3927.8............	4063.8	0.0
3930.4............	4071.9	0.0
3940.3............	4101.6	+0.3
3950.5............	4132.3	−0.1
3954.4............	4144.0	0.0
3982.0............	4227.2	−0.3
4014.7............	4325.9	0.0
4033.8............	4383.5	+0.2
4040.8............	4404.6	+0.3
4044.2............	4414.8	+0.5

Evidently no further evidence is needed to establish the truth of Mr. Baxandall's conclusions.[1]

Mount Wilson Solar Observatory
October 8, 1917

[1] Absence from the Observatory and the pressure of other duties have unduly delayed the re-examination of the original photographs and the publication of this note, which has experienced the fate of several other astronomical manuscripts, essentially completed many months ago, but still unpublished.

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

By HENRI BUISSON

Attention has been called to this subject in the two recent papers by H. E. Ives[1] and by H. N. Russell.[2] In closing his paper Russell expressed the desire that experiments on this point should be made in the laboratory.

I recently had occasion to make some determinations of the limits of sensibility of the eye, and I believe that it would be interesting to communicate these results in spite of the fact that they are not very numerous or extensive.

I undertook photometric measures on phosphorescent screens similar to those employed to make visible in darkness certain objects, such as the hands of a watch. These screens are covered with ZnS rendered luminous by the addition of a salt of radium. These are very faint, and, since their color corresponds nearly to the maximum of sensibility of the eye in the spectrum, they readily serve for determining the limit of sensibility.

The photometric measures were made as relative comparisons, but it has been easy to modify them in connection with absolute measures. The phosphorescent screen was compared with a film diffusing by transmission, which had been previously studied, so that its light was known when it was illuminated by a source of known intensity. It was thus found that the brightness of the phosphorescent screen varied, according to the screen, from 2 to 4×10^{-6} candles per sq. cm.

The screens were circular disks of 2.5 mm and of 5 mm diameter and were examined in darkness at increasing distances, the maximum distance at which they were seen being measured. The necessary precautions were taken to avoid errors of imagination: an assistant moved the screen, and the observer announced the motion that had been made, if he perceived it; or the two disks were placed on the same sheet of cardboard, and the observer stated the direction of the line joining them.

[1] *Astrophysical Journal*, 44, 124, 1916. [2] *Ibid.*, 45, 60, 1917.

The observations have been made in only a limited number, and by a single person having ordinary vision, after remaining in darkness for about 15^m. Table I gives the results, together with the computed distance at which a candle would be still visible.

TABLE I

Diameter of Disk	Brightness in Candles per sq. cm	Intensity in Candles	Maximum Distance of Visibility	Distance of Visibility of a Candle
2.5 mm..........	1.7×10^{-6}	8.4×10^{-8}	7.7 m	26.5 km
5.06.............	4.15×10^{-6}	83.0×10^{-8}	26.0	23.5
5.12.............	2.77×10^{-6}	57.2×10^{-8}	20.0	26.4

The results indicate directly the maximum distance at which a candle is visible. They are, furthermore, in excellent accord, yielding a mean of 27 km. That distance is much greater than has heretofore been accepted (about 11 km). It should be stated, moreover, that an actual candle would not be visible at that distance, as much because of the atmospheric absorption as because of the Purkinje phenomenon, which is so appreciable with the red color of the candle.

It is also easy to express the results in terms of stellar magnitude: assuming that a candle at one km is of magnitude 0.82, we deduce that the limit of visibility would be a star of magnitude 8.0, a value very close to that given by Russell, but obtained by an entirely different method.

It should be added that the luminous surfaces used were seen, at the limit of visibility, under very small angles—that is to say, under conditions which are similar to those under which we observe the stars. If we express in energy the quantity of light received by the eye at the limit of visibility, adopting the same values that Russell did, namely, 1.59 ergs per second per sq. cm for an illumination of one lux, and 0.57 sq. cm for the area of the pupil dilated widely in the darkness (diameter = 8.5 mm), we shall obtain 1.25×10^{-9} ergs per second, a value which is a little higher than that given by Russell.

MARSEILLE
August 1917

SOLAR HYDROGEN "BOMBS"[1]

By FERDINAND ELLERMAN

Visual and photographic observations of a solar phenomenon which had previously escaped our attention have been carried on at this observatory during the past two years.

On September 21, 1915, while the writer was observing the Hα line for reversals and distortions in an active spot-group, there suddenly appeared a very brilliant and very narrow band extending four or five angstroms on either side of the line, but not crossing it. In a couple of minutes it faded away and was not seen again. A month later, on October 21, more observations were recorded and a spectrogram was secured.

On the first occasion the appearance was so extraordinary that it seemed hardly real; after the second observation, however, the existence of such phenomena as part of the solar activity seemed established, and a search has been made for them whenever conditions of seeing and other work have permitted.

There are two conditions essential for observation—good seeing and a large solar image—as the area of the phenomenon, even with the 16-inch image of the sun at the 150-foot tower telescope, is so small that only with difficulty is the point of disturbance kept on the slit.

The appearance of the phenomenon indicates something in the nature of an explosion, in which hydrogen seems to be the only element playing a part. The duration is only a few minutes—from one to three on the average, and from five to ten minutes rarely. This sudden performance suggested the name of hydrogen "bomb," which we have adopted to designate it.

In the *Astrophysical Journal*, **30**, 78, 1909, Dr. Walter M. Mitchell gives an account of solar observations made at Haverford College Observatory, together with a drawing which illustrates the appearance of Hα in the spectrum of a "bomb," and from his

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 141.

description there seems to be no doubt of its being such. This is the only recorded observation of the kind that has come to our notice, and it must have been an unusually large one to have been seen with the instrument used by Dr. Mitchell. Furthermore, he mentions D_3 as showing the band, while our observations have failed to detect it.

While the average width of the Hα band is about 8 A, in an exceptional and probably an extreme case it was as wide as 30 A. In general the band appears symmetrical on each side of the line, but in a few measured cases the extension was greater on the violet side than on the red. In none have we found greater extension on the red side.

The regions where the "bombs" are likely to appear are around and among active spot-groups, especially groups which are developing and composed of many members. In such groups they appear at the outer edge of the penumbra, or at points between the spots, and also at small distances to one side. When a "bomb" appears and fades away, the point of disturbance should be kept under observation, as it is very probable that others will appear in the same place. At times they seem to follow one another like the balls of a Roman candle at intervals varying from ten to twenty minutes or more.

They frequently appear in the faculae so that their spectra are superposed on those of the faculae, thus giving the appearance of great extension to the bright "bomb" band. On rarer occasions they are superposed on bright reversals of Hα over eruptive regions, but this is an uncommon occurrence, and the distinction is easily made between the two phenomena by the flickering of the "bomb" band compared to the Hα reversal, due to the effect of seeing.

The observations have been confined mainly to Hα, although D_1, D_2, D_3; b_1, b_2, b_4; Hβ and Hγ have been examined. None but the hydrogen lines have shown the band. Hβ has been photographed and shows the band fairly well, but not as well as Hα nor of as great width. Hγ also shows the band, but less clearly than Hβ.

The level at which the "bombs" occur must lie well below the reversing layer, as the hydrogen absorption line is not affected by them, nor do they seem to affect the other Fraunhofer lines.

In Plate XVI, *a* and *b*, are shown spectra of "bombs" which appeared in regions where the Hα line shows considerable distortion due to radial velocities of the hydrogen flocculi. It will be noted that the "bomb" bands do not extend as far on each side of Hα as mentioned above, but this is to be expected, as it is impossible to guide and keep the image of the "bomb" on the slit perfectly during the entire exposure, and the fainter extensions which are easily observed visually are lost in the photograph. Plate XVI, *c*, shows the "bomb" band superposed on the spectrum of a facula, and in *d* are shown two bright reversals of Hα near a small spot, to illustrate the difference in appearance between ordinary eruptive reversals of Hα and the bright band due to "bombs."

MOUNT WILSON SOLAR OBSERVATORY
October 1917

REVIEWS

The Principles of Aërography. By ALEXANDER MCADIE, Professor of Meteorology, Harvard University, and Director of the Blue Hill Observatory. Chicago and New York: Rand McNally & Co., 1917. Figs. 114. 8vo, pp. xvi+318. \$3.00.

The usual book review generally falls into one of three fairly distinct classes. In the first place, there is the colorless review in which merely the number of pages, the number of illustrations, the headings of the various chapters, and similar information are given. Then there is the extremely eulogistic review which reads like an advertisement of the book and leads one to believe that it is the best which has been or could be written on the subject. Lastly, there is the extremely critical review in which faults of arrangement and sins of omission and commission are much emphasized. Should these be lacking, then typographical errors are called in, and, should these be in small number, then the incorrect spelling of a single foreign name is dwelt upon to such an extent that the book is made to appear inaccurate and the author unscholarly. The paragraphs which follow may suggest these three kinds of reviews.

The book under consideration consists of 16 pages of preliminary material, 278 pages of text, an appendix of 14 pages, and an unusually long index of 26 pages. It is divided into 18 chapters with the headings: "A Brief History of Meteorology"; "Units and Symbols"; "Temperature-Scales"; "Thermodynamics of the Atmosphere"; "Stratosphere and Troposphere"; "The Circulation of the Atmosphere"; "The Major Circulations"; "The Minor Circulations"; "Forecasting Storms"; "The Winds"; "The Water Vapor of the Atmosphere"; "Condensation"; "Dust and Microbes"; "Atmospheric Electricity"; "Precipitation"; "Floods and Notable Storms"; "Frosts"; "Solar Influences." There are 114 figures, of which a little less than half are illustrations. The rest are charts and diagrams, and they are all in the text. The appendix consists mostly of conversion tables for changing from one set of units to another. The index is simply an index and contains no biographical or additional information.

The title of the book, *The Principles of Aërography*, includes a word which will cause many to wonder if it contains anything different from

what would be expected if the title had been "The Principles of Meteorology." Frankly, it does not. Fifty or a hundred years ago the word "aërography" was perhaps as commonly used as the word "meteorology," but at present it is "meteorology" which is used for the treatment of all atmospheric phenomena, and it is too firmly intrenched to be easily forced to give ground. The author, however, would seem to make a distinction, for in the preface it is stated: "Thus aërography resembles geography in the larger sense, while meteorology, according to the general acceptation of the term, remains the science of recording diverse atmospheric conditions. The chief purpose of aërography is exploration with a view to utilizing the knowledge gained to insure human safety and to expedite progress."

The book is well printed on paper calculated to make the illustrations appear at their best. In fact, the illustrations are one of the chief charms of the book, for they are refreshingly new. They are not the common ones which have been so frequently used. The index is particularly well arranged and useful.

In reading the book one will be struck by the many things which deserve particular commendation. The treatment of the clouds is especially good and well illustrated. Two chapters, comprising 46 pages, are practically given up to them, and, in accord with the author's statement in the preface, they have been treated from the point of view of origin rather than appearance. The long chapter on atmospheric electricity, which is devoted almost entirely to the consideration of the thunder shower and lightning, is again an excellent one. The pictures of lightning flashes taken with a stationary and with a revolving camera are extremely good, and the spectrum of lightning has been given a very extended discussion. It should perhaps be noted that there are many pages in this chapter quoted from an article by another writer which appeared about three years ago. Full credit is, however, always given. The first chapter, on a brief history of meteorology, is very interesting and well written. In the fifth chapter, on the stratosphere and troposphere, will be found a particularly valuable summary of recent upper-air research, a subject which is very inadequately treated in any of the much-used textbooks on meteorology. The treatment of ice storms in chapter xv is far more complete and better illustrated than in any other book.

To what class of readers will the book particularly appeal? There are, in general, three classes of readers to whom a book on meteorology may appeal. These are, first, the general reader of scientific tastes;

second, the college undergraduate or advanced high-school student who is using the book as a textbook; third, the advanced student or reader who has already carefully studied other books on the subject. The book under consideration has numbered paragraphs, each with a definite heading, and marginal comments on the sides of the pages. All these are earmarks of the textbook and lead one to think that the author hoped it might be so used. Judged as a textbook, and thus the first book to be used by a student or general reader, it has many shortcomings. The material is not well arranged; there are too many omissions and not enough elementary detail in the treatment of many subjects. A single example will suffice. There are practically no descriptions or illustrations of the apparatus used for making meteorological observations. But the author says in the preface: "However, no attempt is made to reproduce *in extenso* weather charts and photographs of common instruments; the former are given in official reports and the latter belong more appropriately to the catalogues issued by instrument makers." But to how many readers of the book will a file of the catalogues of instrument-makers be accessible? The author states in the preface: "It is thought advisable that an effort be made to present this new knowledge [of about the last ten years] in a convenient form, even if considerably condensed. . . . . The present book, therefore, aims to give prominence to recent work that has been done in exploration of the air. . . . . Another important reason for offering this volume is the desire to further the use of the c.g.s. system of units. Throughout the book preference is given to absolute units, in the hope that the student will forget as soon as possible the old, arbitrary, and irrational units." The book, then, is to be considered as a reference work for recent research and as an advocate of absolute units. Viewed in this light, the book is all too short and many subjects deserve a more extended treatment. Weather prediction is covered in a single chapter of ten pages, and two of these are given up to a description of tornadoes and two to waterspouts—elementary and not recent material. Practically nothing is said about the weekly forecast of the Weather Bureau or long-range prediction. The semi-permanent highs and lows, often called centers of action, receive fairly brief treatment, as does the whole subject of correlation. Atmospheric optics, including halos and related phenomena, are treated in two pages and that in the chapter on dust and microbes—a grouping of subjects destined to fill one at first sight with dismay. It should, however, be said that in any book an author always expands the things in which he is especially interested, and a reviewer looks for those things in which he

is interested and judges the book by their presence or absence. Recent books and articles are frequently mentioned in the text, but one misses a classified bibliography at the end of each chapter or at the end of the book. The insistent use of absolute units will meet the warm approbation of a few. It will be a matter of indifference or annoyance to the majority of the readers of the book. There is as yet no perfect international agreement as to what absolute units are to be used or what they shall be called. The book thus lays stress on what is still more or less of a controversial point.

In conclusion, it can be stated without hesitation that the book as a whole is a very valuable contribution to meteorological literature, and this would be expected from the position and wide experience of the author. If a list of the ten best modern books on meteorology in English were being made, it would without question be included. How high it would stand in such a list would be a matter of individual judgment.

W. I. M.

THE

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A DIFFERENTIAL SPECTRO-PHOTOMETER

By G. A. SHOOK

The novel feature of this spectro-photometer is the differential double slit, shown in Fig. 1. Two slits A and B are actuated by means of a single micrometer M. The lower jaw of the upper slit A and the upper jaw of the lower slit B are rigidly connected by a movable piece C, as shown.

When the micrometer screw is turned clockwise, C moves downward, making A wider and B narrower. When it is turned counterclockwise, the spring S causes C to follow the screw. The sum of the two slit-widths can be varied by means of the capstan screws D. The lower jaw of the lower slit is held in position by means of a spring, so that the slit will not be injured if the screw is turned too far.

In the instrument, as used by the writer, each slit is 1 mm wide when the other is entirely closed; that is, when both slits are equal in width each is 0.5 mm wide. The micrometer-head is divided into 100 divisions and the pitch of the screw is 1 mm, so that when A is open the micrometer reads 100, when closed, 0, and when both slits are equal in width it reads 50.

The method used for bringing the two photometric fields together is shown in Figs. 2 and 3.

The light after passing through the slits *A* and *B* is rendered parallel by means of two lenses, as shown in Fig. 3. The upper beam passes directly through a photometric cube *C* and thence through an ordinary spectrometer prism *P*. By means of a third lens this beam is brought to a focus at the ocular slit *E*. The photometric cube is a double prism with a strip of silver on the hypotenuse face of the upper prism. When the eye is placed at *E* it sees a field divided into three parts, as shown, the outer parts being illuminated by the light from the upper slit.

The light from *B*, after passing through the lens, is reflected from *D*, then from the silver strip *S*, and finally reaches the ocular slit *E*. The central part of the field is therefore illuminated by the light from *B*.

The telescope is rigidly fastened to the support, to which the several prisms are attached and the various wave-lengths are brought into the field by means of a micrometer-screw attached to the ocular slit.

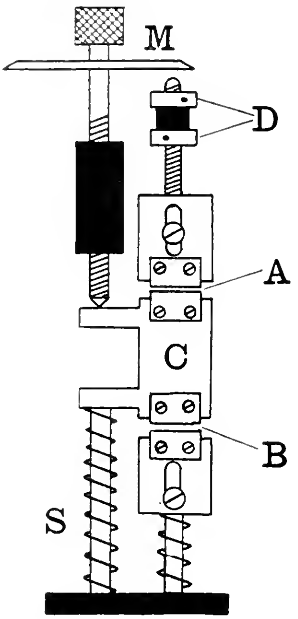


FIG. 1.—The differential slit

I. AS A SPECTRO-PHOTOMETER

In order to make spectro-photometric measurements with this instrument it is necessary to place a reflecting prism in front of one or both of the slits. The slits are 2 inches apart in the instrument constructed by the writer, but if they were 3 inches or more two sources of light might be compared without the use of reflecting prisms. Due to the fact that the light from *B* is absorbed more than the light from *A* (on account of the reflecting prism *D*, Fig. 3) it is necessary to apply a correction when the intensities of two lights are compared.

The instrument was constructed primarily, however, for obtaining coefficients of reflection and for measuring the concentration of colored solutions, and in these cases the assymetry of the two optical paths is taken care of by the proper adjustment of the light-sources.

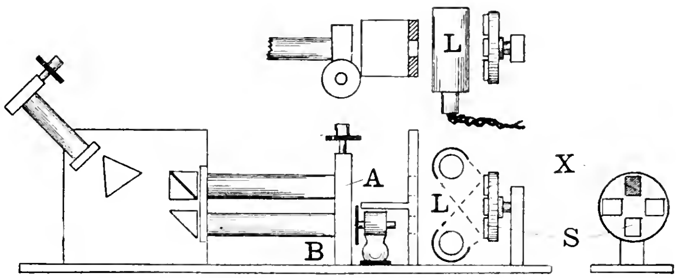


FIG. 2.—The spectro-photometer as used in determining coefficients of reflection

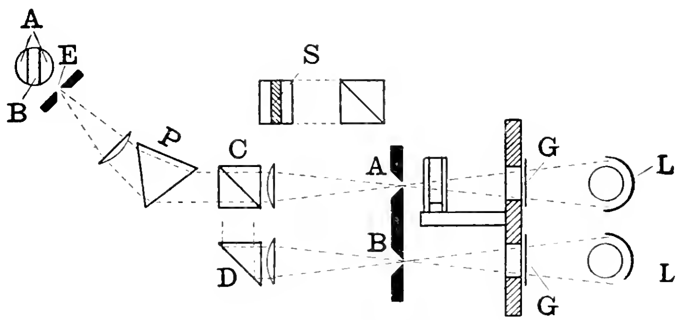


FIG. 3.—The optical system of the differential spectro-photometer

If I_A and I_B are the intensities of two light-sources placed in front of slit A and slit B, respectively, the ratio of the intensities is manifestly given by the following equation,

$$\frac{I_A}{I_B}=\frac{100-R}{R}, \qquad (1)$$

where R is the reading of micrometer.

A few values of this ratio are given in Table I.

TABLE I

Reading of Screw R	Width of A in mm	Width of B in mm	Ratio I_A/I_B
90	0.90	0.10	0.111
80	.80	.20	.250
70	.70	.30	.429
60	.60	.40	.666
50	.50	.50	1.000
40	.40	.60	1.500
30	.30	.70	2.333
20	.20	.80	4.000
10	0.10	0.90	9.000

II. AS A SPECTRO-COLORIMETER

When the instrument is used for colorimetric measurements, the slits are illuminated as indicated in Fig. 3. Two tubular lamps L and L, provided with reflectors, are placed directly in front of the slits and the light is diffused by means of two ground-glasses G and G. The lamps are provided with suitable clamps, so that they may be easily adjusted to produce equality of brightness when the slits have the same width, i.e., when the micrometer reads 50. If a colored liquid is to be examined it is placed in a glass cell, which in turn is placed in front of A. The initial photometric balance in this case is made with the cell in position and filled with the solvent to be used.

Applying Beer's law to equation (1), the concentration C, in terms of the slit-reading R, becomes

$$C = A \log \frac{1}{R}. \tag{2}$$

The *absorption ratio* A is a constant which depends upon the solution in question and the wave-length of light used.

The variation of C with $\log \frac{1}{R}$ is shown in Table II.

The *concentration scale* for this instrument is somewhat more uniform than that of the ordinary form of double-slit spectro-

TABLE II

Reading of Screw R	$\log \frac{1}{R}$	Reading of Screw R	$\log \frac{1}{R}$
80	0.602	60	0.176
75	.477	55	.087
70	.368	50	0.000
65	0.269		

colorimeter or the Koenig-Martens polarization spectro-photometer. The three *concentration-curves* plotted to the same scale are shown in Fig. 4.

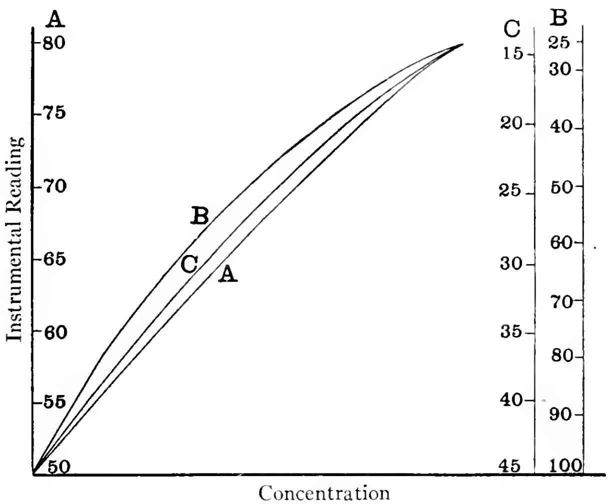


FIG. 4.—Relation between the instrumental reading and the concentration for the differential spectro-photometer A, double-slit spectro-photometer B, polarization spectro-photometer C.

III. AS A PYROMETER

The principle of the differential slit could evidently be adapted to a pyrometer, in which case it might take the form shown in Fig. 5.

Light from the body whose temperature is desired enters A and light from a suitable standard lamp S enters B. The ocular slit E

would in this case be adjusted for a particular wave-length, preferably 0.65 μ.

While the temperature-scale in this case is more uniform than that which obtains for a polarizing instrument (Wanner or Scimatco Pyrometer), the limits of measurement of temperature are not nearly so great, as a slit-width ratio of 1 to 10 could hardly be depended upon.

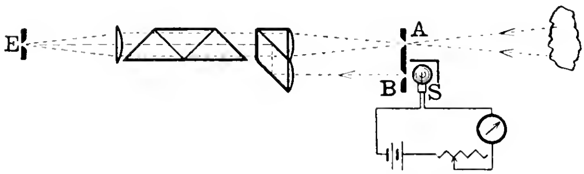


FIG. 5.—Differential-slit pyrometer

Suppose the standard lamp is so adjusted that when the observed temperature is 1000° C. a balance obtains for $R=50$, then applying Wien's law the relation between R and the temperature t is that given in columns 1 and 2 of Table III.

TABLE III

Reading of Screw R	Temperature t C.	Rotation of Analyzer	Temperature t C.
90	860°	9°	775°
80	900	18	855
70	940	27	910
60	970	36	955
50	1000	45	1000
40	1030	54	1050
30	1060	63	1110
20	1110	72	1195
10	1180	81	1350

In a similar manner if the Wanner pyrometer standard lamp is adjusted so that 1000° C. corresponds to 45° of rotation of the Nicol analyzer, the relation between the degrees of rotation and the temperature t is that given in columns 3 and 4 of Table III.

By means of suitable absorption screens several scales could, to be sure, be provided and the instrument might prove a useful pyrometer.

Any pyrometer utilizing a prism for obtaining monochromatic light has clearly certain advantages over an instrument using the so-called "monochromatic red" glass. The question of selective absorption of the absorption screens would not enter in.

IV. DETERMINATION OF COEFFICIENTS OF REFLECTION

Suppose it is desired to determine the percentage of light reflected from a given colored paint or paper in terms of some white standard. The method of procedure is as follows:

The ground-glasses are removed and the colored paint X and the white standard S are placed as shown in Fig. 2. The circular plate upon which these specimens are placed may be rotated about an axis of the instrument and thereby the position of X and S may be easily interchanged. By means of this device any small inequality of illumination of the two surfaces may be eliminated, and therefore it is not necessary to obtain an accurate initial balance.

To make the initial balance, two additional white surfaces are placed on the two remaining quadrants of the plate, as shown. The plate is now rotated until both slits are illuminated by light reflected from the two standard surfaces. With the micrometer set at 50 the two lamps, which are arranged as shown, are adjusted until a balance obtains.

In measurements of this sort it is, of course, necessary to make a number of observations and by means of this rotating plate half of these may be made with the surfaces in one position and half of them with the surfaces interchanged so that only an occasional checking of the initial balance is necessary.

When the unknown surface X is in front of A, the upper slit, the percentage of light reflected from X for a particular wave-length is

$$r_\lambda = \frac{100-R}{R}. \tag{3}$$

When X is in front of B, r_λ becomes

$$r'_\lambda = \frac{R}{100-R}. \tag{4}$$

The true value is the mean, provided that the difference $r_\lambda - r'_\lambda$ is small.

In order to measure small reflection coefficients without narrowing down slit *B* too much a sector disk is used. This disk is rotated by a small motor, which is an integral part of the apparatus (Fig. 2). Several disks were made, but for most purposes one disk with a transmission of 0.2 is sufficient. When not in use, it is only necessary to rotate it until the opening is opposite the slit. The reflection coefficients for several values of *R* are shown in Table IV.

TABLE IV

Reading of Screw *R*	Percentage of Light Reflected (without Disk)	Percentage of Light Reflected (with Disk)	Reading of Screw *R*	Percentage of Light Reflected (without Disk)	Percentage of Light Reflected (with Disk)
70	0.429	0.086	45	....	0.244
65	.538	.108	40	....	.300
60	.666	.133	35	....	.371
55	.818	.164	30	....	0.466
50	1.000	0.200			

It is thus seen that reflection coefficients from 0.1 to 1.0 may be measured without changing the disk and furthermore without using a ratio of slit-width much greater than 1 to 2.

A second motor, not shown in Fig. 2, was provided with a similar disk, so that it could be used in front of slit *A* when the position of the reflecting surfaces were interchanged.

WILLIAMS COLLEGE
WILLIAMSTOWN, MASS.
October 29, 1917

THE LUMINOSITIES AND PARALLAXES OF FIVE HUNDRED STARS[1]

FIRST LIST

By WALTER S. ADAMS and A. H. JOY

The determination by means of a study of their spectra of the luminosities and parallaxes of the stars under investigation for radial velocity at the Mount Wilson Observatory has been continued as a regular part of the stellar spectroscopic work. The methods employed are essentially those described by one of us in a series of papers[2] published somewhat more than a year ago. So far as these methods have been modified or extended in the course of the present investigation, reference will be made accordingly.

Although the observational material and the number of parallax determinations have accumulated rapidly, it has seemed preferable to us to delay the publication of the results until enough were available to make possible a detailed comparison with the trigonometrical parallaxes. Moreover, we have found that the accuracy of the estimation of relative line-intensities depends to a considerable extent upon the number of photographs. A single spectrogram, owing to under- or over-exposure or peculiarities in the plate-grain, will sometimes give an inaccurate value which subsequent plates will correct. Accordingly we have attempted as far as possible to secure at least three photographs of the spectrum of each star.

Another important consideration is the fact that measured parallaxes[3] of a high degree of accuracy are being published at frequent intervals. Since the spectroscopic determinations depend upon empirical curves connecting line-intensity with absolute magnitude, and since this absolute magnitude is calculated from the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 142.

[2] Adams, *Mount Wilson Communications*, Nos. 23–26; *Proceedings of the National Academy of Sciences*, **2**, 143, 1916.

[3] The term "measured parallaxes" is used in this article to designate those determined trigonometrically. In the absence of a better term "spectroscopic parallax" is also employed. This should not be confused with the radial-velocity method used for parallax determinations in double-star systems, and employed, for example, by Wright in the case of α Centauri.

measured parallax, it is desirable from time to time to readjust the curves with the aid of such new material as is available.

In the list of results for 500 stars which is given in this communication the attempt has been made to include as many stars as possible with measured parallaxes. Since many of the recent values by Mitchell and Miller are for relatively bright stars, we have added to our observing list the stars of the *American Ephemeris* with types between F0 and M. In addition there are included many stars of small proper motion observed by van Maanen. The measured parallaxes of a number of these have been used in the establishment of the relationships between line-intensity and absolute magnitude and have proved of great value. For the most part they are stars of high luminosity and so are particularly useful in fixing the lower portions of the magnitude-curves. A number of the results of Mitchell have been equally valuable in this respect. About 80 stars in all have been used in determining the reduction formulae for the various types.

There are three classes of stars in the list for which modifications have been made of the methods of reduction used previously. These are the early F-type stars and the giants and dwarfs of type M. It was soon found in the case of the F0 to F5 stars that the line λ 4455 of calcium showed but little variation with absolute magnitude. On the other hand, the strontium line at λ 4215 still remained available, and a search was made for additional lines to supplement it. Such lines were found in the similar strontium line at λ 4077, and the line at λ 4290, which probably is due, at least mainly, to enhanced titanium. All three of these lines increase in intensity with increase of luminosity. They are compared in intensity with three neighboring iron lines, λ 4072, λ 4250, and λ 4271, respectively, and the reduction-curves have been formed in the usual way by determining these differences for stars of measured parallax. This method gives satisfactory results for stars with spectra between A8 and F5. For stars later than F5 the usual lines λ 4215 and λ 4455 are employed.

The giant M-type stars have proved most difficult of treatment. The plan finally adopted in their case has been to compare the spectrum of each star with that of α Orionis, which probably is the

most highly luminous star of this type of which we have any certain knowledge, and to estimate the differences in intensity of certain selected lines in their spectra. The lines chosen are λ 4077 of strontium, a line of uncertain origin at λ 4207, λ 4215 of strontium, and the two hydrogen lines $H\delta$ and $H\gamma$. All of these lines increase in intensity with the luminosity of the star. They are nearly as strong in the spectrum of α Scorpii as in that of α Orionis, and this star has served as one of the fundamental points for the reduction curves. Its parallax as determined from group-motion has been furnished to us by the kindness of Professor Russell. Additional points for the reduction-curves are provided by the parallaxes of M-type stars of van Maanen and Mitchell. This method of reduction, though not entirely satisfactory, is the best which we have as yet been able to devise.

The recent determinations of the parallax of the star of large proper motion discovered by Barnard indicate that its absolute magnitude is about 13.3. The reduction-curves previously used for stars of this type depended upon extreme values of about 11.0, and were nearly asymptotic to the magnitude axis at this point. Accordingly it became essential to extend the curves to include fainter stars of this type. The calcium line at λ 4455 reaches so great an intensity in all dwarf M-type stars that it becomes difficult to make use of it for the determination of relatively small differences. An examination, however, of the low-temperature line of strontium at λ 4607 showed a marked increase of intensity in the spectrum of Barnard's star as compared with other stars of this class, and the character of the line is such as to make estimates of its intensity comparatively simple. We have therefore used this line in determining the relative absolute magnitudes of the various dwarf M stars. It is worthy of note that in the case of both classes of M-type stars the modifications introduced into the reduction method have been for the purpose of securing more accurate determinations of magnitude within each class. They may therefore be regarded as an attempt at a second approximation, with the separation into the two classes as the first.

The results of our determinations are given in Table I. The values for spectral types and absolute magnitude are the means of

TABLE I

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum.	π	Trig. π	Auth
1	Lal. 17231[1]	8.0	K2	$0^{h}\ 0^{m}.4$	+45° 10′	0″.00	4	K7	K7	+9.3	0.019	0″.120	+0″.096	Y, S, Fox
2	Σ 3062 Br	6.9	F	0 1.0	+57 53	0.266	3	G4	G2	+5.2	0.832	0.046	+0.036	Mil.
3	Σ 3062 Ft	8.0		0 1.0	+57 53	0.266	2	G9	G8	+5.3	0.759	0.029		
4	12 Ceti	6.0	K	0 24.9	− 4 31	0.011	3	K5	K2	+1.7	20.9	0.014		
5	Boss 96	5.7	G5	0 26.2	+52 17	0.056	3	K0	K0	+0.3	75.9	0.008	+0.008	vM
6	Boss 106	6.4	G5	0 27.5	+27 44	0.014	3	G6	G7	+0.4	69.2	0.006	+0.005	vM
7	Pi $0^{h}130$	5.7	K	0 32.2	−25 19	1.39	3	G8	G6	+4.8	1.20	0.066		
8	ε Androm	4.5	G5	0 33.3	+28 46	0.336	3	G3	G4	+1.3	30.2	0.023	+0.075	S and M, Mil.
9	δ Androm	3.5	K0	0 34.0	+30 19	0.162	2	K3	K3	+1.4	27.5	0.038		
10	Lal. 909	7.4	G	0 34.0	+ 2 35	0.79	3	G3	G1	+5.2	0.832	0.036	+0.033	Y, Jw.
11	α Cassiop	{2.2 {2.8–	K0	0 34.8	+55 59	0.060	2	G8	G8	−0.9	229.	{0.018– {0.024	+0.023	F, M
12	Lal. 1015	7.5	K	0 35.3	+39 39	0.81	3	K3	K3	+5.8	0.470	0.046	+0.017	Y
13	η Cassiop. Br	3.6	F8	0 43.0	+57 17	1.242	3	F9	G0	+5.0	1.00	0.190	+0.191	5
14	η Cassiop. Ft	7.6		0 43.0	+57 17	1.242	1	K4	K0	+8.7	0.033	0.166		
15	Groom. 145	8.0	K	0 43.3	+69 54	0.54	3	K2	K2	+6.2	0.331	0.044	+0.028	Y
16	A. G. Hels 914	8.7		1 0.3	+63 24	1.62	2	K7	K6	+8.6	0.036	0.096		
17	Lal. 1966	7.9	F5	1 3.3	+61 1	0.69	3	F5	F5	+5.1	0.912	0.028	+0.022	Y, S and M
18	β Androm	2.4	Ma	1 4.1	+35 5	0.216	2	Ma	G8	+1.6	22.0	0.069	+0.058	F, M
19	W.B. $1^{h}161$	8.0	K	1 13.5	− 1 23	0.54	5	K0	G9	+6.1	0.363	0.042	−0.011	Y
20	Lal. 2387	8.5	G	1 14.0	− 9 27	0.57	2	G1	G1	+4.7	1.32	0.017	+0.026	Y
21	Lal. 2450	8.1	F8	1 16.0	+18 10	0.57	3	G0	G1	+5.7	0.525	0.033	+0.017	Y
22	ω Androm	5.0	F5	1 21.7	+44 53	0.356	1	F0	F1	+3.4	4.37	0.048	+0.037	F, Jw., M
23	α Urs. Min	2.1	F8	1 22.6	+88 46	0.044	8	F8	F8	+0.6	57.5	0.050	+0.041	10
24	Lal. 2682	7.8	G5	1 23.6	+21 13	0.53	6	K3	K2	+6.5	0.251	0.055	+0.009	Y
25	η Piscium	3.7	G5	1 26.1	+14 50	0.031	2	G5	G5	+0.3	75.9	0.021	+0.056	M
26	40 Cassiop	5.5	K	1 30.5	+72 32	0.008	3	G6	G5	+0.2	83.2	0.009		
27	υ Androm	4.2	G0	1 30.9	+40 54	0.418	2	F9	F9	+4.0	2.51	0.091	+0.057	M
28	Lal. 2966	7.6	G5	1 34.2	+66 25	0.76	3	G4	G2	+6.2	0.331	0.052	+0.036	Y
29	Lal. 3128	5.3	G8	1 37.1	+19 47	0.718	4	K1	K1	+5.5	0.631	0.110	+0.129	Y
30	τ Ceti	3.6	K0	1 39.4	−16 28	1.922	3	G7	G5	+6.2	0.331	0.331	+0.317	4
31	Boss 405	5.8	F	1 43.0	+32 11	0.354	3	F6	F8	+3.9	2.75	0.042		
32	Boss 420	5.6	K	1 47.3	+40 14	0.009	3	K2	K1	+0.6	57.5	0.010	+0.006	vM
33	Lal. 3922	7.5	G	2 2.5	− 1 5	0.50	3	F9	G0	+3.8	3.02	0.018	−0.042	Y
34	55 Cassiop	6.2	F5	2 6.6	+66 3	0.003	3	F5	F7	+1.0	17.4	0.014		
35	6 Persei	5.4	K0	2 7.0	+50 36	0.386	1	G6	G3	−0.3	132.	0.008	−0.001	L and J
36	Lal. 3987	7.8	K	2 7.5	+67 13	0.56	3	K2	K2	+6.0	0.398	0.044	+0.097	Y
37	W.B. $2^{h}95$	8.3	G	2 9.5	− 1 40	0.97	5	F8	F0	+4.4	1.74	0.017	+0.014	Y
38	Lal. 4141	6.9	G	2 9.7	+23 49	0.60	4	G7	G4	+4.5	1.58	0.033	+0.020	Y
39	Lal. 4268	5.8	F5	2 12.8	+ 1 17	0.527	3	F7	F8	+4.0	2.51	0.044		

40	Boss 556	6.5	G	2 22.1	+ 9 45	0.355	4	F5	F6	+4.5	1.58	0.040	....	....
41	Boss 592	7.4	F	2 31.2	+24 13	0.152	3	F5	F7	+3.7	3.31	0.018	+0.010	vM
42	Boss 593	6.6	F5	2 31.2	+24 13	0.140	6	F3	F5	+3.9	2.75	0.028	+0.030	vM
43	Lal. 4855	7.2	G	2 32.6	+30 24	0.60	3	F8	F8	+4.7	1.32	0.032	+0.020	Y, F
44	θ Persei	4.2	Go	2 37.4	+48 48	0.351	3	F7	F7	+4.2	2.09	0.100	+0.088	S
45	SU Cassiop.	{5.9– 6.3}	F	2 43.0	+68 28	0.016	7	F5	F5	+1.0	39.8	0.010	+0.010	vM
46	η Persei	3.9	Ko	2 43.4	+55 29	0.030	2	K3	G4	+0.5	63.1	0.021	....	....
47	τ Persei	4.1	Gop	2 47.2	+52 21	0.008	2	G1	G1	+1.5	25.1	0.030	....	....
48	20 Persei	5.3	F	2 47.4	+37 56	0.099	4	Fo	F2	+3.3	4.79	0.040	+0.013	S and M, Mil.
49	Boss 672	5.6	G5p	2 53.0	+46 49	0.035	3	G3	G3	+1.5	25.1	0.015	−0.007	vM
50	α Ceti	2.8	Ma	2 57.1	+ 3 42	0.078	2	Ma	G8	+0.5	63.1	0.035	+0.022	M
51	ρ Persei	{3.4– 4.2}	Mb	2 58.9	+38 27	0.173	2	Mb	G7	−0.5	158.	{0.011– 0.017}	+0.047	R, M
52	ι Persei	4.2	G	3 1.8	+49 11	1.269	3	Go	Go	+4.1	2.29	0.096	+0.126	4
53	δ Arietis	4.5	Ko	3 5.9	+19 21	0.152	2	Ko	G9	+1.0	30.8	0.020	....	....
54	α Persei	1.9	F5	3 17.2	+49 30	0.039	3	F5	F7	+0.3	75.9	0.048	+0.016	F, S and M
55	Lal. 6320	7.9	K5	3 20.1	− 5 42	0.86	3	K5	K5	+7.1	0.145	0.069	+0.062	Y, M, Jw.
56	ε Eridani	3.8	K	3 28.2	− 9 48	0.972	4	K2	K1	+6.1	0.363	0.288	+0.317	Y, F, M
57	10 Tauri	4.4	G5	3 31.8	+ 0 5	0.536	4	F8	F9	+3.8	3.02	0.076	+0.074	Y, Jw.
58	Boss 831	5.9	A	3 34.1	− 5 57	0.204	2	Ko	Ko	+3.2	5.25	0.029	....	....
59	W B. 3h617	7.2	F8	3 35.3	− 3 32	0.79	3	F6	F7	+4.6	1.45	0.030	−0.073	Y
60	ν Persei	3.9	F5	3 38.4	+42 16	0.009	2	F5	F6	+0.7	52.5	0.023	....	....
61	δ Eridani	3.7	K	3 38.4	−10 6	0.749	3	G9	G9	+2.9	6.92	0.069	+0.147	Y, F, M
62	B D. +23°535	7.0	....	3 41.4	+23 25	....	3	F5	F5	+3.9	2.75	0.016	+0.007	Y
63	γ Eridani	3.2	K5	3 53.4	−13 48	0.130	2	K6	Ko	+1.5	25.1	0.046	....	....
64	Lal. 7413	8.6	G	3 56.5	+35 2	2.19	3	Ko	G8	+6.7	0.209	0.042	+0.040	6
65	Boss 933	5.7	Comp.	3 58.4	+23 50	0.022	3	F5p	F7	+1.4	27.5	0.014	+0.014	vM
66	A Tauri	4.5	Ko	3 58.8	+21 49	0.113	2	Ko	G9	+1.1	36.3	0.021	....	....
67	43 Tauri	5.7	G	4 3.3	+19 21	0.114	2	K1	Ko	+1.8	19.1	0.017	+0.024	M
68	o² Eridani	4.5	G2	4 10.7	− 7 49	4.085	4	K1	Ko	+6.0	0.398	0.200	+0.182	4
69	γ Tauri	3.9	Ko	4 14.1	+15 23	0.120	3	G7	G8	+0.1	91.2	0.017	....	....
70	Boss 1014	6.1	K	4 16.4	+20 35	0.011	3	Ma	G5	+0.5	63.1	0.008	+0.001	vM
71	δ Tauri	3.9	Ko	4 17.1	+17 18	0.115	2	G8	G9	+0.8	47.9	0.024	....	....
72	ι Tauri	3.6	Ko	4 22.8	+18 58	0.120	4	G9	G9	0.0	100.	0.019	....	....
73	θ¹ Tauri	4.0	Ko	4 22.9	+15 44	0.108	3	G9	G8	+0.9	43.7	0.024	....	....
74	A. Oe. 4961	8.5	....	4 29.9	+52 42	0.48	4	K6	K5	+8.6	0.036	0.105	+0.099	S and M
75	α Tauri	1.1	K5	4 30.2	+16 18	0.201	6	K5	K2	+0.9	43.7	0.091	+0.070	5
76	Groom. 864	7.3	G	4 34.5	+41 56	0.73	2	G2	G1	+6.1	0.363	0.058	+0.028	Y, F
77	Boss 1128	5.8	Mb	4 42.7	+63 20	0.111	4	Mb	Ko	+2.2	13.2	0.019	....	....
78	Boss 1131	6.8	G5	4 42.8	+18 33	0.440	6	F9	Go	+4.9	1.10	0.042	....	....
79	π³ Orionis	3.3	F8	4 44.4	+ 6 47	0.471	2	F5	F6	+4.4	1.74	0.166	+0.156	M
80	Lal. 9001	7.0	F5	4 45.7	+10 54	0.10	3	F9	F8	+4.6	1.45	0.033	+0.007	F, Mil.
81	Lal. 9109	6.7	F5	4 46.2	+13 29	0.13	3	F7	F6	+3.9	2.75	0.027	+0.007	F
82	ι Aurigae	2.9	K2	4 50.5	+33 0	0.028	2	K3	Ko	−0.1	110.	0.025	−0.011	F
83	A. G. Leid. 1812	8.0	....	4 51.3	+34 7	0.76	3	G9	G7	+7.7	0.083	0.087	....	....
84	β Camelop.	4.2	Go	4 54.5	+60 18	0.013	2	F7p	F8	−0.9	229.	0.010	....	....

TABLE I—*Continued*

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum.	π	Trig. π	Auth.
85	ε Aurigae	3.0–4.5	F5p	$4^h54^m.8$	+43° 41′	0″.014	2	F5p	F7	−0.3	132.	0″.011–0.022	+0″.060	Jw.
86	ζ Aurigae	3.9	Kop	4 55.5	+40 56	0.032	2	G8	G1	+1.1	36.3	0.027	−0.028	Ab.
87	ε Leporis	3.3	K5	5 1.2	−22 30	0.072	2	K4	K1	+1.6	22.9	0.046	....	....
88	Boss 1217	6.3	G	5 2.3	+ 9 21	0.381	3	Go	Go	+5.0	1.00	0.055	....	....
89	α Aurigae	0.2	Go	5 9.3	+45 54	0.437	4	G1	Go	+0.3	75.0	0.105	+0.075	F, Y, Jw.
90	λ Aurigae	4.8	Go	5 12.1	+40 1	0.846	2	G3	G2	+4.8	1.20	0.100	+0.089	S
91	Lal. 9960	8.5	K	5 14.1	− 3 11	0.73	3	K5	K4	+6.8	0.191	0.046	+0.074	Y
92	17 Camelop	5.8	Ma	5 20.7	+62 59	0.005	3	Ma	G8	+1.3	30.2	0.013	....	....
93	β Leporis	3.0	Go	5 24.0	−20 50	0.094	2	G1	G1	+1.1	36.3	0.042	....	....
94	Groom. 966	6.4	K5	5 26.3	+74 59	0.019	3	Ma	G8	+1.7	20.9	0.012	....	....
95	W.B. 5^h592	8.9	Ma	5 26.4	− 3 42	2.23	5	Ma	Ma	+9.0	0.011	0.158	+0.176	F, Sc., Jw.
96	α Leporis	2.7	Fo	5 28.3	−17 54	0.003	2	F4p	F5	+0.4	69.2	0.035	+0.014	M
97	Groom. 990	8.1	K	5 30.4	+51 23	0.56	3	K2	Ko	+6.0	0.398	0.038	+0.028	Y
98	ϕ^2 Orionis	4.4	K	5 31.4	+ 0 14	0.321	3	G7	G6	+1.3	30.2	0.024	+0.024	S and M, Mil.
99	γ Leporis Ft	6.4	K	5 40.3	−22 27	0.461	1	K5	K3	+7.5	0.100	0.166	+0.145	M
100	γ Leporis Br	3.8	F8	5 40.3	−22 29	0.468	2	F7	F7	+4.7	1.32	0.151	+0.108	M
101	δ Leporis	3.9	Ko	5 47.0	−20 53	0.696	2	G7	G6	+1.1	36.3	0.027	....	....
102	δ Aurigae	3.9	Ko	5 51.3	+54 17	0.153	3	G7	G8	+0.7	52.5	0.023	+0.025	Ab., M
103	η Leporis	3.8	F5	5 51.9	−14 11	0.138	2	F3	F3	+2.1	14.5	0.046	....	....
104	A.G. Ber. 1866	9.0	....	5 57.2	+19 23	0.76	3	F9	Go	+5.2	0.832	0.017	....	....
105	1 Gemin	4.3	G5	5 58.0	+23 16	0.108	2	G3	G3	+2.9	6.92	0.052	....	....
106	Boss 1549	6.1	K	6 6.3	+22 56	0.027	4	Ma	G4	+0.7	52.5	0.008	+0.003	vM
107	η Gemin	3.2–4.2	Ma	6 8.8	+22 32	0.065	2	Ma	G6	+1.7	20.9	0.032–0.050	+0.005	R, M
108	W.B. 6^h128	8.4	G	6 9.5	+44 45	0.413	3	G8	G8	+8.0	0.063	0.083	+0.102	Ram.
109	ψ^1 Aurigae	5.1	K2	6 17.2	+49 20	0.014	2	K3	G5p	+1.1	36.3	0.016	....	....
110	Boss 1627	6.0	G	6 22.1	+58 14	0.33	5	G9	G7	+1.6	22.9	0.013	−0.006	Ab.
111	23 Hev. Cam	5.6	F8	6 29.2	+79 40	0.637	3	F8	F7	+4.6	1.45	0.063	+0.037	Y
112	ε Gemin	3.2	G5	6 37.8	+25 14	0.020	2	G8	G2	−1.0	251.	0.014	....	....
113	ψ^5 Aurigae	5.3	Go	6 39.5	+43 41	0.158	2	F8	F8	+4.0	2.51	0.055	+0.078	Jo.
114	ξ Gemin	3.4	F5	6 39.7	+13 0	0.231	2	F4	F4	+1.0	39.8	0.033	+0.042	M
115	Lal. 13284	6.9	K	6 47.4	− 5 3	0.57	5	K5	K4	+7.2	0.132	0.115	+0.121	Y
116	A.G. Bonn 5621	8.3	....	6 49.5	+40 13	0.57	3	K4	K3	+7.1	0.145	0.058	....	....
117	θ Can. Maj	4.2	K2	6 49.5	−11 55	0.138	2	K4	K1	+1.5	25.1	0.029	+0.017	L and J
118	W.B. 6^h1500	7.7	G	6 51.4	+ 1 18	0.57	3	G6	G3	+6.0	0.398	0.046	+0.000	Y
119	ζ Gemin	3.7–4.3	Go	6 58.2	+20 43	0.009	2	F8p	Go	+0.5	63.1	0.017–0.023	+0.025	Ab., Mil.
120	Lal. 13849	6.5	G5	7 4.2	+21 25	0.52	3	Ko	G7	+4.0	2.51	0.032	+0.052	Y, Mil.
121	Boss 1868	5.9	K	7 9.7	+28 4	0.020	3	Ma	K2	+0.8	47.9	0.010	+0.008	vM

122	Lal. 14146	7.3	G	7 11.3	−12 53	0.55	3	F8	F9	+5.9	0.437	0.052	+0.080	Y
123	ι Gemin.	3.9	Ko	7 19.5	+28 0	0.145	2	G9	G8	+0.8	47.9	0.024	+0.013	M
124	Boss 1971	5.6	G5	7 26.0	+17 18	0.089	3	K1	K1	+0.6	57.5	0.010	0.000	vM
125	Comp. α Gemin.	9.0	......	7 28.2	+32 6	......	3	K9p	......	+9.0	0.025	0.100	......	......
126	α Can. Min.	0.5	F5	7 34.1	+ 5 29	1.242	6	F3	F4	+3.1	5.75	0.331	+0.309	5
127	Boss 2020	5.8	Ma	7 36.4	+14 27	0.015	3	Mb	G5	+0.4	69.2	0.008	+0.006	vM
128	γ Monoc.	4.1	K	7 36.5	− 9 19	0.079	2	G9	G9	+0.7	52.5	0.021	+0.005	M
129	σ Gemin.	4.3	K	7 37.1	+29 8	0.247	1	G9	G7	+1.1	36.3	0.023	+0.030	Jo., L and J
130	W.B. 7^h1029	6.8	......	7 38.2	+39 49	0.52	3	F6	F7	+4.6	1.45	0.036	......	......
131	β Gemin.	1.2	Ko	7 39.2	+28 16	0.623	4	G9	G8	+0.8	47.9	0.083	+0.064	Y
132	28 Hev. Cam.	6.5	G5	7 39.8	+80 31	0.50	3	Ko	G5	+4.2	2.09	0.035	+0.049	Y, Gr.
133	Lal 15290	8.2	G	7 47.2	+30 55	1.07	3	F7	F8	+5.9	0.437	0.035	+0.045	4
134	Lal 15394	8.2	......	7 49.1	+19 31	0.50	4	K3	K3	+6.4	0.275	0.044	......	......
135	Boss 2098	6.0	K	7 51.3	+16 3	0.056	3	K3	K2	+1.4	27.5	0.012	......	......
136	Lal. 15547	8.6	F8	7 53.7	+21 8	0.59	3	G9	Ko	+5.9	0.437	0.029	+0.089	Y
137	ω Cancri	5.9	K	7 54.9	+25 40	0.014	3	G9	G7	−0.2	120.	0.006	......	......
138	Boss 2148	6.4	K5	8 2.5	+42 43	0.073	4	K4	K2	+1.3	30.20	0.009	−0.030	Ab.
139	3 H. Urs. Maj.	5.5	G5	8 2.9	+68 46	0.006	3	G5	G4	+0.2	83.2	0.009	......	......
140	ζ Cancri Br.	5.2	Go	8 6.5	+17 57	0.155	2	F8	F9	+3.4	4.37	0.044	+0.031	Mil., F
141	ζ Cancri Ft.	6.0	......	8 6.5	+17 57	0.155	2	Go	Go	+4.1	2.29	0.042	+0.035	Mil., F
142	β Cancri	3.8	K2	8 11.1	+ 9 30	0.075	2	K3	K1	+1.1	36.3	0.029	0.000	M
143	W. B. 8^h181	8.6	K	8 12.0	+30 56	0.90	3	K6	K6	+7.0	0.158	0.048	+0.014	Y, F
144	Boss 2236	6.3	F5	8 20.6	+45 59	0.362	3	G4	Go	+1.3	1.91	0.040	......	......
145	Groom. 1450	6.0	Ko	8 26.4	+38 22	0.204	2	K2	K1	+1.3	30.2	0.011	......	......
146	32 Lyncis	6.1	F	8 26.9	+36 47	0.139	3	F3	F4	+4.5	1.58	0.048	+0.088	Jo.
147	Boss 2275	6.4	G	8 27.1	+24 25	0.091	3	G9	G7	+0.3	75.9	0.006	......	......
148	A.G. Cam. 3181	8.6	......	8 28.2	+54 4	0.93	3	G7	G7	+2.1	14.5	0.005	......	......
149	W. B. 8^h596	8.6	......	8 28.8	+42 6	0.76	3	K4	K2	+6.7	0.209	0.042	+0.023	S and M
150	Lal. 16904	8.1	G5	8 33.1	+56 2	0.44	3	G3	G2	+6.4	0.275	0.040	−0.042	Y
151	Lal. 17161	8.4	G	8 38.6	+42 3	0.69	3	K3	K3	+7.2	0.132	0.058	−0.073	Y
152	δ Cancri	4.2	Ko	8 39.0	+18 31	0.240	2	Ko	G9	+1.0	39.8	0.023	+0.053	M
153	Boss 2338	6.1	G	8 39.2	+31 4	0.020	3	G3	G3	+0.8	47.9	0.008	+0.007	vM
154	ι Hydrae Br.	3.5	F8	8 41.5	+ 6 47	0.196	5	F9	Go	+2.5	10.0	0.063	+0.004	Mil.
155	ι Hydrae Ft.	7.5	......	8 41.5	+ 6 47	0.196	3	F4	F6	+3.6	3.63	0.016		
156	Fed. 1384^1	9.2	K2	8 46.0	+71 11	1.40	3	K8	K6	+9.7	0.013	0.126	+0.086	F, Gr
157	Fed. 1384^2	9.2	......	8 46.0	+71 11	1.38	1	K7	K5	+9.2	0.021	0.100	+0.086	F, Gr.
158	Boss 2410	6.6	Ma	8 53.5	+18 31	0.080	3	Mc	G8	+1.6	22.9	0.010	......	......
159	10 Urs. Maj.	4.1	F	8 54.2	+42 11	0.504	1	F4	F5	+4.4	1.74	0.115	+0.062	F, Y, Mil.
160	81 π^1 Cancri	6.4	G	9 6.8	+15 24	0.576	4	G8	G6	+4.9	1.10	0.050	−0.008	Y
161	Lal. 18115^1	7.9	K5	9 7.6	+53 7	1.69	3	K8	K7	+8.6	0.036	0.138	+0.152	5
162	Lal. 18115^2	7.9	......	9 7.6	+53 7	1.69	3	K8	K5	+9.2	0.021	0.182	+0.152	5
163	Lal. 18286	7.3	K5	9 12.0	+29 0	0.52	3	K3	K1	+6.9	0.174	0.083	+0.067	Y, F, Mil.
164	83 Cancri	6.6	F5	9 13.4	+18 8	0.180	2	F6	F7	+4.9	1.10	0.040	......	......
165	Boss 2528	6.3	G5	9 20.0	+17 1	0.070	3	Ko	G9	+1.1	36.3	0.009	+0.015	vM
166	Lal. 18672	7.7	......	9 21.7	+ 6 5	0.54	3	K4	K3	+5.9	0.437	0.044	......	......
167	d Urs. Maj.	4.6	Go	9 25.6	+70 16	0.090	1	G3	G2	+2.7	8.32	0.012	+0.048	Y
168	θ Urs. Maj.	3.3	F8	9 26.2	+52 8	1.092	3	F5	F5	+3.5	3.98	0.110	+0.098	4

TABLE I—*Continued*

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum	π	Trig. π	Auth
169	11 Leo. Min	5.5	K	9^h29^m7	+36° 16′	0″745	3	K1	G6	+ 4.4	1.74	0″060	+0″179	Y, Jo., Mil.
170	o Leonis	3.8	F5p	9 35.8	+10 21	0.150	2	F4p	F5	+ 1.3	30.2	0.032	+0.034	M
171	Boss 2603	6.4	K	9 35.9	+26 22	0.049	2	K1	G8	+ 0.6	57.5	0.007	+0.012	vM
172	W.B. 9^h954	9.3	......	9 46.2	−11 49	1.95	1	K8	......	+ 9.7	0.013	0.120	......	......
173	Groom. 1596	8.2	G5	9 54.9	+56 5	0.49	3	G5	G5	+ 5.6	0.575	0.030	+0.069	Y
174	Comp. α Leonis	7.6	G	10 3.0	+12 27	0.247	3	K2	K0	+ 6.0	0.398	0.048	......	......
175	Lal. 19780	7.5	G	10 3.3	−19 15	0.38	3	G2	G0	+ 5.3	0.759	0.036	+0.057	F
176	Lal. 19821	8.5	G5	10 6.3	+24 15	0.42	3	G2	G1	+ 5.0	1.00	0.020	+0.049	Y
177	A.Oe. 10640	9.2	......	10 7.5	+53 1	0.70	1	K5	K4	+ 8.1	0.057	0.060	+0.074	L and J
178	Lal. 19896	7.7	G5	10 8.9	+ 3 39	0.47	3	G0	G0	+ 5.4	0.692	0.035	−0.048	Y
179	39 Leonis	5.8	F5	10 11.7	+23 36	0.44	4	F5	F6	+ 4.6	1.45	0.057	+0.092	Y
180	Munich II 5178	8.8	......	10 15.7	− 0 58	0.70	2	K5	K4	+ 7.8	0.076	0.063	......	......
181	Brad. 1433	5.9	G	10 16.2	+41 44	0.194	3	F7	F8	+ 3.6	3.63	0.035	+0.090	Jo., Mil.
182	μ Urs. Maj	3.2	K5	10 16.4	+42 0	0.084	2	K5	G8	+ 1.8	19.1	0.052	+0.051	Y
183	Groom. 1646	6.5	F8	10 21.9	+49 19	0.898	4	G1	G1	+ 6.1	0.363	0.083	+0.078	K, R, F
184	Pi. 10^h96	7.6	F8	10 27.7	+49 42	0.30	3	F6	F6	+ 5.4	0.692	0.036	+0.045	K
185	W.B. 10^h520	5.7	F	10 31.6	−11 42	0.734	3	F6	F6	+ 4.4	1.74	0.055	+0.001	Y, F
186	38 Leo. Min	5.8	K	10 33.4	+38 26	0.227	7	F9	F8	+ 2.8	7.59	0.025	+0.029	Ab.
187	B.D. +47°1806	(9.0)	......	10 38.1	+46 49	......	2	F8	F8	+ 4.1	2.29	0.010	......	......
188	Lal. 20670	7.7	G5	10 38.1	+46 44	0.280	7	F9	G0	+ 4.6	1.45	0.024	−0.004	K
189	Boss 2881	6.3	K	10 43.4	+29 57	0.103	3	K0	G9	+ 1.1	36.3	0.009	+0.036	vM
190	Groom. 1697	6.1	K	10 46.7	+70 23	0.403	3	K1	K1	+ 2.9	6.92	0.023	−0.011	Y
191	46 Leo. Min	3.7	K0	10 47.7	+34 45	0.304	4	K1	G9	+ 1.1	36.3	0.030	+0.092	Ab.
192	Boss 2910	5.9	K	10 50.2	+34 2	0.113	3	K0	G9	+ 0.5	63.1	0.008	+0.014	Ab.
193	Lal. 21008	8.5	G	10 50.9	+28 17	0.50	3	G5	G5	+ 5.2	0.832	0.022	+0.050	Y
194	Boss 2921	6.2	Ma	10 54.0	+36 38	0.095	3	Ma	G9	+ 3.6	3.63	0.030	+0.079	vM
195	Lal. 21185	7.6	Ma	10 57.9	+36 38	4.77	3	Ma	K8	+10.8	0.005	0.437	+0.422	7
196	χ Leonis	4.7	F0	10 59.9	+ 7 53	0.350	2	F4	F5	+ 0.1	91.2	0.012	+0.015	M
197	51 Leo. Min	7.5	G	10 59.9	+25 45	0.409	3	F5	F6	+ 4.6	1.45	0.026	+0.039	Y
198	Lal. 21258	8.9	Ma	11 0.5	+44 2	4.47	3	Ma	Ma	+11.2	0.003	0.288	+0.203	5
199	ψ Urs. Maj	3.2	K0	11 4.0	+45 2	0.069	2	G9	G9	+ 1.6	22.9	0.048	......	......
200	Lal. 21368	8.9	K5	11 5.6	+31 0	0.58	3	K6	K4	+ 8.2	0.052	0.072	+0.048	Y
201	Fed. 1831 Br	7.7	K5	11 8.7	+74 1	0.45	3	K5	K5	+ 7.0	0.158	0.072	+0.088	Bg., F
202	Groom. 1757	6.0	K	11 11.1	+50 1	0.092	4	K0	G9	+ 1.6	22.9	0.013	−0.081	Y
203	Boss 2983	5.4	Mb	11 12.1	+ 2 34	0.159	3	Ma	K2	+ 2.8	7.59	0.030	......	......
204	ξ Urs. Maj. Br	4.0	G	11 12.9	+32 6	0.732	3	F9	F9	+ 5.0	1.00	0.158	+0.158	Y, Ab., F
205	ξ Urs. Maj. Ft	4.9	......	11 12.9	+32 6	0.732	2	G1	G1	+ 5.6	0.575	0.138		
206	Lal. 21565	7.2	K	11 13.2	− 4 31	0.79	5	G8	G5	+ 6.1	0.363	0.060	+0.006	Y
207	δ Crateris	3.8	K0	11 14.3	−14 14	0.230	2	G8	G7	+ 0.3	75.9	0.020	......	......
208	A.Oe. 11677	9.2	......	11 14.8	+66 23	3.03	3	Ma	K8	+10.9	0.004	0.219	+0.198	Bg.

209	83 Leonis Br.	6.5	K	11 21.7	+ 3 33	0.743	3	G9	G7	+ 4.9	1.10	0.048	+0.023	Y, R
210	83 Leonis Ft.	7.6	......	11 21.7	+ 3 33	0.736	3	K4	K5	+ 6.3	0.302	0.055	+0.023	Y, R
211	Boss 3025	6.0	K	11 24.5	+15 58	0.066	3	K3	K2	+ 1.5	25.1	0.013	......	......
212	Boss 3030	5.7	K	11 25.3	+18 58	0.078	3	K0	G8	+ 0.7	52.5	0.010	+0.024	vM
213	Boss 3032	6.2	G	11 26.6	+14 55	0.387	4	F9	F9	+ 4.9	1.10	0.055	......	......
214	Groom. 1812	6.3	F	11 33.5	+45 40	0.583	2	F9	F9	+ 3.7	3.31	0.030	+0.038	K
215	Boss 3078	5.7	F	11 36.4	+32 18	0.347	3	F2	F3	+ 3.9	2.75	0.044	+0.042	vM
216	Groom. 1822	7.9	G	11 40.3	+48 14	0.66	3	F9	F9	+ 4.4	1.74	0.020	+0.024	K
217	Boss 3089	4.2	Ma	11 40.7	+ 7 5	0.188	3	Ma	G6	+ 1.9	17.4	0.035	......	......
218	93 Leonis	4.5	F8	11 42.8	+20 46	0.155	2	F5p	F7	+ 2.9	6.92	0.048	+0.024	M
219	β Virginis	3.8	F8	11 45.5	+ 2 20	0.793	2	F8	F8	+ 3.8	3.02	0.100	+0.116	Y, M
220	Groom. 1830	6.5	G5	11 47.2	+38 26	7.037	4	G9	G5	+ 6.0	0.398	0.079	+0.102	6
221	Lal. 22585	5.6	G	11 55.6	− 9 53	0.491	5	G9	G7	+ 4.2	2.09	0.052	+0.118	Y
222	Pi. 11^h218	6.8	K	11 57.4	+43 39	0.67	3	G9	G6	+ 3.5	3.98	0.022	+0.017	K, F
223	Lal. 22701	8.4	G	12 0.2	− 0 57	0.51	3	G9	G6	+ 8.1	0.057	0.087	+0.110	Y
224	Boss 3169	6.1	K	12 4.6	+ 2 28	0.190	4	K2	K1	+ 1.8	19.1	0.014	......	......
225	W.B. 12^h69	7.3	G	12 7.4	− 2 32	0.73	3	G4	G2	+ 4.4	1.74	0.020	+0.011	Y, F
226	Lal. 22954	6.1	F5	12 10.0	− 9 44	1.03	3	F3	F4	+ 4.7	1.32	0.052	+0.026	Y, F
227	Boss 3233	6.1	K	12 20.2	+24 29	0.072	3	G9	G9	+ 0.5	63.1	0.008	+0.005	vM
228	Lal. 23361	8.5	G	12 24.6	− 2 46	0.69	4	G2	G3	+ 7.0	0.158	0.050	+0.095	Y
229	Boss 3274	6.5	K	12 28.0	+10 51	0.003	3	G8	G8	+ 0.3	75.9	0.006	......	......
230	β Can. Ven.	4.3	G0	12 29.0	+41 54	0.758	3	G0	G0	+ 4.4	1.74	0.105	+0.096	Y, F
231	Boss 3294	6.0	Ma	12 33.3	+ 2 24	0.090	3	Mb	G9	+ 2.3	12.0	0.018	......	......
232	γ Virginis N	3.7	F0	12 36.6	− 0 54	0.564	3	F0	A8	+ 3.1	5.75	0.076	+0.068	Bel, R, F
233	γ Virginis S	3.7	F0	12 36.6	− 0 54	0.564	3	F0	A8	+ 3.2	5.25	0.079	......	......
234	Boss 3321	6.0	F	12 40.3	+39 49	0.375	3	F6	F7	+ 4.9	1.10	0.060	......	......
235	33 Virginis	5.9	K	12 41.3	+10 6	0.531	3	K2	G9	+ 3.3	4.79	0.030	+0.002	Y, Mil.
236	Lal. 23917	8.3	F8	12 44.6	+ 1 45	0.68	3	G7	G4	+ 6.5	0.251	0.044	+0.029	Y, Sc.
237	δ Virginis	3.7	Ma	12 50.6	+ 3 56	0.479	3	Mb	G7	+ 1.3	30.2	0.033	−0.030	Y
238	Lal. 24168	7.7	G	12 53.9	− 9 18	0.82	3	K1	G9	+ 7.0	0.158	0.072	−0.015	Y
239	ε Virginis	3.0	K0	12 57.2	+11 30	0.273	2	G6	G6	+ 0.4	69.2	0.030	+0.034	F
240	Boss 3398	5.9	Mbp	13 1.5	+23 9	0.066	4	Mc	G9	+ 1.8	19.1	0.015	+0.043	vM
241	Lal. 24414	6.9	G	13 3.8	+ 5 46	0.72	3	G4	G2	+ 3.4	4.37	0.020	+0.008	Y
242	Lal. 24504	8.5	G	13 6.4	+10 9	0.54	3	F8	G2	+ 4.4	1.74	0.015	+0.028	Y
243	β Comae	4.3	G0	13 7.2	+28 23	1.184	3	G1	G0	+ 4.6	1.45	0.115	+0.116	P, F, Y
244	A.G. Ber. 4855	7.8	....	13 7.5	+18 3	0.60	3	F6	F8	+ 5.3	0.759	0.032	......	......
245	Lal. 24652	7.0	......	13 11.9	+17 35	0.70	3	K3	K1	+ 6.7	0.209	0.087	......	......
246	W.B. 13^h216	8.8	G5	13 15.7	+ 4 39	0.58	5	K4	K3	+ 6.0	0.398	0.028	−0.021	Y
247	Lal. 24774	8.3	K	13 16.1	+43 38	0.45	3	K0	K0	+ 5.7	0.525	0.030	+0.179	Y
248	70 Virginis	5.2	G5	13 23.5	+14 19	0.634	2	G4	G4	+ 3.1	5.75	0.038	+0.149	Y
249	Lal. 25012	7.5	G	13 26.6	− 1 49	0.91	3	G9	G7	+ 7.5	0.100	0.100	+0.053	Y
250	Boss 3500	6.2	K	13 26.9	+42 37	0.091	3	K0	K0	+ 1.6	22.9	0.012	......	......
251	Boss 3534	5.2	Ma	13 36.4	− 8 12	0.108	3	Ma	G9	+ 1.0	39.8	0.014	......	......
252	A.G. Ber. A 4999	9.6		13 40.2	+18 21	1.92	1	K9	Ma	+10.2	0.008	0.132	+0.091	Sc., R
253	Lal. 25372	8.7	K	13 40.7	+15 26	2.32	3	Ma	Ma	+10.4	0.007	0.210	+0.176	I
254	Boss 3554	6.3	F5	13 42.0	+ 6 51	0.499	4	F9	G0	+ 4.7	1.32	0.048	......	......
255	τ Bootis	4.5	F5	13 42.5	+17 57	0.487	2	F6	F6	+ 3.1	5.75	0.052	+0.040	Y

TABLE I—*Continued*

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum.	π	Trig. π	Auth.
256	Boss 3605	6.0	K	13h53m8	+15° 8′	0″087	3	K4	K2	+1.1	36.3	0″010	.....	.....
257	d Boötis	4.8	F5	14 5.8	+25 34	0.076	2	F7	F8	+2.6	9.12	0.036	.....	.....
258	κ Virginis	4.3	Ko	14 7.6	−9 48	0.130	2	K2	G9	+1.6	22.9	0.029	.....	.....
259	Boss 3647	6.3	F5	14 9.1	−5 29	0.317	3	F8	F8	+4.1	2.29	0.036	.....	.....
260	Boss 3650	5.5	F	14 9.3	+13 26	0.267	3	F7	F7	+4.1	2.29	0.052	+0″097	vM
261	α Boötis	0.2	Ko	14 11.1	+19 42	2.282	6	Ko	G9	+0.9	43.7	0.138	+0.075	F, Y
262	Lal. 26294	8.3	.....	14 17.7	+30 6	0.56	3	K5	K4	+8.7	0.033	0.120	.....	.....
263	Ber. B. 5072	9.5	.....	14 21.1	+24 6	1.38	1	K8	K6	+9.0	0.025	0.079	+0.072	R, L and J
264	θ Boötis	4.1	F8	14 21.8	+52 19	0.471	2	F6	F6	+3.4	4.37	0.072	.....	.....
265	ρ Boötis	3.8	Ko	14 27.5	+30 49	0.149	2	K3	K1	+0.9	43.7	0.026	.....	.....
266	σ Boötis	4.5	Fo	14 30.3	+30 11	0.228	2	Fo	F2	+3.5	3.98	0.063	.....	.....
267	Lal. 26630	6.2	G	14 31.7	−11 53	0.946	3	F5	F5	+4.3	1.91	0.042	+0.049	Y
268	Boss 3736	6.0	K	14 33.6	+18 44	0.096	3	Ko	Ko	+1.1	36.3	0.011	+0.057	vM
269	μ Virginis	4.0	F5	14 37.8	−5 13	0.339	2	F1	F2	+2.0	6.92	0.060	.....	.....
270	ε Boötis Br	2.7	Kop	14 40.6	+27 30	0.049	3	G8	G8	−0.7	191.	0.021	+0.050	F
271	Lal. 27026	7.7	G5	14 46.0	−23 53	1.02	3	K5	K4	+6.6	0.229	0.060	+0.041	F
272	ξ Boötis Br	4.7	G5	14 46.8	+19 31	0.168	3	G6	G6	+5.5	0.631	0.145	+0.230	M
273	ξ Boötis Ft	6.6	.....	14 46.8	+19 31	0.168	2	K3	K3	+7.6	0.091	0.158		
274	Lal. 27137	6.3	.....	14 48.8	+19 33	0.53	3	G9	Ko	+5.8	0.479	0.079	.....	.....
275	β Urs. Min	2.2	K5	14 51.0	+74 34	0.030	2	K4	K1	+0.6	57.5	0.048	+0.014	F
276	Boss 3810	5.6	K	14 51.3	−11 0	0.002	5	K3	Ko	+2.2	13.2	0.021	.....	.....
277	Pi. 14h212 Br	5.8	K	14 51.6	−20 58	2.039	3	K5	K6	+6.8	0.191	0.158	+0.174	Sitter, F, M
278	Pi. 14h212 Ft	8.7	.....	14 51.6	−20 58	2.07	1	Ma	.....	+10.1	0.009	0.190	+0.212	M
279	Boss 3816	5.7	G5	14 52.4	+0 14	0.069	3	K1	K1	+1.4	27.5	0.014	+0.044	vM
280	β Boötis	3.6	G5	14 58.2	+40 47	0.063	2	G5	G4	+0.4	69.2	0.023	.....	.....
281	Lal. 27558	8.7	.....	15 3.0	+9 16	0.54	3	F6	F7	+4.7	1.32	0.016	.....	.....
282	A.Oe. 14318	9.6	K5	15 4.7	−15 59	3.76	1	Ko	G8	+6.2	0.331	0.021	+0.035	R, L and J
283	A.Oe. 14320	9.2	G	15 4.7	−15 54	3.75	2	G8	G5	+6.0	0.398	0.023	+0.054	R, L and J
284	Boss 3867	6.0	Mb	15 7.5	+19 21	0.004	3	Mb	G6	+0.4	69.2	0.008	+0.005	vM
285	Lal. 27742	6.8	G	15 8.2	+19 39	0.68	3	G6	G3	+3.9	2.75	0.026	+0.018	R, Y
286	Lal. 27743	7.6		15 8.2	+19 40	0.62	4	G9	G5	+4.9	1.10	0.029	−0.071	R
287	3 Serpentis	5.2	Ko	15 10.2	+5 19	0.023	2	G9	G6	+0.2	83.2	0.009	.....	.....
288	δ Boötis	3.5	Ko	15 11.4	+33 41	0.155	2	G5	G4	+0.2	83.2	0.022	.....	.....
289	5 Serpentis	5.2	G	15 14.2	+2 9	0.644	2	F7	F7	+4.2	2.09	0.063	+0.094	Y
290	Lal. 27958	8.0	G	15 14.8	+26 4	0.51	3	G8	G6	+4.3	1.91	0.018	+0.031	Y
291	Boss 3900	6.2	G	15 15.2	−17 48	0.069	4	G9	G6	+0.6	57.5	0.008	.....	.....
292	6 Serpentis	5.5	K	15 15.9	+1 5	0.125	3	K5	K2	+2.5	10.0	0.025	+0.133	F
293	W.B.15h268	8.7	K5	15 17.7	+1 47	0.52	3	K4	K4	+6.3	0.302	0.033	+0.118	Y
294	η Coronae	5.6	G	15 19.1	+30 39	0.237	3	Go	Go	+4.8	1.20	0.069	+0.078	S
295	μ¹ Boötis	4.5	Fo	15 20.7	+37 44	0.169	3	Fo	.....	+3.1	5.75	0.052	+0.039	Mil.

296	μ^2 Boötis	6.7	K	15 20.7	+37 44	0.169	3	G0	G0	+ 5.3	0.759	0.052	+0.055	F, Mil.
297	τ^1 Serpentis	5.5	Ma	15 21.2	+15 47	0.033	2	K6	K1	+ 1.1	36.3	0.013	....	....
298	β Cor. Bor	3.7	Fp	15 23.7	+29 27	0.190	3	F2p	F2	+ 1.2	33.1	0.032	....	....
299	Lal. 28358	6.9	F8	15 26.6	+57 47	0.29	3	F5	F5	+ 4.2	2.09	0.029	+0.029	F
300	Boss 3952	4.8	K	15 28.7	− 9 43	0.389	3	K2	G9	+ 1.5	25.1	0.022	....	....
301	γ Librae	4.0	K0	15 29.9	−14 27	0.065	2	G9	G8	+ 0.4	69.2	0.019	....	....
302	Boss 3963	6.1	K	15 31.0	+17 59	0.081	5	G7	G6	+ 0.8	47.9	0.009	....	....
303	α Serpentis	2.8	K0	15 39.3	+ 6 44	0.130	3	K1	K1	+ 0.2	83.2	0.030	+0.151	F
304	κ Serpentis	4.3	K5	15 44.2	+18 27	0.110	2	K7	K0	+ 0.9	43.7	0.021	....	....
305	39 Serpentis	6.2	G	15 48.5	+13 31	0.586	4	G1	G1	+ 5.5	0.631	0.072	+0.040	Y
306	Boss 4048	5.8	K5	15 50.2	+20 36	0.083	3	K4	G8	+ 0.9	43.7	0.011	+0.040	vM
307	γ Serpentis	3.9	F8	15 51.8	+15 59	0.133	2	F4	F5	+ 3.9	2.75	0.100	+0.066	4
308	W.B. 15ʰ1323	8.0	....	15 54.5	+28 1	0.85	3	G8	G9	+ 6.4	0.275	0.048	....	....
309	Lal. 29085	5.5	F	15 54.7	−16 14	0.75	6	F5	F5	+ 4.2	2.09	0.055	....	....
310	ρ Cor. Bor	5.4	F	15 57.2	+33 36	0.81	3	F8	F9	+ 3.9	2.75	0.050	+0.031	Y
311	Lal. 29307	7.5	....	15 59.9	+25 31	0.87	3	K0	G8	+ 5.4	0.692	0.038	+0.058	S
312	θ Draconis	4.1	F8	16 0.0	+58 50	0.459	2	F7	F8	+ 3.6	3.63	0.070	+0.066	Kostinsky
313	Lal. 29330	8.5	....	16 1.2	+10 57	0.50	7	K0	G9	+ 7.4	0.110	0.060	....	....
314	Lal. 29381	6.8	G5	16 1.5	+30 26	0.55	3	G8	G6	+ 6.7	0.209	0.096	+0.069	Y
315	Boss 4096	5.6	Ma	16 2.0	−26 4	0.122	3	Ma	G8	+ 2.0	15.8	0.019	....	....
316	Lal. 29430	9.2	G5	16 2.9	+38 55	0.60	3	K4	K2	+ 7.1	0.145	0.038	+0.014	Y, K
317	κ Herculis	5.3	G5	16 3.6	+17 19	0.046	2	G6	G4	+ 0.1	91.2	0.009	−0.004	L and J
318	τ Cor. Bor	4.9	K	16 5.3	+36 45	0.325	4	K0	K0	+ 2.0	15.8	0.026	+0.102	F
319	Boss 4130	5.6	G5	16 8.3	+ 5 17	0.046	3	K4	K2	+ 1.1	36.3	0.012	+0.008	vM
320	δ Ophiuchi	3.0	Ma	16 9.1	− 3 26	0.161	3	Ma	G8	+ 2.2	13.2	0.069	....	....
321	Lal. 29617	5.6	F	16 10.2	− 8 6	0.563	5	F9	G1	+ 4.4	1.74	0.058	+0.039	Y
322	σ Coronae Ft	6.8	....	16 10.9	+34 7	0.302	4	F9	F9	+ 5.1	0.912	0.046	+0.031	F
323	σ Coronae Br	5.8	G0	16 10.9	+34 7		4	F8p	G1	+ 4.4	1.74	0.052		
324	Lal. 29917	8.6	K2	16 16.5	+67 29	0.50	3	K7	K5	+ 8.0	0.063	0.076	+0.151	Y
325	ν^1 Cor. Bor	5.4	Ma	16 18.6	+34 2	0.049	2	Ma	G9	+ 0.9	43.7	0.013	....	....
326	ν^2 Cor. Bor	5.3	K5	16 18.7	+33 56	0.055	2	K6	K2	+ 1.6	22.9	0.018	....	....
327	η Draconis	2.9	G5	16 22.6	+61 44	0.062	3	G6	G5	+ 0.9	43.7	0.040	+0.019	F, M
328	α Scorpii	1.2	Map	16 23.3	−26 13	0.034	2	Ma	G2	− 3.0	1584.	0.014	+0.030	Finlay
329	Lal. 30024	7.0	G5	16 24.5	+18 37	0.53	3	K2	K2	+ 7.2	0.132	0.110	+0.088	Y
330	Lal. 30044	7.3	G	16 25.6	+ 4 26	1.46	3	F6	F6	+ 5.5	0.631	0.044	+0.011	F, Y
331	β Herculis	2.8	K0	16 25.9	+21 42	0.107	2	G6	G7	− 0.1	110.	0.026	+0.020	F
332	Boss 4222	5.9	F	16 31.1	− 2 7	0.546	3	G9	G9	+ 5.4	0.692	0.079	....	....
333	Lal. 30271	7.2	G	16 32.6	+31 9	0.49	3	F6	F6	+ 5.2	0.832	0.040	+0.012	Y
334	Gro. 19 Area VII 308	8.6	....	16 34.1	+51 45	0.115	3	G2	G2	+ 4.5	1.58	0.015	+0.017	K
335	ζ Herculis	3.0	G0	16 37.5	+31 47	0.608	3	G1	G1	+ 2.1	14.5	0.066	+0.118	4
336	Boss 4262	5.8	Ma	16 40.8	+15 56	0.054	3	Mb	G9	+ 1.8	19.1	0.016	+0.022	vM
337	W.B. 16ʰ906	8.8	K	16 50.1	− 8 9	1.27	4	Md	....	+10.0	0.010	0.174	+0.150	Y, L and J
338	Boss 4303	6.5	G	16 50.2	−16 39	0.089	3	K0	K0	+ 3.0	6.31	0.020	....	....
339	Boss 4307	5.5	F	16 50.6	+21 7	0.059	3	G7	G6	+ 0.9	43.7	0.012	+0.001	vM
340	Boss 4326	6.4	K	16 56.0	−18 44	0.052	3	G9	G8	+ 0.9	43.7	0.008	....	....
341	Boss 4371	5.2	K	17 9.2	−26 27	1.224	3	K2	K1	+ 6.3	0.302	0.166	+0.152	F
342	α Herculis	{3.1– 3.9}	Mb	17 10.1	+14 30	0.030	2	Mc	G5	− 0.7	191.	{0.012– 0.017}	....	....

TABLE I—*Continued*

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum.	π	Trig. π	Auth.
343	Boss 4393	5.8	K	17h14m9	+28° 56′	0″045	4	G8	G8	+ 0.6	57.5	0″009	+0″015	vM
344	72 Herculis	5.4	G	17 16.9	+32 36	1.06	4	F8	G1	+ 5.8	0.479	0.120	+0.123	F, Y
345	Lal. 32047	6.3	K	17 25.3	+67 23	0.52	3	G9	G8	+ 5.9	0.437	0.083	+0.062	Y, Gr
346	B.D. +1°3450	7.2	K2	17 26.5	+ 1 45	......	3	K3	K4	+ 1.8	19.1	0.008	+0.039	Y
347	λ Herculis	4.5	K0	17 26.7	+26 11	0.021	3	K2	K1	+ 0.5	63.1	0.016	....	....
348	W.B. 17h514	8.6	G	17 29.9	+ 6 4	0.58	4	F5	F6	+ 4.4	1.74	0.014	−0.004	Y
349	Anon	9.1	....	17 33.5	+18 37	1.29	1	K8	K6	+10.0	0.010	0.151	+0.128	S and M, L and J
350	26 Draconis	5.3	F	17 34.0	+61 57	0.563	2	G0	G0	+ 4.7	1.32	0.076	+0.087	Y, Mil.
351	B.D. +18°3424	9.2	....	17 34.3	+18 37	0.09	1	K1	K0	+ 6.5	0.251	0.029	+0.008	S and M
352	Lal. 32322	8.4	....	17 36.3	+37 16	0.88	3	F7	F9	+ 5.0	1.00	0.021	....	....
353	A.Oe. 17415	9.1	K	17 37.0	+68 26	1.30	5	Mb	Mb	+10.6	0.006	0.200	+0.254	4
354	Boss 4480	6.5	F8	17 37.0	+24 34	0.052	3	K1	K1	+ 0.6	57.5	0.007	+0.020	vM
355	Lal. 32390	7.5	....	17 39.0	+21 40	0.61	3	G8	G6	+ 5.9	0.437	0.048	....	....
356	μ Herculis	3.5	G5	17 42.5	+27 47	0.817	3	G5	G4	+ 3.2	5.25	0.087	+0.095	4
357	Comp. μ Herculis	9.7	....	17 42.5	+27 47	0.817	1	Mb	....	+ 9.6	0.014	0.096	+0.056	R
358	Barnard's P.M. Star	9.7	....	17 52.9	+ 4 25	10.3	3	Mb	....	+13.3	0.0005	0.50	+0.50	Sc. (Allegheny)
359	γ Draconis	2.4	K5	17 54.3	+51 30	0.027	3	K5	K1	+ 1.4	27.5	0.063	+0.015	4
360	Boss 4555	5.9	K	17 57.1	+45 30	0.044	3	K6	K3	+ 1.5	25.1	0.013	+0.026	vM
361	B.D. +26°3151	7.3	....	17 58.4	+26 20	0.66	3	K0	K0	+ 6.1	0.363	0.058	....	....
362	70 Ophiuchi Br.	4.1	K0	18 0.4	+ 2 31	1.131	3	K0	G9	+ 5.6	0.575	0.200	+0.187	6
363	70 Ophiuchi Ft.	6.0	....	18 0.4	+ 2 31	1.131	3	K5	K7	+ 7.6	0.091	0.209	+0.170	M
364	99 Herculis Br.	5.2	F8	18 3.2	+30 33	0.113	3	F4	F6	+ 5.3	0.759	0.105	+0.074	F
365	Lal. 33439	6.7	G5	18 6.3	+38 27	0.62	3	K2	K2	+ 6.4	0.275	0.087	+0.045	Y
366	Boss 4629	5.5	K	18 15.1	+24 24	0.016	3	K2	K0	+ 0.4	69.2	0.010	+0.002	vM
367	η Serpentis	3.4	K0	18 16.1	− 2 55	0.898	3	G8	G7	+ 1.8	19.1	0.048	+0.057	Y, Jw.
368	Boss 4647	6.2	K	18 17.6	+51 18	0.062	3	K1	K0	+ 0.5	63.1	0.008	+0.036	vM
369	109 Herculis	3.9	K0	18 19.4	+21 43	0.324	2	K2	K1	+ 1.0	39.8	0.026	+0.029	Jo.
370	Gro. 19, Area II 66	7.7	G5	18 21.4	+ 8 44	0.498	3	G5	G4	+ 6.3	0.302	0.052	+0.037	K
371	Gro. 19, Area II 75	8.1	G5	18 21.6	+ 8 34	0.498	3	G8	G8	+ 4.3	1.91	0.018	....	....
372	λ Sagittarii	2.9	K0	18 21.8	−25 29	0.197	2	K0	G9	+ 2.3	12.0	0.076	+0.069	F
373	Gro. 19, Area II 90	8.6	G5	18 22.0	+ 8 3	0.157	3	G1	G0	+ 5.3	0.759	0.022	+0.015	K
374	χ Draconis	3.7	F8	18 22.9	+72 41	0.638	4	F5	F6	+ 5.0	1.00	0.182	+0.105	Y, Gr., M
375	1 Aquilae	4.1	K0	18 29.8	− 8 19	0.318	2	K4	K2	+ 1.1	36.3	0.025	....	....
376	A.G. Leid. 6797	8.7	....	18 37.1	+31 28	0.82	3	K3	K2	+ 7.3	0.120	0.052	....	....
377	110 Herculis	4.3	F5	18 41.4	+20 27	0.345	3	F4	F4	+ 3.5	3.98	0.069	+0.080	M, Jo.
378	Pos. Med. 2164[1]	8.9	K	18 41.7	+59 29	2.28	2	Mb	Mb	+10.7	0.005	0.229	+0.290	R, Sc.
379	o Draconis	4.8	K0	18 49.7	+59 16	0.086	3	G8	G4	− 0.2	120.	0.010	....	....
380	R Lyrae	{4.0– 4.7}	Mb	18 52.3	+43 49	0.077	3	Mc	G8	+ 0.8	47.9	{0.016– 0.023}	+0.028	Ab.
381	Munich 18180	9.4	Ma	18 53.1	+ 5 48	1.24	3	K7	K6	+ 9.2	0.021	0.091	+0.069	R, Sc.

382	Brad. 2388	5.2	G	18 53.3	+32 46	0.23	3	F7	F9	+ 4.7	1.32	0.079	+0.116	F, Jw., Mil.
383	Boss 4860	5.8	K	19 1.2	+31 36	0.085	3	K5	K1	+ 1.4	27.5	0.013	+0.026	vM
384	Munich 18816	9.3	K	19 2.2	+ 7 29	0.80	3	K5	K3	+ 8.0	0.063	0.055	+0.040	Sc.
385	Boss 4892	6.8	K	19 9.5	+49 40	0.63	4	G4	G3	+ 5.5	0.631	0.055	+0.038	R, M, vM
386	Boss 4893	6.6	K	19 9.5	+49 40	0.654	4	G6	G2	+ 5.5	0.631	0.060	+0.052	R, M, vM
387	δ Draconis	3.2	Ko	19 12.5	+67 29	0.133	2	G9	G7	+ 0.5	63.1	0.029	....	....
388	θ Lyrae	4.5	K	19 12.9	+37 57	0.012	5	G9	G7	− 0.8	209.	0.009	....	....
389	Boss 4915	6.3	A	19 13.3	−15 43	0.285	4	K2	G9	+ 1.7	20.9	0.011	....	....
390	Boss 4920	6.0	F	19 14.0	+46 49	0.285	3	F4	F5	+ 3.2	5.25	0.027	....	....
391	κ Cygni	4.0	Ko	19 14.8	+53 11	0.133	3	G7	G7	+ 0.9	43.7	0.024	+0.002	Ab.
392	3 Cygni	6.2	K	19 21.3	+24 44	0.658	2	F6	F4	+ 4.1	2.29	0.038	+0.060	Y, Jo., F
393	Boss 4976	4.6	Ma	19 24.5	+24 28	0.170	4	Ma	K1	+ 1.6	22.9	0.025	....	....
394	Groom. 2875	6.7	G5	19 29.5	+58 23	0.66	3	K3	K2	+ 6.1	0.363	0.076	−0.055	Y
395	Lal. 37120	6.6	G	19 29.7	+32 59	0.52	4	F9	F9	+ 4.6	1.45	0.040	+0.058	Y
396	σ Draconis	4.8	K	19 32.6	+69 29	1.840	3	G8	G7	+ 5.4	0.692	0.132	+0.198	4
397	θ Cygni	4.6	F5	19 33.8	+49 59	0.249	3	F3	F4	+ 3.7	3.31	0.066	+0.073	Y, F, M, Ab.
398	54 Sagittarii	5.4	Ko	19 35.0	−16 31	0.085	2	K1	Ko	+ 0.8	47.9	0.012	....	....
399	Boss 5037	6.3	F	19 39.2	+50 18	0.217	3	G1	G1	+ 5.3	0.759	0.063	+0.048	S and M
400	Boss 5038	6.4	F5	19 39.2	+50 17	0.220	4	G4	G3	+ 4.4	1.74	0.040	+0.033	S and M
401	Boss 5043	6.0	K	19 40.4	+41 32	0.017	3	K6	K1	+ 0 9	43.7	0.009	+0.006	vM
402	Boss 5044	5.1	K	19 40.5	−20 0	0.165	3	Ko	G9	+ 0 1	91.2	0.010	....	....
403	γ Aquilae	2.8	K2	19 41.5	+10 22	0.014	2	K3	G9	+ 0.3	75.9	0.032	−0.009	F
404	δ Sagittae	3.8	Map	19 42.9	+18 17	0.009	3	Ma	G1	+ 1.8	19.1	0.040	....	....
405	η Aquilae	{3.7– 4.4}	Go	19 47.4	+ 0 45	0.012	3	(Fo)	(Go)	+ 0.5	63.1	{0.010– 0.023}	+0.006	M
406	β Aquilae	3.9	Ko	19 50.4	+ 6 9	0.484	4	G8	G7	+ 3.2	5.25	0.072	+0.071	M
407	Lal. 38100	6.3	F8	19 54.4	−10 13	0.488	3	F9	F8	+ 4.6	1.45	0.046	−0.004	Y
408	Lal. 38139	7.3	....	19 55.5	−12 31	0.50	3	F4	F5	+ 4.9	1.10	0.033	....	....
409	Lal. 38287	7.2	G	19 58.0	+15 20	0.60	3	G8	G5	+ 6.6	0.229	0.076	+0.088	Y
410	Lal. 38380	5.7	G5	19 59.5	+29 38	0.86	3	G9	G7	+ 4.0	2.51	0.046	+0.050	Y, F
411	Boss 5146	5.9	G	19 59.6	+16 48	0.557	3	F9	Go	+ 5.2	0.832	0.072	+0.104	Y, Jo
412	Lal. 38383	7.2	G	19 59.7	+23 5	1.39	3	K2	Ko	+ 7.6	0.091	0.120	+0.099	Jw., Y, F
413	Boss 5157	5.5	K	20 2.6	+35 42	0.496	3	G9	G6	+ 3.0	6.31	0.032	....	....
414	Groom. 3042	5.7	A	20 3.6	+52 52	0.334	4	F4	F4	+ 4.5	1.58	0.057	+0.066	Y, Ab.
415	Lal. 38683	7.3	G5	20 6.6	+15 53	0.55	3	K1	K1	+ 6.7	0.209	0.076	+0.077	Y
416	Lal. 38711	5.7	G2	20 9.0	−27 20	1.256	3	K3	K2	+ 4.9	1.10	0.069	....	....
417	Boss 5184	5.7	F	20 9.9	+61 47	0.155	3	F6	F4	+ 2.5	10.0	0.023	+0.037	vM
418	o² Cygni	4.0	Kop	20 10.5	+46 26	0.002	3	G9p	G1	+ 1.2	33.1	0.028	+0.003	M
419	24 Vulpeculae	5.4	Ko	20 12.5	+24 22	0.027	3	G7	G5	+ 0.7	52.5	0.012	....	....
420	A.G. Cam. 6486	7.4	....	20 15.9	+55 5	0.81	4	F3	F4	+ 3.7	3.31	0.018	....	....
421	Groom. 3150	6.1	G	20 16.5	+66 32	0.557	3	F7	Go	+ 6.1	0.363	0.100	+0.120	Y, Jw.
422	A.Oe. 20452	8.1	F8	20 17.7	−21 40	1.22	4	F6	F7	+ 4.8	1.20	0.022	....	....
423	γ Cygni	2.3	F8p	20 18.6	+39 56	0.003	3	F9p	Go	+ 1.3	30.2	0.063	+0.072	F, Y
424	Boss 5263	5.8	G	20 26.9	−10 12	0.317	6	G4	G2	+ 4.3	1.91	0.050	+0.035	F
425	Lal. 39704	7.0	G5	20 29.3	+41 33	0.50	3	Ko	G7	+ 6.4	0.275	0.076	+0.007	Y, Jo.
426	β Delphini	3.7	F5	20 32.9	+14 15	0.113	3	F3	F3	+ 1.6	22.9	0.038	+0.017	F, M, Mil.
427	Lal. 39866	8.4	K	20 34.6	+ 4 37	0.85	6	K5	K3	+ 8.0	0.063	0.083	+0.068	Y, Sc.

TABLE I—*Continued*

No.	Name	Harv. Visual Mag.	Harv. Spec.	α 1900	δ 1900	μ	No. Pl.	Spectrum Est.	Spectrum Meas.	Abs. Visual Mag.	Lum.	π	Trig. π	Auth
428	Groom. 3263	6.0	F	20h38m2	+60° 9′	0″.186	3	F5	F6	+ 4.4	1.74	0″.048	+0″.049	Y, Ab.
429	γ Delphini Br.	4.5	G5	20 42.0	+15 46	0.207	3	K1	K1	+ 1.2	33.1	0.022	+0.076	M
430	ϵ Cygni	2.6	K0	20 42.2	+33 36	0.483	2	G8	G7	+ 0.8	47.9	0.044	+0.016	4
431	η Cephei	3.6	K0	20 43.3	+61 27	0.826	3	G9	G8	+ 2.4	11.0	0.058	+0.116	Y, Ab., S and M
432	Fed. 3638	7.8	G5	20 52.4	+74 23	0.70	3	G6	G4	+ 5.2	0.832	0.030	−0.020	Y, F
433	ϵ Equulei	5.3	F5	20 54.1	+ 3 55	0.191	4	F2	F3	+ 3.2	5.25	0.038	+0.048	M
434	Groom. 3357	6.6	G	20 56.1	+39 51	0.33	3	F7	F8	+ 4.6	1.45	0.040	+0.077	Jo.
435	W.B. 20h1454	8.0	F8	20 59.1	+ 2 36	0.55	3	F6	F6	+ 5.0	1.00	0.025	−0.066	Y
436	Lal. 40844	8.8	K	21 0.4	+ 6 41	0.57	4	K6	K4	+ 8.7	0.033	0.096	+0.167	Y
437	61¹ Cygni	5.6	K5	21 2.4	+38 15	5.271	6	K7	K7	+ 7.9	0.069	0.288	+0.313	16
438	61² Cygni	6.3	K5	21 2.4	+38 15	5.157	6	K8	K7	+ 8.7	0.033	0.302	+0.309	16
439	B.D. +38°4362	7.9	K2	21 5.2	+38 19	......	3	K2	G9	+ 1.2	33.1	0.005	−0.031	R
440	W.B. 21h97	7.3	F8	21 7.4	+17 21	0.91	3	F3	F2	+ 5.8	0.479	0.050	+0.018	Y, F, Jw.
441	ζ Cygni	3.4	K0	21 8.7	+29 49	0.059	3	G6	G7	− 0.4	145.	0.017	+0.048	M
442	δ Equulei	5.0	F5	21 9.6	+ 9 36	0.306	4	F4	F5	+ 4.0	2.51	0.063	+0.067	F, Hus.
443	τ Cygni	3.8	F0	21 10.8	+37 37	0.455	3	F1	F1	+ 1.9	17.4	0.042	+0.031	4
444	Lal. 41363	6.5	G	21 14.0	−26 46	0.67	2	G9	G5	+ 5.1	0.912	0.052	....	....
445	Boss 5519	5.7	F	21 23.9	+31 47	0.134	3	F0	F1	+ 3.7	3.31	0.040	+0.032	vM
446	W.B. 21h502	9.1	....	21 24.5	−12 56	1.06	2	K5	K6	+ 8.6	0.036	0.079	....	....
447	g Cygni	5.3	K0	21 25.8	+46 6	0.114	3	G9	G8	+ 1.0	39.8	0.014	+0.042	Ab., Mil.
448	ρ Cygni	4.2	K	21 30.2	+45 9	0.095	2	G6	G4	+ 2.1	14.5	0.038	+0.042	Y, Ab.
449	Boss 5559	6.8	F5	21 34.4	− 0 30	0.236	4	F6	F6	+ 5.2	0.832	0.048	+0.017	F, Jw.
450	ϵ Pegasi	2.5	K0	21 39.3	+ 9 25	0.025	3	K0	G4	− 0.3	132.	0.028	+0.041	F
451	A.G. Ber. 8366	8.8	....	21 39.7	+24 53	0.68	3	K1	G7	+ 6.3	0.302	0.032	....	....
452	κ Pegasi	4.3	F5	21 40.1	+25 11	0.033	3	F2	F3	+ 3.4	4.37	0.066	+0.040	5
453	Boss 5602	6.5	K	21 41.8	+25 6	0.148	4	K3	K2	+ 1.5	25.1	0.010	....	....
454	Lal. 42883	6.8	F5	21 54.2	+29 21	0.54	4	F6	F4	+ 5.9	0.437	0.066	+0.041	Y, F, Jw.
455	α Aquarii	3.2	G0	22 0.6	− 0 48	0.015	2	G0p	F8	+ 0.2	83.2	0.025	+0.010	M
456	Boss 5682	(7.1)	F8	22 1.8	+82 23	0.142	3	F6	F7	+ 3.8	3.02	0.022	+0.007	vM
457	ι Pegasi	4.0	F5	22 2.4	+24 51	0.299	3	F3	F4	+ 4.0	2.51	0.100	+0.097	M, Mil.
458	Boss 5691	5.8	F	22 2.7	+18 59	0.126	3	F1	F3	+ 3.3	4.79	0.032	+0.013	vM
459	Lal. 43205	8.0	G5	22 3.1	+52 39	0.61	3	K0	G7	+ 3.6	3.63	0.013	+0.051	4
460	Lal. 43492	6.9	G	22 12.2	+12 24	0.84	3	G1	G0	+ 5.8	0.479	0.060	+0.015	4
461	Boss 5772	6.1	G	22 18.8	+20 21	0.338	6	F5	F5	+ 4.5	1.58	0.048	+0.060	vM
462	Boss 5786	5.8	G	22 21.5	+ 3 53	0.295	4	F5	F6	+ 4.1	2.29	0.046	+0.067	F, Jw.
463	Boss 5790	4.9	K	22 22.8	+ 4 12	0.325	4	G9	G9	+ 1.3	30.2	0.019	....	....
464	Krüger 60	9.2	K5	22 24.5	+57 12	0.94	4	Mb	Mb	+11.2	0.003	0.251	+0.256	Sc., R, Bar.
465	η Pegasi	3.1	G0	22 38.3	+29 42	0.036	2	G2	G3	+ 1.2	33.1	0.042	−0.028	F
466	Boss 5868	6.1	K	22 39.6	+38 56	0.016	3	K4	K3	+ 0.9	43.7	0.009	+0.011	vM
467	Pi. 22h214	6.5	G5	22 40.9	+29 55	0.47	4	K0	G8	+ 4.4	1.74	0.038	+0.015	Y, L and J

468	λ Pegasi	4.1	Ko	22 41.7	+23 2	0.060	2	G7	G6	− 0.5	158.	0.012	....	....
469	ι Cephei	3.7	Ko	22 46.1	+65 40	0.139	2	Ko	G9	+ 1.1	36.3	0.030	+0.036	M
470	σ Pegasi	5.3	F	22 47.3	+ 9 18	0.516	3	F4	F5	+ 3.9	2.75	0.052	+0.033	Y, F
471	Boss 5920	6.5	F5	22 53.5	+ 8 50	0.420	3	Go	F9	+ 4.4	1.74	0.038	....	....
472	Lal. 45028	7.8	K	22 56.6	− 4 23	0.54	4	K2	K3	+ 5.8	0.479	0.040	+0.053	Y, M
473	β Pegasi	2.6	Ma	22 58.9	+27 32	0.234	3	Mb	G7	+ 1.3	30.2	0.055	....	....
474	Fed. 4371	7.5	G	23 1.2	+67 52	0.60	3	G5	G1	+ 7.0	0.158	0.079	+0.056	Y
475	Lal. 45292	8.2	K	23 4.0	− 2 48	0.59	3	K5	K3	+ 6.6	0.229	0.048	+0.035	Y
476	Boss 5972	5.8	A	23 5.8	+43 0	0.275	3	F5	F4	+ 4.1	2.29	0.046	....	....
477	Boss 5976	5.6	K	23 8.5	+56 37	2.105	3	K5	K6	+ 6.6	0.229	0.158	+0.172	4
478	Lal. 45455	8.6	....	23 8.9	− 9 28	0.56	4	F4	F6	+ 5.1	0.912	0.020	....	....
479	Lal. 45456	9.0	F5	23 8.9	− 9 29	0.55	4	Go	G2	+ 5.8	0.470	0.023	+0.019	Y
480	ψ¹ Aquarii	4.5	Ko	23 10.6	− 9 38	0.367	4	Ko	G9	+ 1.1	36.3	0.021	....	....
481	γ Piscium	3.8	Ko	23 12.0	+ 2 44	0.753	3	G5	G4	+ 1.2	33.1	0.030	−0.041	Y
482	Boss 5989	5.6	F	23 12.1	+52 40	0.270	3	F9	F8	+ 4.1	2.29	0.050	+0.049	vM
483	Lal. 45638	8.2	G5	23 13.8	+ 4 52	0.52	3	K2	G9	+ 6.6	0.229	0.048	+0.016	Y, Jw., M
484	W.B. 23^h257	8.5	....	23 15.0	+28 19	0.57	3	K1	K1	+ 6.8	0.191	0.046	+0.060	S
485	Lal. 45755	7.6	F8	23 16.8	+43 33	0.67	3	Ko	G7	+ 6.3	0.302	0.055	+0.039	Y, R
486	υ Pegasi	4.6	Go	23 20.4	+22 51	0.191	3	F6	F6	+ 2.5	10.0	0.038	....	....
487	A.Oe. 25685	7.5	....	23 26.5	+58 37	1.08	3	Ko	Ko	+ 5.8	0.479	0.046	+0.075	Y
488	B.D. +62°2244	7.3	G5	23 28.2	+62 36	0.44	3	G5	G3	+ 4.3	1.91	0.025	−0.012	Y
489	ι Piscium	4.3	Go	23 34.8	+ 5 5	0.574	3	F5	F7	+ 4.3	1.91	0.100	+0.142	Y, Jw.
490	γ Cephei	3.4	Ko	23 35.2	+77 4	0.169	2	K1	K1	+ 2.4	11.0	0.063	+0.071	S and M
491	Lal. 46495	7.0	G	23 38.5	+57 31	0.62	3	Go	F9	+ 4.9	1.10	0.038	−0.062	Y,F
492	Boss 6097	6.6	G	23 39.9	+55 15	0.029	3	G6	G6	+ 0.7	52.5	0.007	+0.008	vM
493	W.B. 23^h819	8.6	....	23 40.0	+29 0	0.85	2	K1	K1	+ 7.4	0.110	0.058	....	....
494	τ Cassiop	5.1	K	23 42.2	+58 6	0.087	4	Ko	G9	+ 0.6	57.5	0.013	....	....
495	Lal. 46650	8.9	Ma	23 44.0	+ 1 52	1.39	4	Ma	Ma	+10.3	0.008	0.190	+0.182	5
496	Boss 6129	6.6	K	23 47.5	+74 59	0.332	7	K3p	K3	+ 7.2	0.132	0.132	....	....
497	ρ Cassiop	5.0	F8p	23 49.4	+56 57	0.007	5	G4p	G3	− 0.4	145.	0.008	....	....
498	B.D. +26°4721	7.8	....	23 54.2	+26 43	....	3	K2	G6	+ 0.1	91.2	0.003	....	....
499	85 Pegasi	5.9	G	23 56.9	+26 33	1.291	5	G1	G1	+ 5.8	0.479	0.096	+0.099	5
500	Lal. 47207	6.2	G	23 59.6	+34 6	0.78	3	Go	Go	+ 4.4	1.74	0.044	+0.008	Y

the results obtained by three observers, Adams, Joy, and Miss Burwell. The successive columns of the table contain the following data: (1) number of the star; (2) name of star; (3) visual magnitude as given by the Harvard observers wherever possible; (4) spectral type from Harvard determinations; for the fainter stars many of the values are taken from the recent list of measured parallaxes compiled by Walkey;[1] (5) right ascension; (6) declination; (7) total proper motion; (8) number of photographs of spectrum; (9) spectrum as determined from estimates of the general characteristics of the lines; (10) spectrum as determined from the intensities of the hydrogen lines; (11) absolute visual magnitude; (12) luminosity in terms of the sun as unity, the value 5.0 being taken as the sun's absolute magnitude; (13) parallax as computed from the absolute magnitude by the formula $5 \log \pi = M - m - 5$; (14) measured or trigonometric parallax; (15) authorities for the measured parallax; when these exceed three a figure is given in this column to indicate the number of determinations. The following abbreviations are used:

Ab.	Abetti	M	Mitchell (McCormick)
Bar.	Barnard	Mil.	Miller
Bel.	Belopolsky	Ram.	Rambaut
Bg.	Bergstrand	R	Russell
F	Flint	S	Slocum (Yerkes)
Gr.	Greenwich	Sc.	Schlesinger (Yerkes)
Hus.	Hussey	S. and M.	Slocum and Mitchell (Yerkes)
Jw.	Jewdokimow		
Jo.	Jost	vM	van Maanen
K	Kapteyn	Y	Yale
L and J	Lee and Joy (Yerkes)		

It is with considerable hesitation that we have decided to give values of the parallax resulting from the absolute magnitudes to three decimal places. From the relation connecting parallax and magnitude it follows that

$$\frac{d\pi}{\pi} = \frac{dM - dm}{2.2}$$

The percentage uncertainty in a calculated parallax is therefore proportional to the uncertainty in the magnitudes; but the abso-

[1] *Journal of British Astronomical Association*, 27, Appendix, 1917.

lute error depends upon the parallax itself. For many stars, particularly those of large parallax upon which even moderate errors in the magnitudes have a marked influence, the third decimal is probably undesirable. On the other hand, because of uniformity and the universal practice in the case of measured parallaxes, it has seemed preferable to retain it. An additional reason is the great number of stars of very small parallax. Attention should be called to the influence of uncertainties in the apparent magnitudes upon the parallaxes. In the case of some of the fainter stars and the components of close pairs this may be very appreciable.

NOTES ON INDIVIDUAL STARS

No. 45. This star is Boss 637. The mean spectral type is given and the mean magnitude is used for the computation of the parallax.

Nos. 69, 71, 72, 73. These are the brightest stars of the Hyades and form part of the moving cluster in Taurus. The parallax of this cluster as computed by Kapteyn is $+0\overset{''}{.}023$.

No. 125. The spectrum contains bright hydrogen lines, but the bands are not sufficiently strong for classification as Md.

No. 248. An unpublished parallax kindly communicated by Professor Mitchell is $+0\overset{''}{.}050$.

No. 303. An unpublished parallax by Mitchell is $+0\overset{''}{.}043$.

No. 357. The apparent magnitude is uncertain.

No. 358. This star is used as a standard for reduction of absolute magnitudes. Hence the parallax agreement is necessary.

No. 405. The mean spectral type is given.

No. 456. The apparent magnitude is uncertain.

No. 496. This star has a composite spectrum and is a spectroscopic binary. The absolute magnitudes show a considerable range on different photographs.

COMPARISON WITH THE MEASURED PARALLAXES

There are in the list 360 stars with measured parallaxes. If we divide these stars according to spectral type and absolute magnitude we obtain the comparison given in Table II. The differences Δ are spectroscopic *minus* measured values. The mean difference,

accordingly, for the 360 stars is +0.″0037, the spectroscopic values being the larger. The very large discordance in the case of the K3–K9 stars under M = +2.0 to 4.9 is due to No. 292 of the list with but a single measured parallax. The average difference taken

TABLE II

Type	M = −3.0 to +1.9		+2.0 to +4.9		+5.0 to +7.9		+8.0 to +13.3		Total	
	No.	Δ	No.	Δ	No.	Δ	No.	Δ	No.	Δ
F0–F8...	12	+0.″003	62	+0.″004	14	+0.″018	.....	....	88	+0.″006
F9–G7...	22	+0.005	35	+0.007	34	+0.007	.....	....	91	+0.007
G8–K2...	38	−0.004	21	−0.011	35	+0.002	2	−0.021	96	−0.004
K3–K9...	16	+0.011	1	−0.108	23	+0.014	17	+0.005	57	+0.008
Ma–Md..	14	+0.004	1	−0.049	.....	....	13	+0.008	28	+0.004
All....	102	+0.002	120	+0.001	106	+0.008	32	+0.004	360	+0.0037

regardless of sign between the spectroscopic and the measured values for all the stars is 0.″026.

If the comparison is limited to the stars with several parallax measurements the systematic difference nearly disappears. Thus

TABLE III

Observer	No.	Δ
Miller..........................	34	−0.″008
Mitchell........................	53	+0.003
Schlesinger.....................	15	+0.005
Slocum, Slocum and Mitchell, and Lee and Joy..................	36	+0.013
van Maanen......................	48	−0.006
Yale............................	148	+0.008

there are 59 stars in the list with parallaxes measured by three or more observers. The mean difference, spectroscopic *minus* measured values, for these stars is +0.″001.

A comparison with the measured parallaxes of various observers may also prove of interest. The results are given in Table III. Most of the Yerkes determinations are combined in one group.

COMPARISON OF SPECTRAL TYPES

Of the 500 stars in the list, 441 have spectral determinations by the Harvard observers. For the faintest of these stars only the even spectral class is given without subdivisions, or at most with divisions of five units such as G5 or K5. Accordingly the comparison is not altogether a fair one. The results are as follows, the unit being one-tenth of a spectral division, the interval, for example, from K0 to K1:

	Δ	Average Difference
Harvard–Mount Wilson estimates..	−1.6	3.4
Harvard–Mount Wilson measures...	−0.7	3.2

These results are influenced greatly by a small number of large discrepancies, such as classifications of A and K for the same star in the two sets of observations.

A more valuable comparison is that of the brighter stars in the list. For these higher dispersion has been employed by the Harvard observers and the spectral division is given more closely. Using the values of the *American Ephemeris* we have in the case of 164 stars:

	Δ	Average Difference
Harvard–Mount Wilson estimates..	+0.5	1.6
Harvard–Mount Wilson measures...	+1.2	2.2

Although the number of stars used is much smaller in this case, the agreement is better than in the previous one. It is, of course, clear from the definition of the estimated and measured determinations of the Mount Wilson values that the estimates should be the more closely comparable with the Harvard results.

INTENSITY OF THE HYDROGEN LINES

The intensity of the hydrogen lines in stars of the same general type of spectrum must be recognized as furnishing valuable evidence as to the position of these stars in the system of stellar development.

The fact that the hydrogen lines are remarkably intense in the giant M stars and weak in the dwarf M stars was found some years ago in the course of classification work at this Observatory,[1] and a tendency in the same direction was recognized for some stars of the K type. Accordingly, in all recent classification work we have made two determinations of type. The first is based upon the general characteristics of the spectrum, such as the high-temperature and low-temperature lines, the intensity of the blue calcium line at λ 4227, the prominence of the bands, and many other features. For this purpose photographs of the spectra of stars typical of each type have been selected, and the spectrum to be classified is compared with these in turn. The second method of classification is based directly upon the intensities of the hydrogen lines and has been described fully elsewhere.[2] If therefore we compare the spectral types as derived by these two methods we obtain a measure of the intensities of the hydrogen lines. When the two determinations of type agree, the hydrogen lines are normal; when the measured value gives an earlier type, the hydrogen lines are exceptionally strong.

The comparison of the estimated spectral types of the stars in Table I with those derived from the hydrogen lines gives results of considerable interest. If we employ as the unit the same spectral subdivision as before, that is, the interval from K0 to K1, for example, and divide the stars into groups arranged according to spectral type and absolute magnitude we obtain Table IV. Under "Abs. Visual Mag." is given the arithmetical mean of the absolute magnitudes, and under "Est.—Meas." the difference in spectral type in the units referred to. Thus a value of $+3.7$ in this column means that the type from the hydrogen lines is 3.7 units earlier than the estimated value. If, for example, the estimated value were K8 the measured value would be K4.3.

It is probable that a difference of at least one unit in the two values for spectral type is due to accidental error or to a systematic deviation in the classification curves. The stars of low

[1] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, **40**, 385, 1914.

[2] *Mount Wilson Communications*, Nos. 23–26; *Proceedings of the National Academy of Sciences*, **2**, 143, 1916.

luminosity therefore show but little difference between the estimated and the measured values throughout all the spectral types, and the same is true for the high-luminosity stars of the earlier types. The hydrogen lines, accordingly, are normal for all of these stars. In the case of the high-luminosity stars, however, the intensity of the hydrogen lines is quite abnormal for the later K and more especially the M stars. There seems to be little doubt that this behavior is related to the well-recognized giant and dwarf division

TABLE IV

	No.	$M \leqq 3.9$		No.	$M \geqq 4.0$	
		Abs. Visual Mag.	Est. − Meas.		Abs. Visual Mag.	Est. − Meas.
F0–F4........	28	2.8	−0.7	10	4.7	−0.9
F5–F9........	33	2.5	−0.8	66	4.7	−0.6
G0–G4........	14	1.6	+0.4	37	5.1	+0.7
G5–G9........	63	0.8	+1.2	38	5.5	+1.7
K0–K3........	58	1.2	+1.6	45	6.2	+1.2
K4–K9........	21	1.3	+3.7	41	7.0	+1.0
M............	30	1.2	+12.5	9	10.8	+0.4

of these stars, and that great intensity of the hydrogen lines is to be associated with high luminosity. They have accordingly been used as criteria for absolute magnitude in the case of the giant M stars.

THE GIANT AND DWARF DIVISIONS

The direct evidence bearing on the division into two classes afforded by the stars in this list may be most clearly indicated by a tabulation of the numbers of stars of the various absolute magnitudes. For this purpose narrow limits of magnitude should be selected. In Table V the results are given for an interval of 0.5 magnitude.

A graphical representation of these results is shown in Fig. 1. The clearly marked separation in the case of the M- and later K-type stars is seen persisting in the form of two strong maxima as far as those of the G type. The crests of these maxima draw more closely together in the successive types. There seem to be the rudiments of the first maximum among even the F stars, though

this is perhaps uncertain. The extraordinary grouping of the F stars around the magnitudes 3.0 to 5.5 is their most characteristic

TABLE V

Absolute Magnitudes	Ma–Md	K9–K4	K3–Ko	G9–Go	F9–Fo
−3.0 to −2.0	1	....	....	....	....
−2.5 −2.1	....	....	....	....	....
−2.0 −1.6	....	....	....	....	....
−1.5 −1.1	....	....	....	....	....
−1.0 −0.6	1	....	....	4	1
−0.5 −0.1	1	....	2	7	1
0.0 +0.4	2	....	6	17	3
+0.5 +0.9	6	6	13	19	4
+1.0 +1.4	5	7	18	11	6
+1.5 +1.9	8	7	10	5	3
+2.0 +2.4	4	....	4	4	1
+2.5 +2.9	1	1	1	3	7
+3.0 +3.4	....	....	3	5	13
+3.5 +3.9	1	....	1	2	23
+4.0 +4.4	....	....	4	14	28
+4.5 +4.9	....	....	1	12	26
+5.0 +5.4	....	....	1	15	15
+5.5 +5.9	....	1	6	15	6
+6.0 +6.4	....	3	16	10	1
+6.5 +6.9	....	6	8	3	....
+7.0 +7.4	....	6	7	2	....
+7.5 +7.9	....	4	2	1	....
+8.0 +8.4	....	5	....	2	....
+8.5 +8.9	....	8	....	1	....
+9.0 +9.4	....	6	....	....	....
+9.5 +9.9	2	2	....	....	....
+10.0 +10.4	3	2	....	....	....
+10.5 +10.9	4	....	....	....	....
+11.0 +11.4	2	....	....	....	....
+11.5 +11.9	....	....	....	....	....
+12.0 +12.4	....	....	....	....	....
+12.5 +12.9	....	....	....	....	....
+13.0 +13.4	1	....	....	....	....

feature. The absolute magnitudes of the maxima of frequency for the various types may be summarized as follows:

	Ma–Md	K9–K4	K3–Ko	G9–Go	F9–Fo
Giant....	+ 1.6	+1.4	+1.3	+0.6	(+1.1)
Dwarf....	+10.8	+7.8	+6.3	+5.3	+4.1

The question whether the selection of the stars observed may be accountable for the absence of stars of magnitude intermediate

between those of the two groups in the case of the M and K types should perhaps be discussed briefly. For the M stars it does

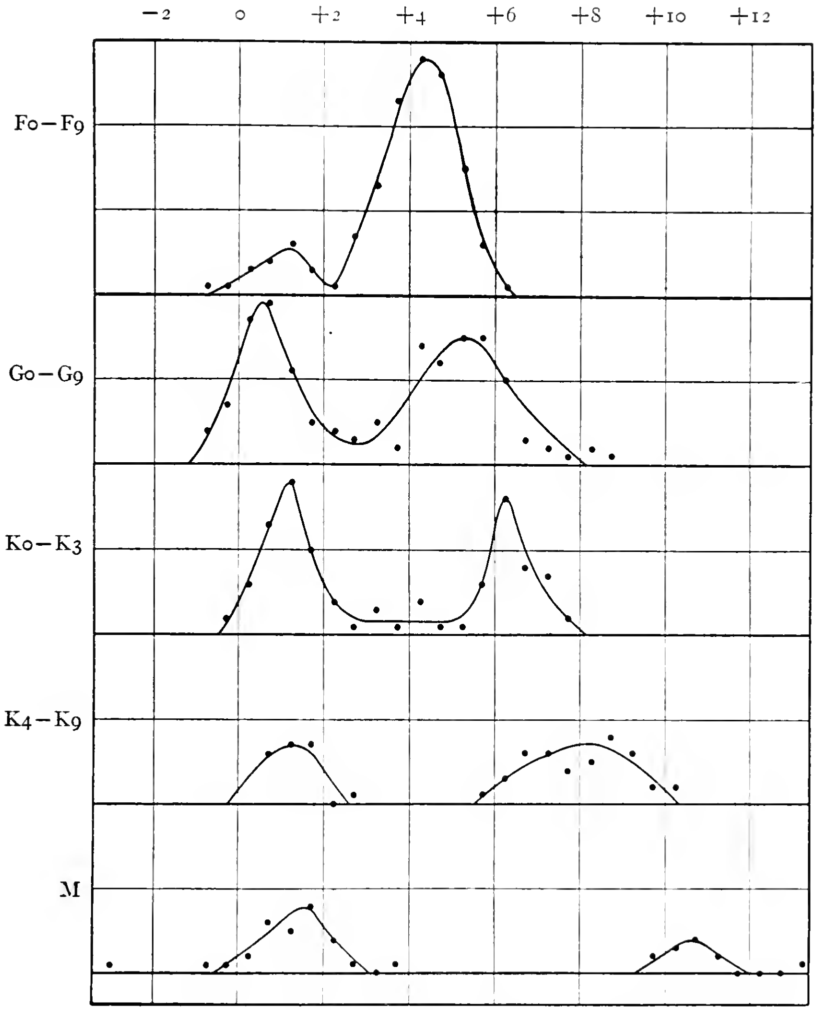


FIG. 1.—Frequency-curves of absolute magnitudes for different spectral types

not seem possible that this can be the case. Our observations have included all of the known faint stars of this type with large proper motions; about 15 stars with small proper motions and apparent

magnitudes between 7.5 and 8.5; and over one-third of the stars of this type listed in the catalogue of Boss with various proper motions. Throughout this list of about 110 stars the spectral differences between the two classes persist clearly, and there is no evidence whatsoever of an intermediate type of spectrum. The interval of over six magnitudes remains without representation.

The same argument applies, but in less degree, to the K-type stars. Among the great number of K stars given in Table I there is a wide variety in proper motion and apparent magnitude. It is difficult to see why this list, which contains representatives of both the giant and the dwarf classes, should not contain a full share of stars of intermediate absolute magnitudes, if such exist.

In the case of the G-type stars it is possible that the basis of selection of the material has affected the results somewhat. The fact that stars of very large proper motion and of very small proper motion have formed a considerable portion of the observing list would tend to give an excess of very bright and very faint stars. This may account in part, but probably not wholly, for the two maxima of the G-type stars shown in Fig. 1. The stars of the *American Ephemeris*, which have not been selected on the basis of proper motion, should tend to smooth out these maxima.

THE BEHAVIOR IN LABORATORY SOURCES OF THE LINES USED FOR ABSOLUTE MAGNITUDE

Three of the most important lines used in the determinations of magnitude are the strontium lines λ 4077, λ 4215, and λ 4607. The first two of these are well-known enhanced lines, being very strong in the spectrum of the electric spark, and, as Mr. King has shown, weak in low-temperature sources such as the electric furnace. The line λ 4607 is of the opposite type and is very strong in the furnace spectrum. A reproduction of photographs of these lines made by Mr. King is given in Plate XVII. The two lines λ 4077 and λ 4215 are identical in behavior in laboratory sources, and it is of interest to note that both are very prominent lines in the spectrum of the solar chromosphere.

The calcium line at λ 4455 is a well-known low-temperature line analogous in behavior to λ 4607 of strontium. The line at λ 4290

is a strongly enhanced titanium line and appears at an early stage in stellar spectra of type A.

Summarizing these results for all of the lines used we have the comparison given in Table VI.

It is probable that the line at λ 4207, like so many of the unidentified lines of Rowland's table, is enhanced. The hydrogen lines $H\gamma$ and $H\delta$ cannot well be classified according to laboratory behavior, although they certainly are not low-temperature lines. The intensity of the hydrogen lines in stellar spectra of types A and B points rather to the opposite conclusion.

TABLE VI

Line	El.	Spectral Types	High-Lumin. Stars	Low-Lumin Stars	Spark	Furnace
4077..........	Sr	A8–F5	Strong	Weak	Strong	Weak
4215..........	Sr	A8–M	Strong	Weak	Strong	Weak
4290..........	Ti	A8–F5	Strong	Weak	Strong	Weak
4455..........	Ca	F6–M	Weak	Strong	Weak	Strong
4607..........	Sr	M	Weak	Strong	Weak	Strong
4207..................		M	Strong	Weak	Unknown	Unknown
$H\delta$, $H\gamma$.......	H	M	Strong	Weak	....	....

The conclusion is obvious that the enhanced lines are relatively strong in the high-luminosity stars and the low-temperature lines strong in the fainter stars. That this is purely a temperature effect, however, is doubtful. The investigations of King with the electric furnace have shown the importance of the influence of vapor-density upon line-intensities, and we should expect wide variations in this respect in the atmospheres of different stars. The observed behavior may well be due to a combination of effects of both temperature and vapor-density in stellar atmospheres. The main consideration, however, is the direct evidence that the use of the absolute-magnitude lines has a definite physical basis and that the lines employed as criteria are as clearly distinguished by their behavior in laboratory sources as in stellar spectra.

GENERAL CRITERIA FOR ABSOLUTE MAGNITUDE

A study of the characteristics of the spectra of stars of very high and very low luminosity makes it clear that the variation of the

lines used as magnitude criteria forms but a portion of a much more general difference. This is most marked in the case of the M-type stars, but it is evident in other types as well. For stars of the same spectral type, the enhanced lines and the hydrogen lines are relatively strong in those of high luminosity and weak in those of low luminosity, while the low-temperature lines are relatively weak in the brighter stars and strong in the fainter stars. To illustrate we may take the case of two stars representing the giant and dwarf classes of M stars:

	α Orionis	Lal. 21185
Hydrogen lines	Very strong (G2)	Weak
Enhanced lines	Strong	Weak
λ 4227 of Ca	Weak	Very strong
Low-temperature lines of Ca, Sr, Fe, Ti, etc.	Weak	Very strong

A very similar result, though somewhat less in degree, is found from a comparison of K-type stars of high and low luminosity. There is little question that λ 4227 of calcium is strongly dependent upon absolute magnitude, being very intense in the low-luminosity stars. We have as yet been unable to employ it satisfactorily, however, because of its great intensity and its rapid change with spectral type.

SUMMARY

1. The absolute magnitudes, luminosities, spectral types, and parallaxes of 500 stars have been determined from investigations of their spectra.

2. A comparison with the directly measured parallaxes of 360 of these stars shows the spectroscopic values to be about $+0\overset{''}{.}004$ larger in the mean. The average difference is $0\overset{''}{.}026$, if all parallaxes are included. A comparison with the 59 parallax values determined by three or more observers gives a difference of $+0\overset{''}{.}001$ in the mean.

3. A method of determining the absolute magnitudes of stars of types A8 to F5 has been derived, and improvements have been made in the methods used for the two classes of M-type stars.

4. A comparison of the determinations of spectral type with those made by the Harvard observers gives a difference of -1.6 spectral divisions in the mean, the Mount Wilson estimates being the later. A comparison for 164 stars listed in the *American Ephemeris* gives a difference of $+0.5$ division.

5. Abnormal intensity of the hydrogen lines is a characteristic of the highly luminous stars of the later types. This is almost certainly related to the giant and dwarf division among these stars.

6. A comparison of the number of stars of each absolute magnitude shows very clearly the giant and dwarf divisions for the M, K, and probably G types of spectrum, with a slight indication even in the case of the F stars. It is almost certain, in the case of the first two of these types at least, that these results cannot be ascribed to the selection of the stars.

7. The lines used for determinations of absolute magnitude are lines which in laboratory sources show marked variations with vapor-density and temperature. The lines which are strong in the highly luminous stars are strong in the electric spark; those strong in the fainter stars are strong in the electric furnace. The correspondence is complete.

8. The high-luminosity stars are characterized by spectra in which both the hydrogen lines and the enhanced lines are abnormally strong. In the spectra of the fainter stars all of the low-temperature lines are strong, including the blue line λ 4227 of calcium. The lines used for determinations of magnitude form but a part of this more general spectral distinction.

Mount Wilson Solar Observatory
September 1917

THE STRUCTURE OF THE MERCURY LINE, λ 2536

By LUCY WILSON

The structure of the line of the mercury arc of wave-length 2536.72 A is of special interest because of its remarkable efficiency in exciting "resonance."[1] For the investigation of the structure of this line, a Fabry and Perot interferometer was selected because of the high resolving power attainable with this instrument under proper conditions. The first part of the problem, therefore, consisted in the study of the relative reflecting power and transmission of various thin films in the region of the 2536 line, in order to find that substance which would be most efficient as a coating for the interferometer plates.

I. THE ULTRA-VIOLET INTERFEROMETER

Any interferometer of the Fabry and Perot type depends for its efficiency upon the number of reflections that it is possible to obtain in the region to be investigated, and upon their relative intensities. These two factors depend only upon the reflecting power of the surfaces, as may be seen from Fig. 1 and Table I.

Let BC and DE (Fig. 1) be the two plates of the interferometer upon which is incident a ray of light whose intensity is represented by 100 after it has traversed the first plate. Assume the reflecting power of the surface to be 30 per cent and the absorption to be zero, then the intensities of the images will be as represented. The first image I_1 is due to the transmitted light which has suffered only one reflection; the second image is produced after three reflections; the third, after five, and so on. The effect on the relative intensities of the images of increasing the reflecting power is apparent from the figures below, which were obtained by calculation from diagrams like Fig. 1.

It is at once evident that, as the reflecting power increases, the ratio of the intensities of the successive images approaches unity.

[1] R. W. Wood, *Philosophical Magazine* (6), **18**, 240, 1909; **23**, 689, 1912; **32**, 329, 1916.

Absorption cuts down the intensity of each transmitted image but does not affect the relative intensities of successive images.

If the reflecting power of the surfaces of the interferometer plates which face each other is f, the relation between the relative

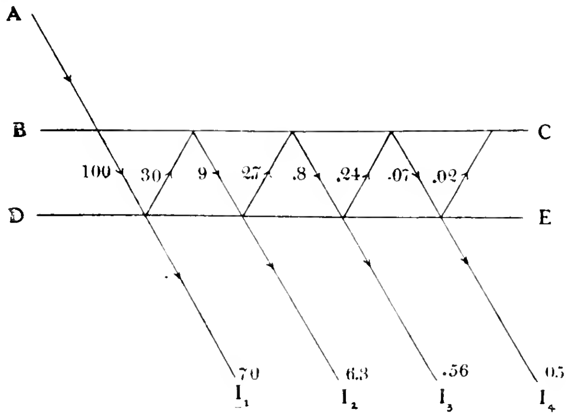


FIG. 1

intensities of the maxima and minima of the fringes produced is given by[1]

$$r=\left(\frac{1-f}{1+f}\right)^2.$$

For example, a value of $f=74$ per cent gives a value of $r=0.02$; i.e., the minima are almost black. Thus the definition of the fringes depends entirely upon the reflecting power.

TABLE I

Reflecting Power	30 Per Cent	50 Per Cent	70 Per Cent	80 Per Cent	90 Per Cent
I_1	70.0	50.0	30.0	20.0	10.0
I_2	6.3	12.5	14.7	12.8	8.0
I_3	0.56	3.13	7.2	8.2	6.6
I_4	0.05	1.6	3.6	5.2	5.9

In using thin films, there are two ways in which the reflecting power can be increased: a thin film of a given substance can be made more highly reflecting by making it thicker up to a certain

[1] Fabry and Perot, *Annales de Chimie et de Physique*, 12, 459, 1897.

limiting point, and a substance of naturally higher reflecting power can be substituted in place of the one under consideration.

Method.—The method of comparing various metals with respect to their efficiency for use in the Fabry and Perot ultra-violet interferometer was a very simple one. Thin films of the substance were deposited on the one-inch quartz plates of the interferometer, and the plates were mounted in the etalon with a wedge-shaped layer of air between them. This was accomplished by inserting a small piece of ordinary writing paper between the plates at a point near their edge, and by bringing the plates into contact at a point 180° removed from the paper, by means of the adjustment screws attached to the mounting of the etalon. Thus multiple images were obtained of any source of light viewed through the plates. The

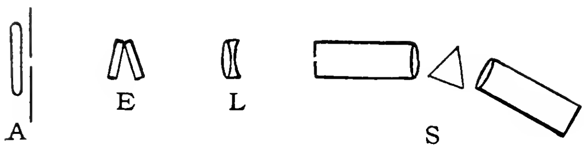


FIG. 2

images in the 2536 line were then photographed. The actual arrangement is represented in Fig. 2. The etalon *E* was mounted at about a meter's distance from the quartz water-cooled mercury arc *A*, in front of which was an aperture of 1 mm diameter. A quartz-fluorite achromatic lens *L*, at a distance of a meter from the etalon, brought the images to a focus on the slit of the quartz spectrograph. In the position of each line of the mercury spectrum, as viewed in the spectrometer and as recorded on the photographic plates, there appeared a series of vertically arranged dots of diminishing intensity. The number of images and the relative brightness of the successive ones were by this means immediately evident. With the films which showed up best by this method, photographs were also taken of the fringes formed when the plates were mounted in the etalon with separations of six and of 10 mm.

Up to this time, silver has been almost universally used in the visible region and nickel in the region of shorter wave-lengths for which the silver failed to give sufficient reflecting power. Cathodi-

cally deposited nickel was used by Fabry and Buisson in their investigation on "Wave-Length Measurements for the Establishment of a System of Spectroscopic Standards," published in the *Astrophysical Journal* of October 1908 (**27**, 169). Their observations extended as far as wave-length 2373. Other observers have also used nickel deposited by cathodic sputtering in the region of wave-lengths slightly longer than that of the 2536 line. The work of Dr. E. O. Hulburt in this laboratory on the "Reflecting Power of Metals in the Ultra-violet Region of the Spectrum"[1] showed that silicon possesses a much greater reflecting power in the regions between 2000 and 3000 A than does nickel. It was, therefore, hoped that silicon might be of use in the present case. Ultimately, films of nickel, silicon, cobalt, aluminum, platinum, silver, gold, and stibnite were prepared and examined.

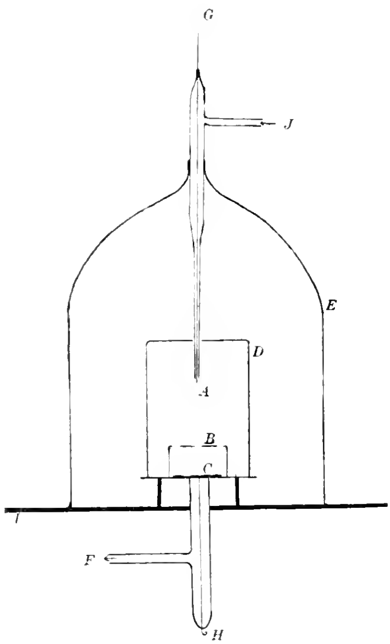


FIG. 3

As many of the films were sputtered cathodically with the same apparatus, a description of it (Fig. 3) in its general form is given here. The large bell-jar *E* was held air-tight against the ground glass plate *I* by means of stopcock grease. To avoid the presence of hydrocarbon vapor, the discharge was confined as nearly as possible to the interior of the inner vessel *D*. The cathode *C* consisted of a large flat piece of the metal to be sputtered (the area was of the order of 10 or 15 sq. cm) and the anode, *A*, was the pointed end of an aluminum rod. The lead wires of both electrodes were incased in glass to prevent the discharge from taking place outside *D*. The

[1] *Astrophysical Journal*, 42, 205, 1915.

surfaces on which the film was to be deposited were mounted on a glass stand at B with their faces toward C. Connections to the pump were made through F and to the transformer by means of G and H. The transformer operated on the 110-volt alternating current and gave approximately 26,000 volts across the secondary. The pointed shape of the anode and the large flat cathode caused the rectification to take place always in this direction. A side tube J allowed any desired gas to be introduced into the vessel.

Silicon.—The silicon films obtained by cathodic sputtering in the atmosphere of mercury vapor yielded disappointing results in that their absorption for the ultra-violet was large. This was probably due to the presence of a trace of carbon from the stopcock grease, which not only made the films faintly brown in color but also caused them to adhere to the surface upon which they were deposited with such tenacity that they could be removed only by repolishing with rouge. Since it was desired to have not only an efficient ultra-violet interferometer but also one that could be easily prepared, the investigation was abandoned in favor of other substances. Later on, however, some beautifully bright silicon films were loaned to us by Mr. W. F. Meggers of the Bureau of Standards. These films were free from the brownish tint, and their absorption was only that which was to be expected from their thickness. Multiple images with them gave, however, a slightly less favorable ratio of intensities than that obtained from our best nickel films.

Nickel.—The nickel films tested were prepared in three different ways:

a) Films of varying thickness were deposited cathodically in an atmosphere of hydrogen and in air. The sputtering apparatus was arranged as represented in Fig. 3. The inflow of the hydrogen was so regulated that the dark space at the cathode was always maintained tangent to the surfaces of the plates upon which the nickel was being deposited. Good nickel films, i.e., those which are hard and gray and bright, were made in this way.

b) Nickel was distilled in a vacuum from a hot tungsten filament on to the quartz surfaces. A piece of fine tungsten wire was wound into a small spiral, and into this spiral was inserted a small sliver of nickel. T (Fig. 4) is the tungsten spiral whose lead

wires A and B connect with a 20-volt circuit containing variable resistance. The quartz plates are at Q and Q'.

In order to obtain films that were fairly hard the receiver was exhausted and the filament was given a preliminary heating before the quartz plates were introduced into the apparatus; this preliminary heating lasted until the nickel melted and a very thin deposit appeared on the walls of the vessel. After the plates were put in, the pump was allowed to run until the McLeod gauge indicated that most of the occluded gases had been given off. The current was then passed through the filament, and its strength increased until the filament reached a white heat. The films produced by this method were of much the same appearance as those deposited cathodically but were not as hard as the best cathodic ones. There was little difference in the behavior of the two sets of films.

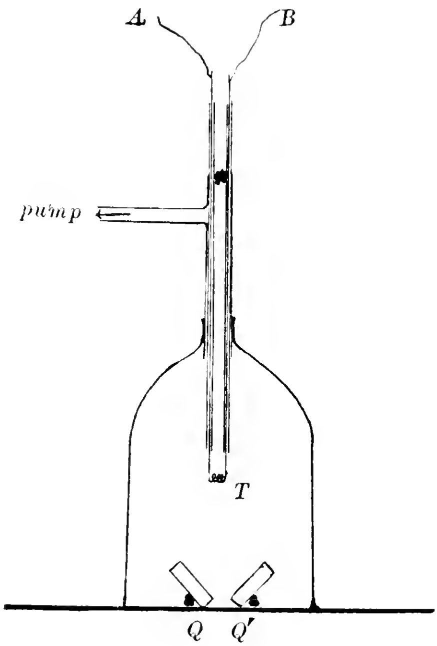


FIG. 4

c) A thin layer of nickel was electroplated from a solution of nickel fluorborate[1] on various foundations. For an initial layer, hard cathodic nickel was found to yield the best results. Nickel on gold and nickel on silver had a fair reflecting power but large absorption. The nickel distilled in vacuo was rather soft and inclined to frill as soon as the plate was removed from the electroplating solution. The films which gave the most favorable ratio of intensities in the multiple images and those which were finally

[1] A. Hollard, *Science Abstracts*, 15, 558, 1912.

used in the work on the determination of the structure of the 2536 mercury line were made by electroplating upon a cathodic foundation. In order to insure hardness in the initial layer, the sputtering apparatus was arranged a little differently. The positions of the cathode and anode were interchanged, and the plates were covered with a piece of thin, sheet tin, and the discharge was allowed to run for two hours until its appearance indicated that little occluded gas was being given off. The tin was then removed by means of an electromagnet and the discharge continued for half an hour.

In the electroplating, the difficulty encountered was the production of a non-uniform layer owing to the greater density of the current near the surface of the liquid than at a short distance below it. This was in part compensated for by inclining the plate and in part by plunging the plate into the solution and slowly drawing it out, repeating the process several times until the film possessed the desired thickness.

This electroplated layer increased the reflecting power without increasing the relative absorption, i.e., a film made according to this last method possessed, with a given absorption, greater reflecting power than one of the same absorption made by either of the other two methods. These films were hard enough to permit polishing with rouge on a bit of cotton, but the process had little effect on the reflecting power.

Cobalt.—Cobalt of three different thicknesses was deposited cathodically in an atmosphere of hydrogen. The method was the same as that described under (*a*) for nickel. A thick film made by sputtering two and three-quarter hours showed good reflecting power but large absorption. A much thinner film, one made by running the discharge only thirty minutes, proved to be almost as efficient in the ultra-violet as the silicon and the electroplated nickel. A film of thickness intermediate between these two (i.e., one for which the sputtering lasted for one and one-half hours) gave less favorable relative intensities than the thin film. An interesting fact noted in connection with these cobalt films was that if a little moisture was deposited on them from the fingers (or elsewhere) the metal frilled and assumed the properties of matt surfaces.

Platinum.—This metal was sputtered cathodically in air and films of two different thicknesses were made and tested. The results showed them to be less desirable than the cobalt.

Aluminum.—From the work of Dr. Hulburt, mentioned above, it appears that as a reflector in the ultra-violet aluminum is second only to silicon. Like silicon, too, it is difficult to obtain in the form of a film. Experiments conducted by V. Kohlschütter and R. Miller[1] and by F. Fischer and O. Hähnel[2] on the "Kathodic Volatilization of Metals in Rarefied Gases" show that the presence of helium accelerates the volatilization of aluminum to a certain extent, and that argon greatly increases the rapidity of the process.

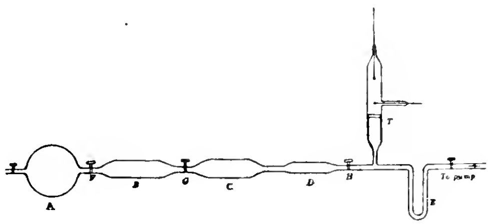


FIG. 5

Therefore the method used in this case was that of cathodic sputtering in argon. An entirely different form of apparatus from that previously described was made use of. It is represented in Fig. 5. The electrodes used in the discharge tube *T* consisted of small lumps of aluminum welded on German silver wire and placed at a distance of about 4 inches apart. The quartz plates (one at a time) were held in position an inch below the lower electrode. (Preliminary experiments with this distance, equal to 1 centimeter, showed decided non-uniformity in the deposit.) This tube was connected with a T-tube, one arm of which joined it to the pump through a U-tube immersed in liquid air and the other arm to the argon bulb *A* through a series of purifying agents. The bulb *B* contained calcium to remove the nitrogen; *C* contained cuprous oxide to remove the oxygen; and *D*, phosphorous pentoxide for absorbing water vapor. The secondary of an induction coil was connected to the electrodes.

[1] *Zeitschrift für Elektrochemie*, 12, 365, 1906.

[2] *Ibid.*, 14, 366, 1908.

The tube *T* was pumped out until the vacuum was such that the dark space was about 3 mm long. Until the final step in the procedure the connections from the induction coil were so made that the upper electrode was the cathode. The stopcocks *G* and *H* were open so that the tubes *B*, *C*, and *D* might be pumped out. When the pressure in *T* had reached its former value (as indicated by the appearance of the discharge), *G* and *H* were once more closed, and *F* opened for an instant to allow a small quantity of argon to flow into *B*. The tube *B* was then heated with a Bunsen burner until the calcium glowed. The stopcock *G* was next opened and the gas allowed to come in contact with the CuO which was also heated and with the P_2O_5. Upon opening the last stopcock *H* the gas flowed into the tube *T*, where it caused a marked change in the appearance of the discharge, due to the increase in pressure and to its characteristic spectrum. Observations with a direct-vision spectroscope showed that hydrogen was always present. This was, in all probability, given off by the aluminum electrodes. The tube was washed out with argon several times in the manner described above, the process lasting from half an hour to an hour. Then a heavy current was applied until the cathode (still the upper electrode) glowed to redness. When this condition was reached, the current was reversed until the lower electrode (now the cathode) reached red heat. The strength of the current in the primary was then reduced to half its former value and so maintained while the deposition took place. The deposit began suddenly, accompanied by marked changes in the appearance of the discharge. Before the cathode reached a temperature sufficient to cause it to glow, the predominating color of the discharge was a pale, whitish lavender; as the temperature of the cathode rose, the luminosity assumed a deeper violet hue; and just before the deposition began the color changed from this deep-blue violet to a bright rose color, which was very brilliant near the cathode, and shaded off to a pale violet in the region of the anode. Three minutes after this marked change took place the films were thick enough.

Though these films were brilliant in visible light and highly reflecting in the ultra-violet, experiments showed them to be slightly less efficient than the electroplated nickel.

Silver and gold.—Films of these two metals were sputtered cathodically in order to be able to make certain comparisons in the visible regions and to gauge some of the governing conditions from the various methods, and in the different forms of apparatus.

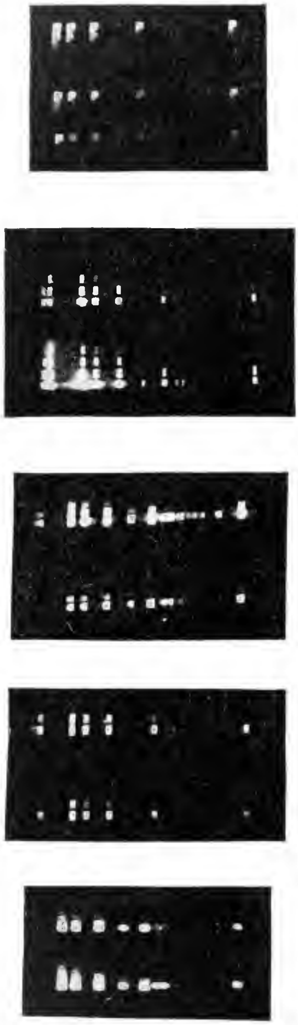
Stibnite.—Because of its remarkable optical properties it was thought that possibly this substance (sulphide of antimony) might prove of value. However, a film of stibnite obtained by cathodic deposit exhibited a very low reflecting power.

From a comparison of the photographs made with the various kinds of films and a consideration of the experimental difficulties involved in preparing the films, it was decided that nickel electroplated upon a cathodic foundation of nickel was best suited for the present investigation. The films used were, therefore, those described on page 345.

Every substance that offered any possibility of possessing high reflecting power in the ultra-violet was tried, but even the best films were most disappointing in the results which they yielded. The largest number of images ever photographed at the 2536 line was four, with the estimated intensity ratio given by a reflecting power of 40 per cent. In Fig. 6, a comparison of the images in the visible lines, photographed with silver films, with those obtained at the 2536 line with the best nickel films shows the relatively poor reflecting power of the latter in the ultra-violet region.

II. THE STRUCTURE OF THE 2536 MERCURY LINE

Method.—The method of using the Fabry and Perot interferometer in order to determine the difference in wave-length of two radiations situated very close together consists simply in finding the difference of path which corresponds to a position of "complete disagreement" or "dissonance" and of ascertaining whether this special path-difference represents the first time this condition has occurred as the plates are separated after being in contact. In the case of the Michelson interferometer, when dissonance occurs for a double line, the fringes disappear completely, if the lines are of equal intensity. In the Fabry and Perot interferometer, dissonance is indicated by a system of fringes with a spacing equal to one-half



Electroplated Nickel

Silicon

Cobalt

Aluminum

Silver

FIG. 6

of that which obtains for zero path-difference. As the path-difference is increased from zero position, the fringes first widen, then double, and finally form an equally spaced system of one-half the original interval.

With visible light, where the wave-lengths of the two radiations are widely different, one can easily watch the fringes get into step and out again, and the position in which one set of fringes is situated exactly between pairs of the other set can be estimated by the eye. For work in the ultra-violet it is, of course, necessary to make use of photography.

The same arrangement of apparatus was used as that indicated in Fig. 2, except that the interferometer was substituted for the etalon, and the screen removed from its position in front of the arc. The distance from the arc to the interferometer plates was about 60 cm, and the lens was placed a meter and a quarter from the interferometer plates. When the fringes were in focus on the slit of the spectrograph $LD = 51$ cm.

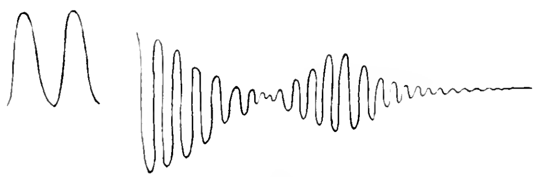
For this work, where it is necessary to have a variable path-distance the etalon was discarded and a mounting substituted by which the plate nearer the arc could be made to approach or recede from the other plate by steps as small as a hundredth of a millimeter. This was accomplished by means of a fixed screw with graduated head and a movable nut on which the one plate of the interferometer was mounted. It was found necessary to diaphragm the interferometer to an aperture of 9 mm because of non-uniformity in the films.

The first step in the procedure consisted in determining the position of contact of the plates. They were then separated by 1 mm and adjustments for parallelism were made for the visible region, i.e., either with a ground glass plate interposed between the arc and the interferometer or with a cell containing neodymium nitrate, which lets through only the green line. When the adjustments were completed, the absorbing screen was removed and the fringes photographed. The same process was repeated for each millimeter up to seventeen. The times of exposure varied from forty-five seconds to five minutes. An exposure of three minutes showed on the average the best contrast.

RESULTS

In the trial experiments with the etalon, the best films of several of the metals showed the fringes in the 2536 line double when the separation was 10 mm. The subsequent work with the electroplated nickel films showed that the fringes in this line were single at a separation of 9 mm and again at 13 mm, but for all intermediate points they were double, and the position of complete disagreement came at a separation of 11.7 mm. The region from zero path-difference up to this point was then examined in half-millimeter steps. No sign of doubling could be detected, and it was therefore concluded that 11.7 mm was the position of the *first* complete disagreement. Investigation was then continued beyond 13 mm up to 2 cm. The fringes gradually faded away into invisibility.

Hence the conclusion is that the line is a doublet, each component of which has a finite width. These results correspond to a Michelson visibility-curve of the form shown in Fig. 7.



Intensity-curve Visibility-curve

FIG. 7

The possibility of the foregoing intensity-curves being due to reversal instead of to a true double line was ruled out by the following experiments. Photographs which were made with a "mercury resonance bulb" duplicated the observations given by the arc. A mercury resonance bulb consists simply of a quartz bulb containing mercury vapor at room temperature, illuminated by light from a quartz mercury arc. Under these conditions the bulb emits a monochromatic radiation of wave-length 2536 which is designated as "resonance radiation." In this case, the mercury vapor present was less dense than in the arc. The water-cooled mercury arc, as it was always used, was flanked on either side by the coils of an electromagnet which deflected the arc forward on to the

quartz window in order to diminish the absorption effect as much as possible. For the purpose of the present experiments, the direction of the current in the magnet was reversed and long exposures (i.e., two to three hours) were made, with the result that no trace of the fringes and only a faint trace of the line itself was discernible. Therefore, a change in possible absorption does not change the intensity-curve, i.e., the relative intensities of the two components remain unaltered.

Calculations of the wave-lengths of the components.—Let the fringes of wave-lengths λ and $(\lambda+a)$ be in position of complete disagreement and let $\Delta=$ the path-difference.

Then $\Delta=2e$, where e is the thickness of the air film between the plates.

If the order of interference is p, then the pth fringe of rays of wave-length $(\lambda+a)$ lies in the center between the pth and the $(p+1)$th fringes of the rays of wave-length λ.

The pth fringe of the rays λ corresponds to a difference of path Δ and the pth fringe of the rays $(\lambda+a)$ corresponds to the path-difference $\left(\Delta+\frac{\lambda}{2}\right)$. Therefore

$$p=\frac{\Delta}{\lambda}=\frac{\Delta+\frac{\lambda}{2}}{\lambda+a},$$

and

$$\frac{a}{\lambda}=\frac{\lambda}{2\Delta}=\frac{1}{2p}.$$

Applying the results obtained from the experiments to this last equation gives

$$\frac{a}{\lambda}=\frac{2536.72\times10^{-7}}{46.8},$$

$$a=\frac{(2536.72\times10^{-7})^2}{46.8}=14\times10^{-10}.$$

Therefore the results given by the Fabry and Perot interferometer lead to the conclusion that the 2536 mercury line is a doublet, the components of which are separated by 0.014 Å. To determine

the probably more complicated structure of this line, an instrument of greater efficiency than the Fabry and Perot interferometer is necessary.

This problem was undertaken at the suggestion of Professor Wood, and the investigation was carried on under his direction. To him I wish to extend my thanks for his unfailing interest as well as for his invaluable help.

I wish to express my gratitude to Professor Ames, whose kindly interest has been continually manifest; and to Dr. Pfund and Dr. Anderson for many helpful suggestions; also to Mr. Meggers for making the silicon films.

JOHNS HOPKINS UNIVERSITY
June 1917

MINOR CONTRIBUTIONS AND NOTES

ON THE TEMPERATURE AND RADIATION OF THE SUN

Messrs. C. G. Abbot, F. E. Fowle, and L. B. Aldrich, in their article in this *Journal* (44, 39, 1916), criticize my paper published here in 43, 197, 1916. They complain that I have not considered the scattering of light, due to which in the earth's atmosphere almost 10 per cent of the sun's light is scattered in all directions, and in the solar atmosphere perhaps even much more.

It is proper to remark that my equation

$$I_z = I p^{F(z)}$$

results from a general differential equation for a ray of energy, i, passing through a medium of thickness, s,

$$di = -\nu i\, ds$$

giving by integration

$$i = I e^{-\nu s}$$

independently of whether we consider the coefficient of transmission as expressing the effect of absorption with the constant ν,

$$p = e^{-\nu s},$$

or the effect of scattering with the constant κ,

$$p = e^{-\frac{\kappa}{\lambda^4}}.$$

Consequently I neglect, as do many others and these authors, in the earth's atmosphere only that part of the scattered light which enters into the direct solar beam from the atmosphere and reaches the observer; that is to say, I neglect the brightness of the earth's atmosphere with respect to the brightness of the sun; and in the solar atmosphere I neglect, as do many others, also only the brightness of the solar atmosphere with respect to the brightness of the photosphere. To what extent this first brightness is negligible with respect to the second can be determined from the black tone of the

nuclei of sun-spots or of the absorption lines in the solar spectrum, because it is the entire radiation of the solar atmosphere, augmented by the radiation of the photosphere transmitted by the absorbing layers, which is projected on the background of these spots or lines.

Let us recall the research by A. Schuster, referred to in the first part of my investigation,[1] where the author examines both the radiation of the photosphere and that of the solar atmosphere. The intensities of the radiation computed by him for the photosphere, as well as the temperatures deduced for it by me, are almost indistinguishable from the intensities and temperatures deduced from the same observed data without considering the radiation of the solar atmosphere.

That the supposition of a black radiation for the photosphere, transmitted by the solar atmosphere practically not radiating compared to the latter, was sufficiently well founded we see finally from the deduced distribution of energy in the spectrum of the solar photosphere, which agrees with the distribution in the spectrum of a black body of known temperature.

With regard to the criticism by these authors as to the relation noted by me between the deduced values for the solar constant and for the coefficients of transmission of the earth's atmosphere observed correspondingly, I reply as follows: The authors criticize my observational data, while they cite many analogous unfavorable atmospheric conditions to justify the uncertainty of their own observations. Moreover, it is notably these unfavorable atmospheric conditions which place more in relief the parallelism noted between the solar constants and the coefficients of transmission, not only in my observations at Warsaw and on Ararat, but also in all the observations by the authors at Washington, on Mount Wilson, and at Bassour. If no relation exists between these constants and coefficients, the probability of a parallel progression in time, direct or reciprocal, between them ought to be expressed by ± 0 per cent. However, this probability is expressed for Mount Wilson and Bassour equally by -27 per cent and -26 per cent (omitting one extreme value), thus showing that in these two places small solar constants correspond to large coefficients of transmission

[1] *Astronomische Nachrichten*, 183, 241, 1910.

and vice versa. For the direct parallel progression between the coefficients of transmission on Mount Wilson and at Bassour a probability of +59 per cent results. For the parallel progression between the solar constants on Mount Wilson and at Bassour there results an equal probability, in sign and size, +58 per cent. It therefore follows that these deduced solar constants depend on observed coefficients of transmission of the earth's atmosphere, affected in their turn by the eruptions of Mount Katmai, because it is not possible that reciprocally the observed eruptions of Mount Katmai should be determined by the solar constants deduced at that time.

The authors of the paper, having chosen in Table I the days of observation without extreme atmospheric conditions, have without doubt diminished the clearness of the relation noted, as is seen in my observations on Mount Ararat as compared with those at Warsaw. Yet, for these data also, if they are construed as in my paper, they show a certain progression, less pronounced, but of the same sign, especially in 1910, 1913, and 1914.

I do not attempt to prove in my investigation that real variations in the radiation of the sun are not possible, but I testify only that it is very difficult to demonstrate it.

FELIX BISCOE

ROSTOV-ON-DON, RUSSIA
June 1917

WAVE-LENGTHS OF THE STRONGER LINES IN THE HELIUM SPECTRUM[1]

I. INTRODUCTION

For both practical and theoretical reasons the spectrum of helium is of considerable importance in spectroscopy and related branches of science. It consists of a comparatively small number of lines well distributed from the ultra-violet to the red, and is conveniently produced in great intensity in an ordinary vacuum tube. Hence it is widely used as a reference spectrum in spectroscopic and optical work of many kinds, but the wave-lengths

[1] *Scientific Papers of the Bureau of Standards*, No. 302; *Bulletin of the Bureau of Standards*, **14**, 159, 1917. By Paul W. Merrill, assistant physicist.

have not been known with sufficient accuracy for the most precise wave-length measurements. It is thought that the present determinations will supply this deficiency and make possible the use of helium lines as standards of wave-length. This investigation is a part of the standard wave-length program of the Bureau of Standards, which has been under way for two years and which is at present chiefly concerned with the spectra of iron, argon, and neon.

Helium is of great interest in astronomical observations, as it is one of the most important elements from a cosmic viewpoint, its spectral lines being prominently observed in such radically different objects as nebulae, early—that is, blue or white—stars, and the sun (flash spectrum).

As an aid to progress in the comprehension of the structure of matter nothing is of greater promise than an accumulation of accurate data in regard to spectral-line series. Thanks to the early investigations of Runge and Paschen, every line observed in the ordinary spectrum of helium may be placed in one of six series. In order to establish the mathematical form of these series and to throw light on their relations to one another as well as to the series of other elements, exact values of the wave-lengths of numerous lines will be required and will grow increasingly valuable as more data become available.

Precise measurements of the stronger helium lines were therefore undertaken in order that they may serve as standards for the determination of other wave-lengths, and to furnish a basis for computations of theoretical interest.

II. MEASUREMENT OF WAVE-LENGTHS

1. *Direct determinations from the red cadmium line.*—Wave-lengths of 21 of the stronger helium lines were measured with the Fabry and Perot type of interferometer, which consisted of two partially reflecting plane surfaces held a few millimeters apart and exactly parallel by an invar separator, using the practice previously adopted by the Bureau of Standards.[1] The interference rings formed by the action of the parallel planes of the interferometer

[1] *Bulletin of the Bureau of Standards*, **12**, 179, 1915; **13**, 245, 1916.

upon the incident light were projected upon the wide slit of a spectrograph having a single prism of rock salt, portions of them accordingly being observed as short bars across the spectral lines. The specially designed achromatic lenses allowed a great range of wave-lengths to be photographed with one exposure. Thus, 2945A and 5875A, (D_3), were obtained on one plate, as were also 3888A and 7281A.

The helium tube employed is similar in every way to the neon tube described by Meggers.[1] It was filled at the Bureau with helium obtained from a London firm. The electrical excitation was furnished by stepping up from commercial alternating current of 110 volts, 60 cycles, to about 10,000 volts (on open circuit). Primary currents from 0.5 to 1.75 a were used, causing from 3 to 11 ma through the tube. The dark space was from 1 to 2 mm in width.

Several of the helium wave-lengths were compared directly with the fundamental standard by photographing the cadmium and helium spectra simultaneously upon the same plate. The cadmium lamp, electrically heated to about 300° C., was in the optical axis of the spectrograph, while the beam of helium light was brought into the axis by reflection from a partially transparent surface through which the cadmium light passed. This surface was formed by a thin film of nickel which had been cathodically deposited upon a quartz disk. Two series of photographs were made in this way, in one case the interferometer films being of copper deposited on glass and in the other of nickel on quartz. The copper interferometers employed were of 5, 10, and 20 mm separation, the nickel of 5, 10, 15, and 20 mm. The plates (Seed 27) were sensitized to the less refrangible rays by treating them with a solution containing dicyanin, pinaverdol, and ammonia. Exposures were made ranging from 4 to 15 minutes and with varying amounts of current through the tube. The results are tabulated in Table I. The agreement of the two series is considered satisfactory except in the case of 7065A. No other reason, however, than accidental error of observation can be assigned for this difference.

[1] *Ibid.*, **12**, 202, 1915.

The very small corrections to reduce the wave-lengths to standard conditions (700 mm, 15° C.) were applied.

TABLE I

HELIUM WAVE-LENGTHS BY DIRECT COMPARISON WITH CD 6438.4696

Copper Films	Nickel Films	Adopted	Copper Films	Nickel Films	Adopted
3888.646	0.646	0.646	(5047.734)	....	....
....	(4026.190)	....	5875.616	0.618	0.617
4471.476	.478	.477	6678.148	.149	.149
4713.143	.143	.143	7065.186	.190	.188
4921.928	.929	.929	7281.350	0.348	0.349
5015.675	0.675	0.675			

2. *Relative measurements.*—The values in Table I served for the determination of other helium lines on photographs of the helium spectrum alone. In this series the nickel films and separations of 3, 5, 7.5, and 15 mm were used with exposures from 20 seconds to 1 hour. With the exception of the three red lines which were not observed in this series, the ratios of wave-lengths in Table I were satisfactorily confirmed, and are apparently accurate to 1 part in 4,000,000. The combined results of all the measurements made in this investigation appear in the first column of Table II.

From the number and internal agreement of the individual determinations it seems that an error larger than 0.003A is scarcely to be expected and that probably most of the errors are smaller than that amount. In the case of the double lines the value, of course, refers to the stronger component. The weaker component seems to have been practically without effect upon the measurements, since accordant values were obtained from a considerable range both of effective exposures and of orders of interference.

This may be illustrated by the values obtained for D_3. It was customary to put several exposures of varying length upon each plate. Determinations of the difference between overexposed and normal or weak images are hence available for several plates. In units of a thousandth of an angstrom they run 0, 0, +1, +1, −3, +4, −5, the last three depending on a poor measurement in each case. The systematic difference is not larger than the accidental error. The photographs of helium alone were taken upon Seed 27

emulsion (not stained), but the longest exposure on each plate (about an hour) shows D_3 faintly. The wave-lengths of this line from four of these underexposed images are practically in agreement with the other determinations, there being possibly a tendency toward a slightly higher value. For 5015A, a single line, the

TABLE II

COLLECTED HELIUM WAVE-LENGTHS (IN I. A.)

Bureau of Standards	Rayleigh (a)	Rayleigh (b)	Eversheim	Runge and Paschen
2945.104				106
3187.743				701
3613.641				641
3705.003				007
3819.606				605
3888.646				638
3964.727				727
4026.189				102
4120.812				821
4143.759				766
4387.928				934
4437.549				549
4471.477	(478)	480	493	475
4713.143	(171)	142	154	074
4921.929	925	928	922	919
5015.675	680	678	683	556
5047.736				641
5875.618	616	623	639	650
6678.149	144	147	151	14
7065.188	189	197	207	22
7281.349				53

difference, strong minus weak exposure, is 0.000 or ±0.001A in 11 cases out of 13. The agreement of different interferometers is shown for two lines by Table III, which refers entirely to the direct comparisons with cadmium. The figures in parentheses give the number of exposures upon which the value depends.

3. *Elimination of difference of phase-change.*—The dispersion of phase-change at reflection from the interferometer mirrors has been referred to as "one of the less agreeable features" of interference measurements. It is customary to find the amount of the differential effect for different wave-lengths by observations with large and small path-differences, and to compute the small corrections to be applied to the measured wave-lengths. This

procedure is, however, by no means necessary, as the whole effect can be eliminated by using differences, as was done by Priest[1] for visual methods. Let us find by the use of the standard line, say Cd 6438A, the double thickness of a large and of a small interferometer. If the difference of these numbers be divided by the difference of the measured orders of interference for another line, it is obvious that the quotient will be the correct wave-length

TABLE III

WAVE-LENGTHS FROM DIFFERENT INTERFEROMETERS

(Comparisons with cadmium)

	5015A		5875A	
	Copper	Nickel	Copper	Nickel
5 mm.........	0.674 (2)	0.677 (2)	0.617 (3)	0.619 (3)
10 mm.........	.673 (3)	.674 (2)	0.616 (3)	.617 (2)
15 mm.........	....	.675 (5)	....	.618 (6)
20 mm.........	0.677 (3)	0.676 (3)	....	0.618 (3)

freed from any effect of difference of phase-change without having found that quantity at all. The difference of the orders will be of about the same accuracy as the larger one, for while the percentage error of the smaller may be greater the actual numerical uncertainty is less.

The same final values should, of course, be arrived at whether the difference of phase-change is eliminated as suggested above or determined by the usual methods and the proper corrections applied. This is the case in the present series of measurements.

III. COMPARISON WITH PREVIOUS VALUES

The present measurements and those by Lord Rayleigh[2] are in quite good agreement, as shown by Table II. Columns 2 (*a*) and 2 (*b*) are separate series of which the second is to be given greater weight. Eversheim's values[3] are greater, but the differences are not uniform. Except for 3187A the grating values of

[1] *Bulletin of the Bureau of Standards*, **6**, 573, 1911.

[2] *Philosophical Magazine* (6), **15**, 548, 1908.

[3] *Zeitschrift für wissenschaftliche Photographie*, **8**, 148, 1909.

the shorter wave-lengths by Runge and Paschen[1] are in good accord with the interference results. The discrepancies among the longer wave-lengths are possibly due in some way to the fact that their measurements depend upon comparison lines in another order (second). This is the case for 4713, 4921, 5015, 5047, 5875, 5678, and 7065A. The mean arithmetic residual, Runge and Paschen minus Bureau of Standards, for these lines is 0.052A, as compared with 0.0065A for the remaining lines, for which the comparisons were in the first order. Omitting 3187A, the mean residual for 12 of the shorter lines is only 0.003A.

Using an interference method which depends upon observing the disappearance of the central ring,[2] Priest found the apparent wave-lengths of certain helium lines to vary with the amount of current through the tube. Variations of this kind were not observed by him for 5015A.[3] Mr. Priest has kindly given me the exact value obtained by him for this line as 5015.679A. This is the mean of several accordant measurements.

IV. SERIES RELATIONS

When our understanding of spectral series is complete, the magnitudes of certain physical quantities can probably be computed from a single series, and from relations between different series, and it may be in this connection that accurate measurements of wave-length will finally be of the most value. In the meantime they can be used to determine the applicability of the various types of formula which have been suggested, to test the so-called "combination principle," etc. For obvious reasons the present investigation includes only a small number of lines at or near the beginning of each series. These lines, although favorably situated for the purpose, will not by themselves give the best values of the series constants, particularly of the convergence frequency. Hence no extensive recomputations have been undertaken at this time, but it was thought of interest to see how closely a three-constant formula based upon three consecutive well-determined lines would represent the remainder of the series.

[1] *Astrophysical Journal*, **3**, 4, 1896.

[2] *Bulletin of the Bureau of Standards*, **6**, 573, 1911.

[3] *Ibid.*, **8**, 1, 1911.

None of the series can be represented exactly by a formula of the type

$$n = A - B/m^2$$

where n is the reciprocal of the wave-length, m represents successive integers, A and B are constants. This equation, however, gives a fairly close representation of the two first subordinate series (5875A, 4471A, etc.; 6678A, 4921A, etc.). The constant B is about the same for both and approximately 0.2 per cent larger than in the Balmer series of hydrogen.

It is well known that the Kayser and Runge formula

$$n = A - B/m^2 - C/m^3$$

will give a fair representation of the helium series. The lines for which $m = 3, 4, 5$, in each of the six series have been measured in

TABLE IV

SERIES CONSTANTS

$$n(\text{vac}) = A - \frac{B}{m^2} - \frac{C}{m^3}$$

	First Group			Second Group		
	Principal	First Subordinate	Second Subordinate	Principal	First Subordinate	Second Subordinate
	CONSTANTS COMPUTED FROM $m=3, 4, 5$, IN EACH SERIES					
A....	38 469.22	29 225.53	29 147.13	32 030.82	27 176.66	27 154.51
B....	110 501.7	109 845.2	102 963.4	109 518.3	109 783.2	107 878.3
C....	13 027.4	152.7	96 030.4	−1 887.8	226.2	38 826.8
	OBSERVED *minus* COMPUTED (ANGSTROMS)					
M....	....	....	....	....	....	....
2....	−18.2	....	....	+124.3	....	....
3....	0	0	0	0	0	0
4....	0	0	0	0	0	0
5....	0	0	0	0	0	0
6....	+ 0.18	+0.021	−1.06	− 0.030	+0.013	−0.34
7....	+ .34	+ .047	−2.2	− .091	+ .036	−0.71
8....	+ .45	+ .078	−3.2	− .103	+ .022	−1.07
9....	+ .56	+. .096	−4.0	− .19	+ .068	−1.30
10....	....	+ .134	....	− .21	+ .069	−1.54
11....	....	....	....	− .23	+ .072	....
12....	....	....	....	....	+ .090	....

the present investigation. The constants A, B, C, as computed from these three lines, appear in Table IV, which also contains the residuals of all the well-observed lines in each series.

In every instance the error of the representation even for $m=6$ is very much larger than the uncertainty of observation, while the residuals show a fairly smooth and converging increase toward the terms of higher order. In some cases there is no improvement as compared with the series computed by Kayser and Runge from less accurate values. This is in agreement with the prevailing opinion that the formula contains the first three terms of a rapidly converging mathematical series which may be regarded as the expansion of a closed expression as yet unknown.

V. SUMMARY

Wave-lengths of 21 of the stronger helium lines have been accurately measured by interference methods. Nine of them were compared directly with the standard cadmium line.

The possibility of eliminating the effect of apparent variation of interferometer thickness with wave-length is noted.

The Kayser and Runge formula for spectral series, based upon three consecutive lines, will not reproduce accurately even the next member in any one of the six helium series.

PAUL W. MERRILL

WASHINGTON, D.C.
March 14, 1917

COMMUNICATION

NAVY DEPARTMENT, U.S. NAVAL OBSERVATORY
WASHINGTON, D.C., November 16, 1917

To the Editor of the Astrophysical Journal:

SIR: The question has recently been raised in England whether the astronomical day should not be set back twelve hours, so as to begin at midnight instead of at noon. It is stated by those advocating the change that the practical consideration of those using the Nautical Almanacs should prevail as against the usage of astronomers. The opinion of American astronomers has been requested, and a committee of the American Astronomical Society has been appointed to collect information for presentation at the next meeting of the Society.

The Committee desires to obtain an expression of opinion on this subject from as large a number as possible of astronomers, geodesists, surveyors, navigators, and all others who have occasion to use Nautical Almanacs.

Communications may be sent direct to Professor W. S. Eichelberger, Director of the Nautical Almanac, U.S. Naval Observatory, Washington, D.C., or possibly better to some journal where a public expression of opinion may stir up further discussion.

Very sincerely,

W. S. EICHELBERGER
Chairman

[The editors are glad to bring the foregoing communication to the attention of the readers of this *Journal*, but suggest that any letters in reply be sent to some other periodical covering this field more closely, as, for instance, *Popular Astronomy*, Northfield, Minnesota.]

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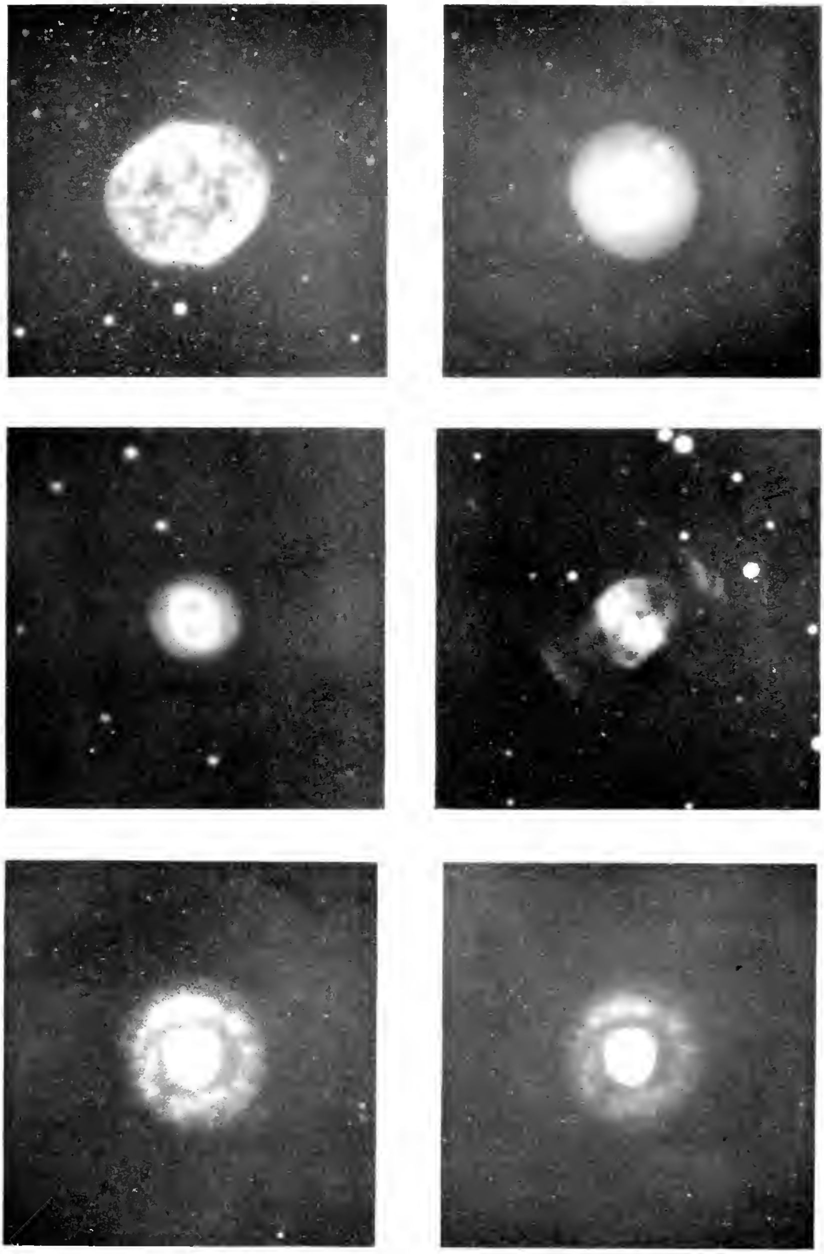
SUBJECTS

INDEX TO VOLUME XLVI

AUTHORS

PLATE I

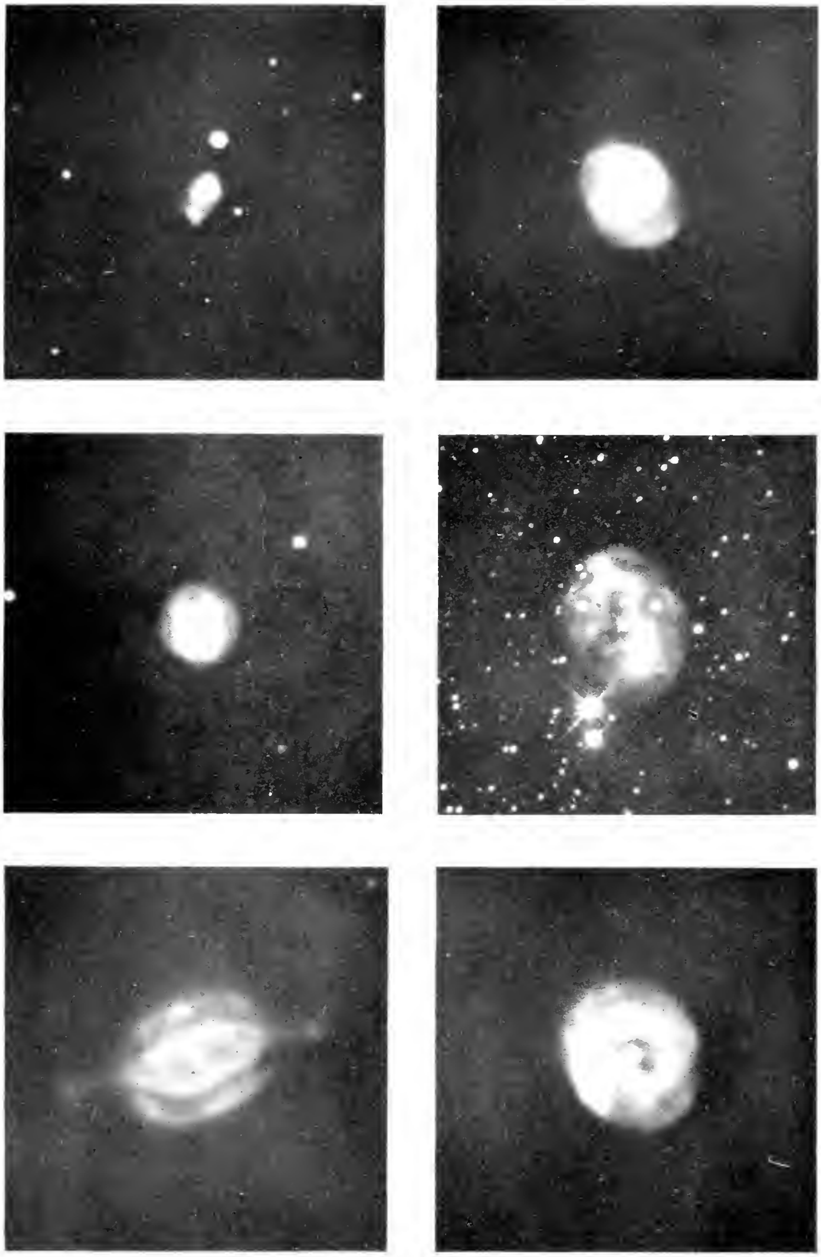


a *b* *c* *d* *e* *f*

a	N.G.C. 1501	Exposure 120^{m}.	Enlargement 10.0.	1 mm = 2.7	
b	1535	85	11.2	2.4	
c	2022	60	11.1	2.5	
d	2371–2	221	5.2	5.2	
e and *f*	2392	120	4.0	2.1	80-ft. focus

e and *f* are reproduced from the same negative with differences in exposure and development.

PLATE II

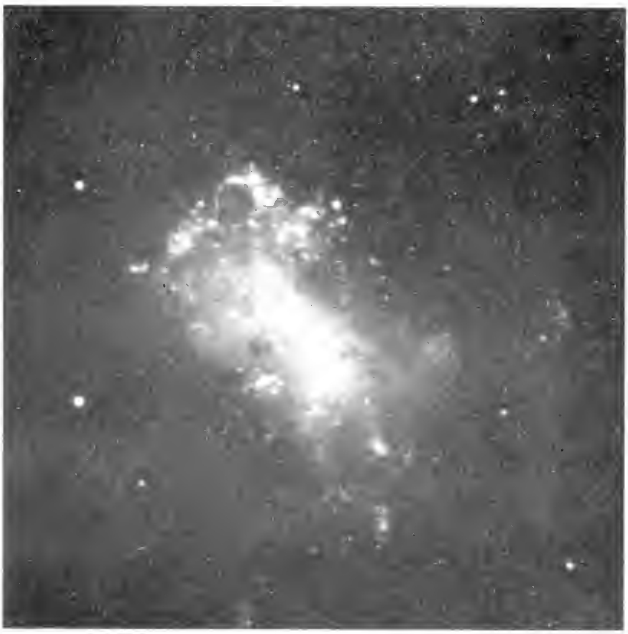


a	N.G.C. 6309	Exposure 90 m.	Enlargement 6.6,	1 mm = 2.8	
b	6543	50	4.5	1.5	100-ft. focus
c	6818	75	2.8	2.4	100-ft. focus
d	7008	180	5.0	4.7	
e	7009	210	4.5	1.5	100 ft. focus
f	7662	90	4.5	1.5	100 ft. focus

PLATE III

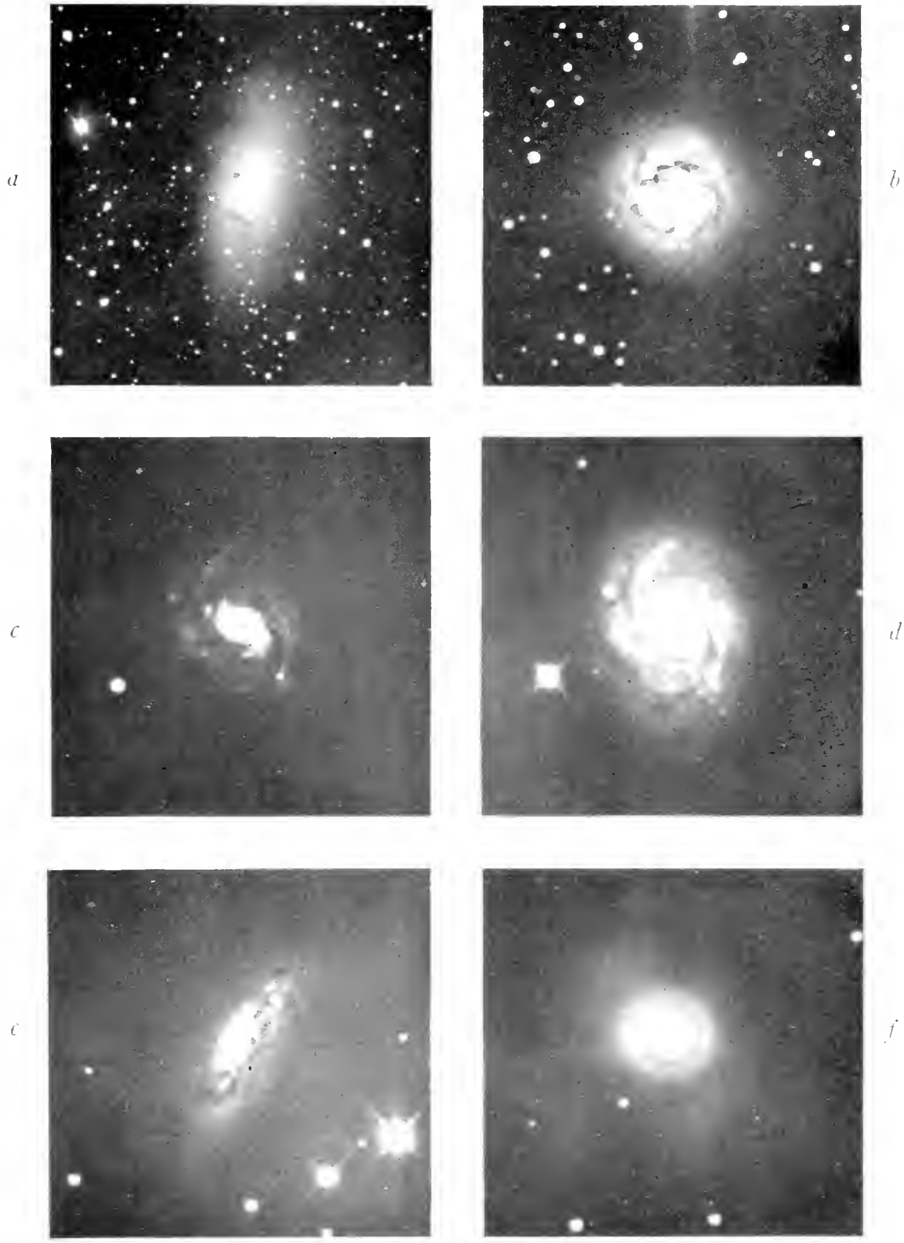
a

b



a	N.G.C. 1570	Exposure 420^m,	Enlargement 3.0,	1 mm = 9″.1
b	4449	300	5.8	4.7

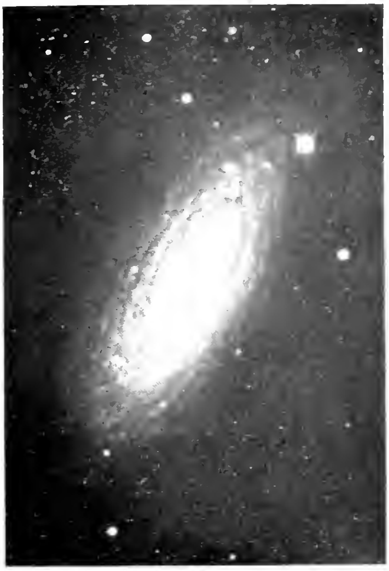
PLATE IV



a	N.G.C. 205	Exposure 230m.	Enlargement 1.5.	1 mm = 10.4
b	278	240	5.8	4.7
c	1068	22	5.5	5.0
d	1068	120	5.5	5.0
e	072	105	[illegible].0	3.0
f	2081	100	7.0	3.4

PLATE V

a



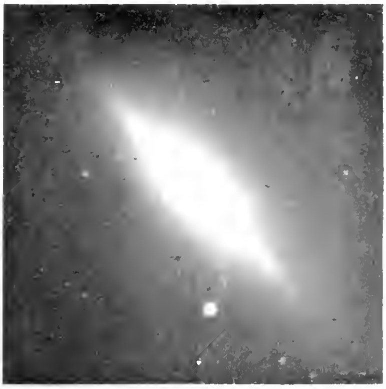
b

c

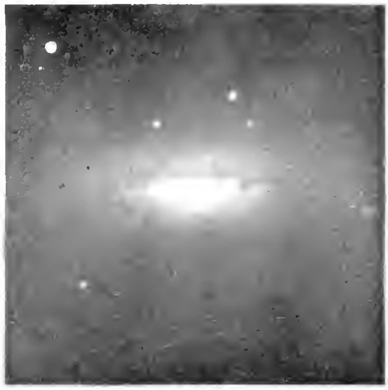
a	N.G.C. 2841	Exposure 120m	Enlargement 3.8	1 mm = 7.2
b	2976	180	4.7	5.7
c	2403	210	2.7	10.0

PLATE VI

a

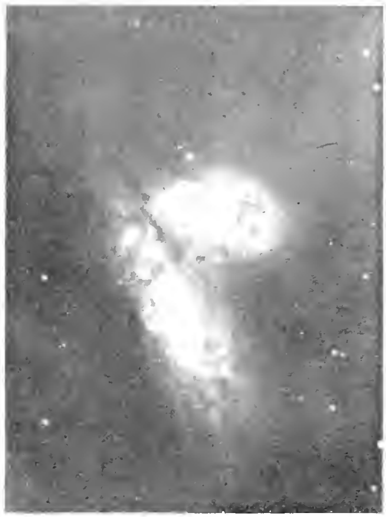


b



c

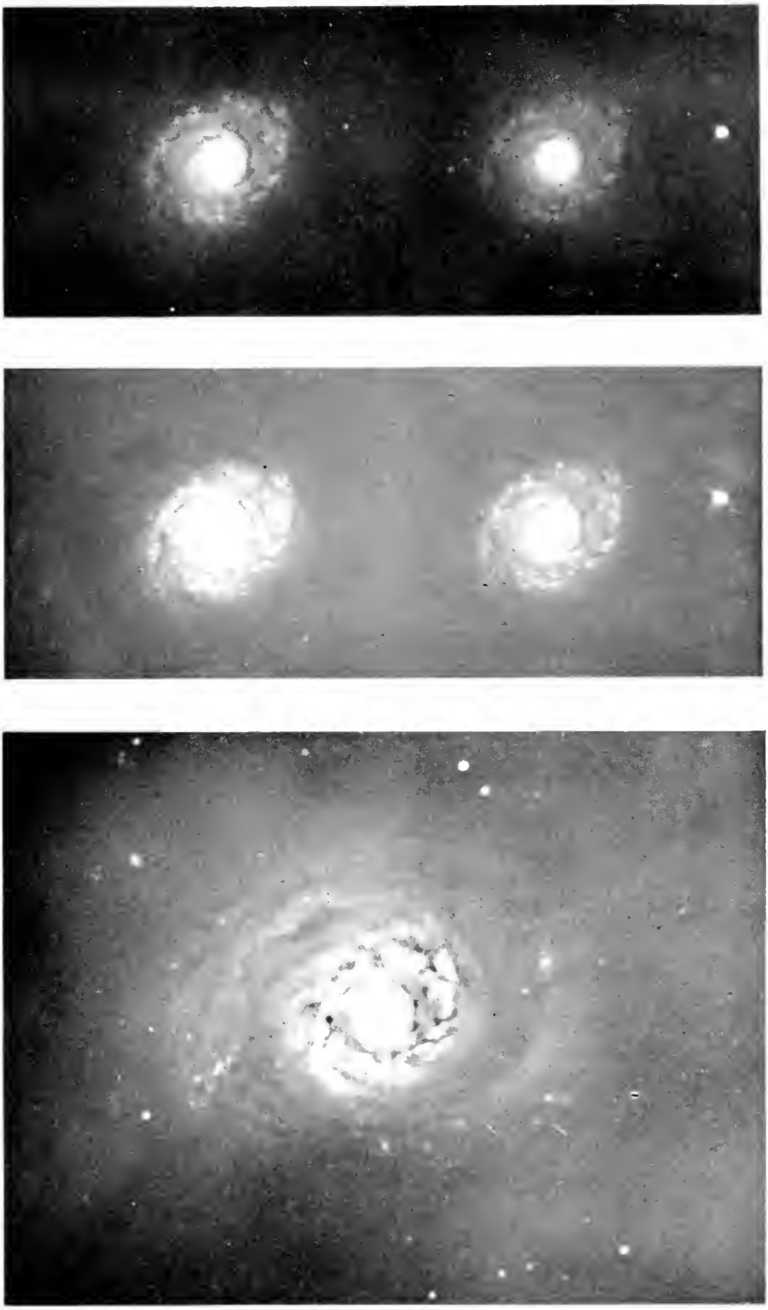
d



e

a	N.G.C. 3115	Exposure 105 [illegible]	Enlargement 5.5.	1 mm [illegible]
b	3593	210	5.5	[illegible]
c	4216	90	3.0	[illegible]
d	4567–8	360	4.0	5.[illegible]
e	4594	132	4.5	[illegible]

PLATE VII

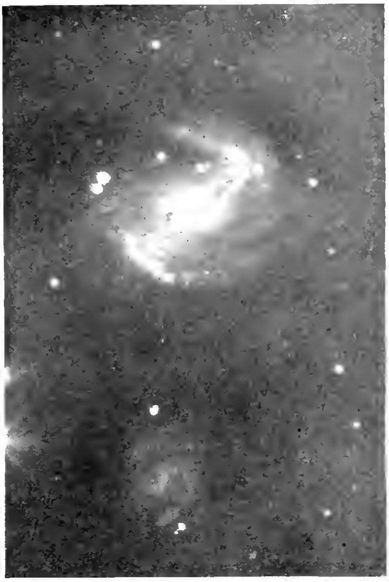
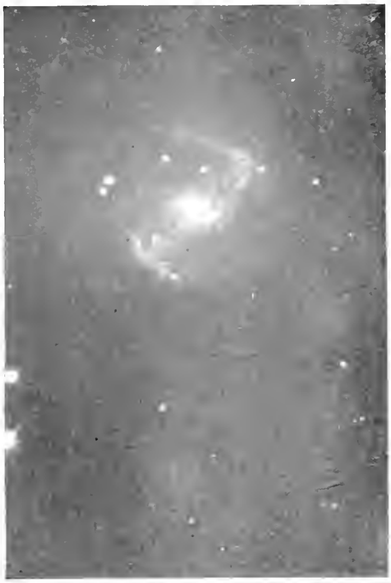
b  *a*

d *c*

e

N.G.C. 4736 Exposure *a*, 5^m; *b*, 10^m; *c*, 20^m; *d*, 40^m; *e*, 225^m.
Enlargement 5 : 1, 1 mm = 5".4

PLATE VIII

b   a

c 

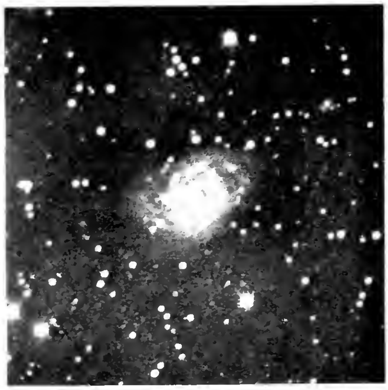
 d

e 

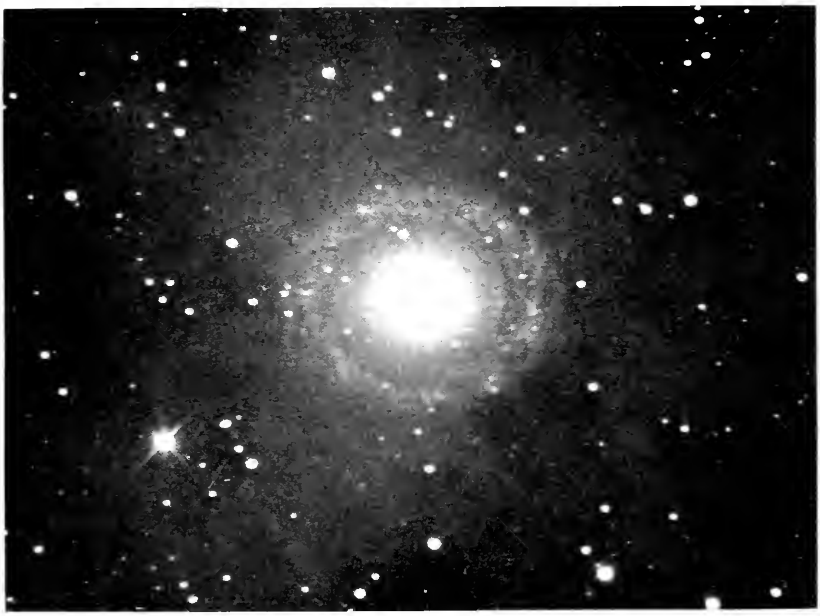
a	N.G.C. 5383	Exposure 180 m.	Enlargement 5.0.	1 mm = 5.4
b	5383	300	4.0	5.5
c	5544–5	300	5.5	5.0
d	5746	300	3.4	7.0
e	5866	105	5.0	5.5

PLATE IX

a



b

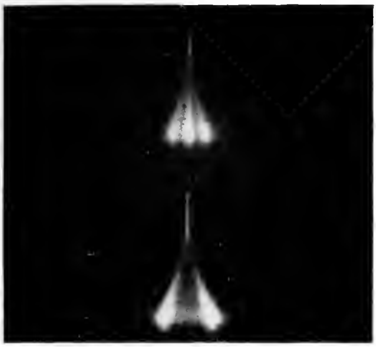


c

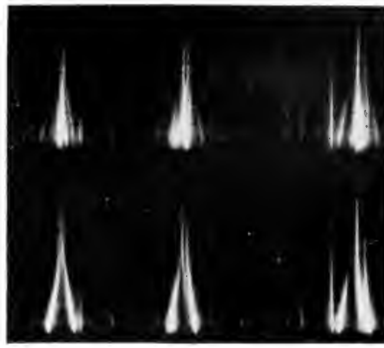
a	N.G.C. 6070	Exposure 150m.	Enlargement 4.2.	1 mm = 6.5
b	6555	300	4.5	6.0
c	7217	330	5.5	5.0

PLATE X

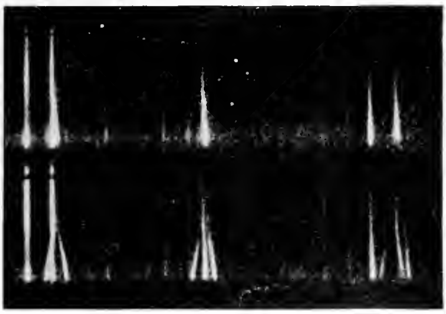
a



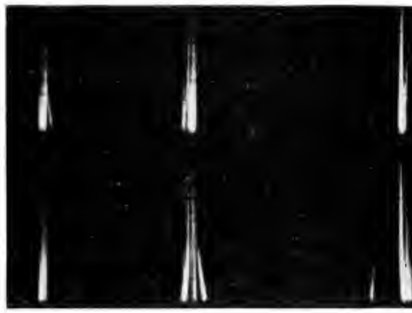
c



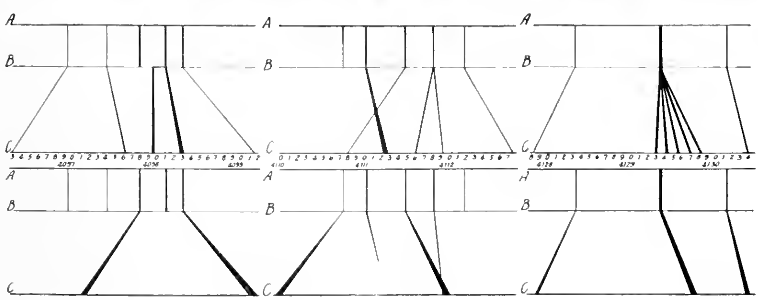
b



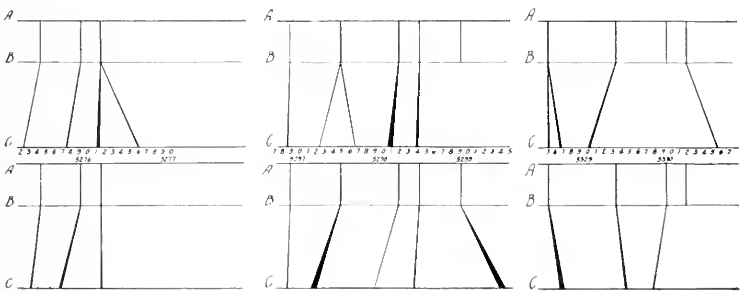
d



c′



d′

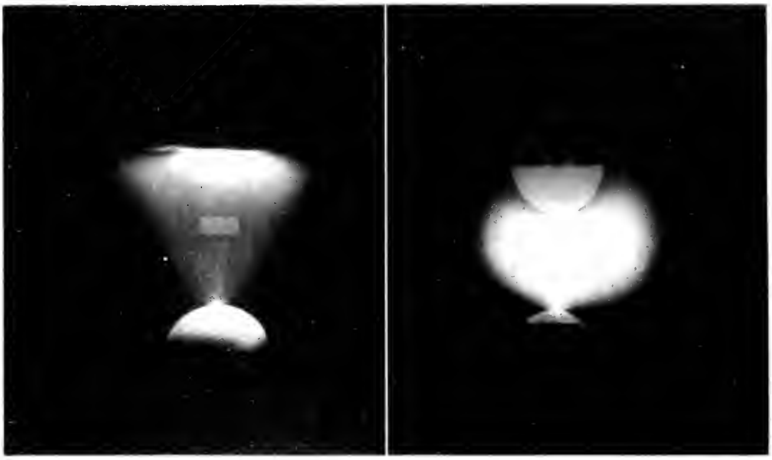


STARK EFFECT

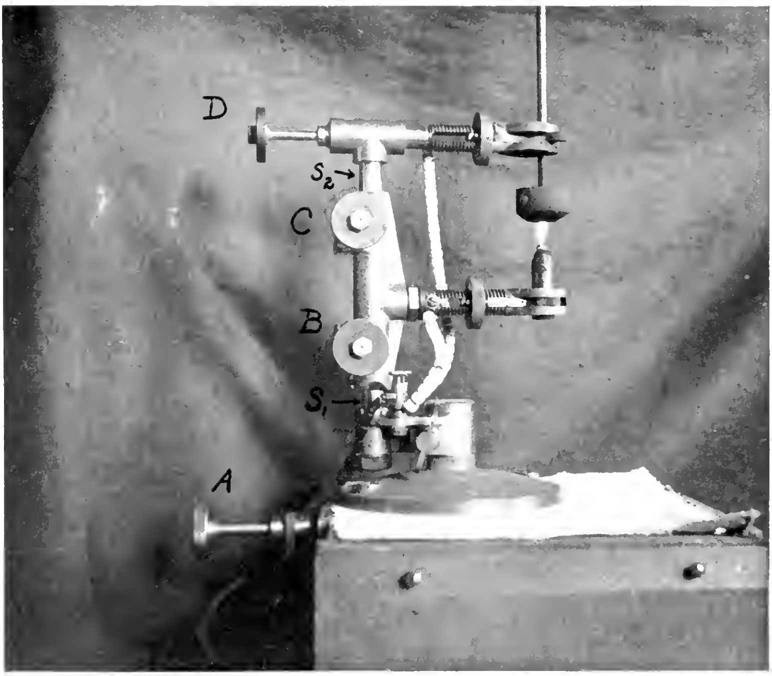
In each illustration the *n*-components are above, the *p*-components below

a, Hγ; *b*, Chromium groups $\lambda\lambda$ 5005, 5027, 5055;
c, Chromium groups at $\lambda\lambda$ 4098, 4111, 4129;
d, Chromium groups at $\lambda\lambda$ 5276, 5298, 5329;
c′, *d′*, drawings representing groups in *c* and *d*,
A–B being the negative glow (field-strength = 0),
C the surface of the cathode (field-strength = 12,000 volts per cm)

PLATE XI

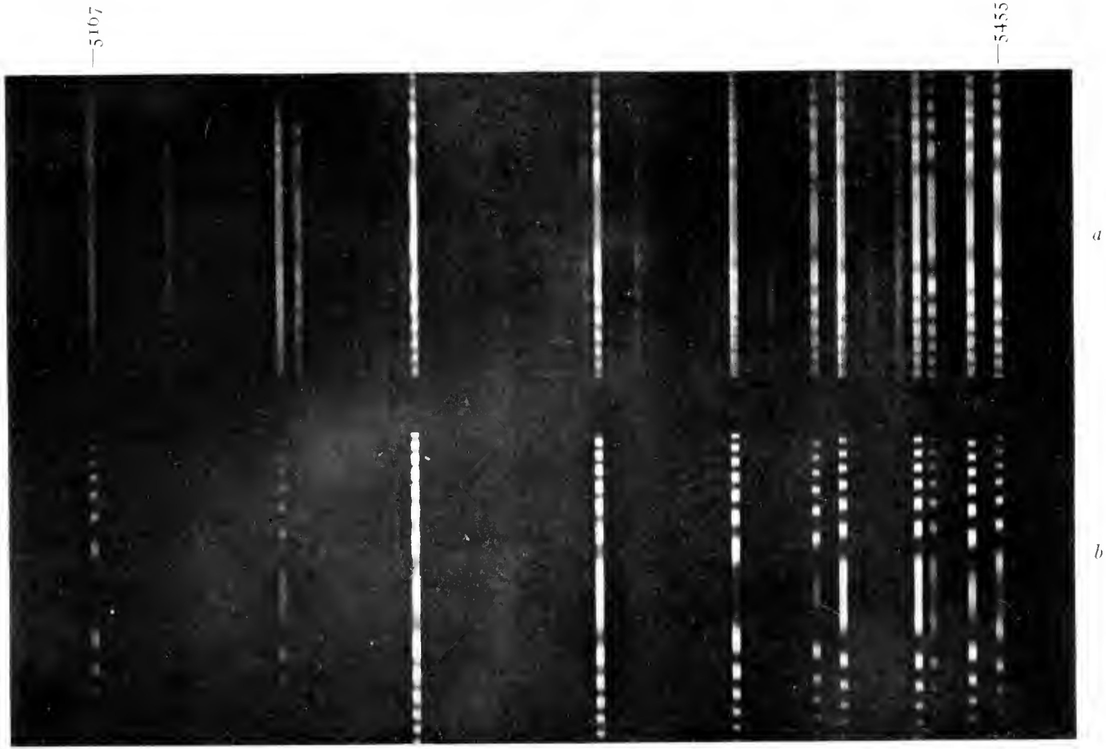
a  *b*

DISTRIBUTION OF LUMINOSITY IN 12-MM, 5-AMP PFUND ARC (*a*) AND IN INTERNATIONAL ARC (*b*) WITH SUPERPOSED IMAGES OF DEFINING DIAPHRAGM



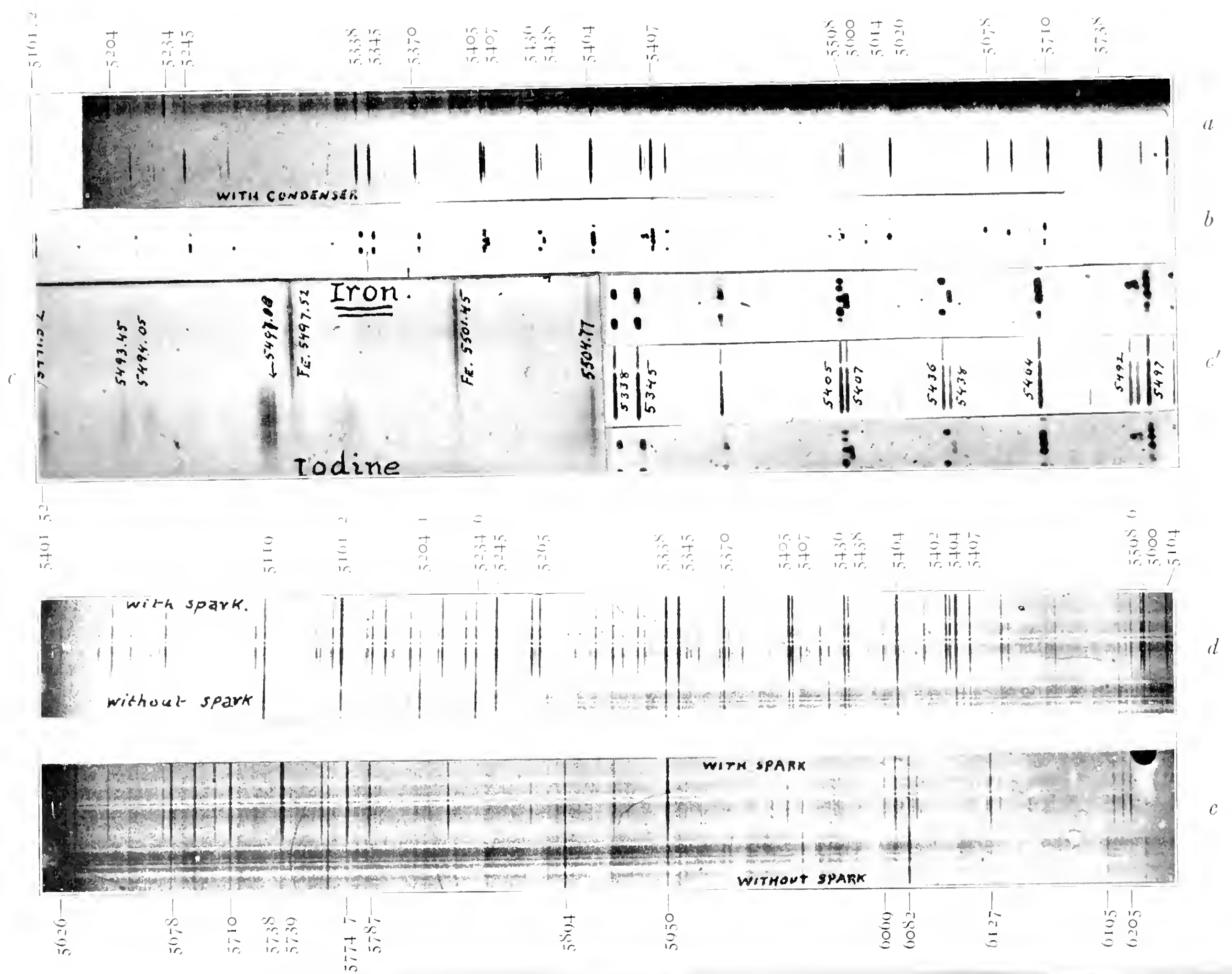
ARC LAMP FOR PRECISION OBSERVATIONS

PLATE XII



Comparative Sharpness of Lines from International Arc (*a*) and Proposed Arc (*b*); 15-mm Etalon

PLATE XIII



WITH CONDENSER
Iron
Iodine
5493.45
5494.05
5497.08
FE. 5497.52
FE. 5501.45
5504.77
5338
5345
5405
5407
5436
5438
5464
5492
5497
with spark.
without spark
WITH SPARK
WITHOUT SPARK
a
b
c'
d
c

PLATE XIV

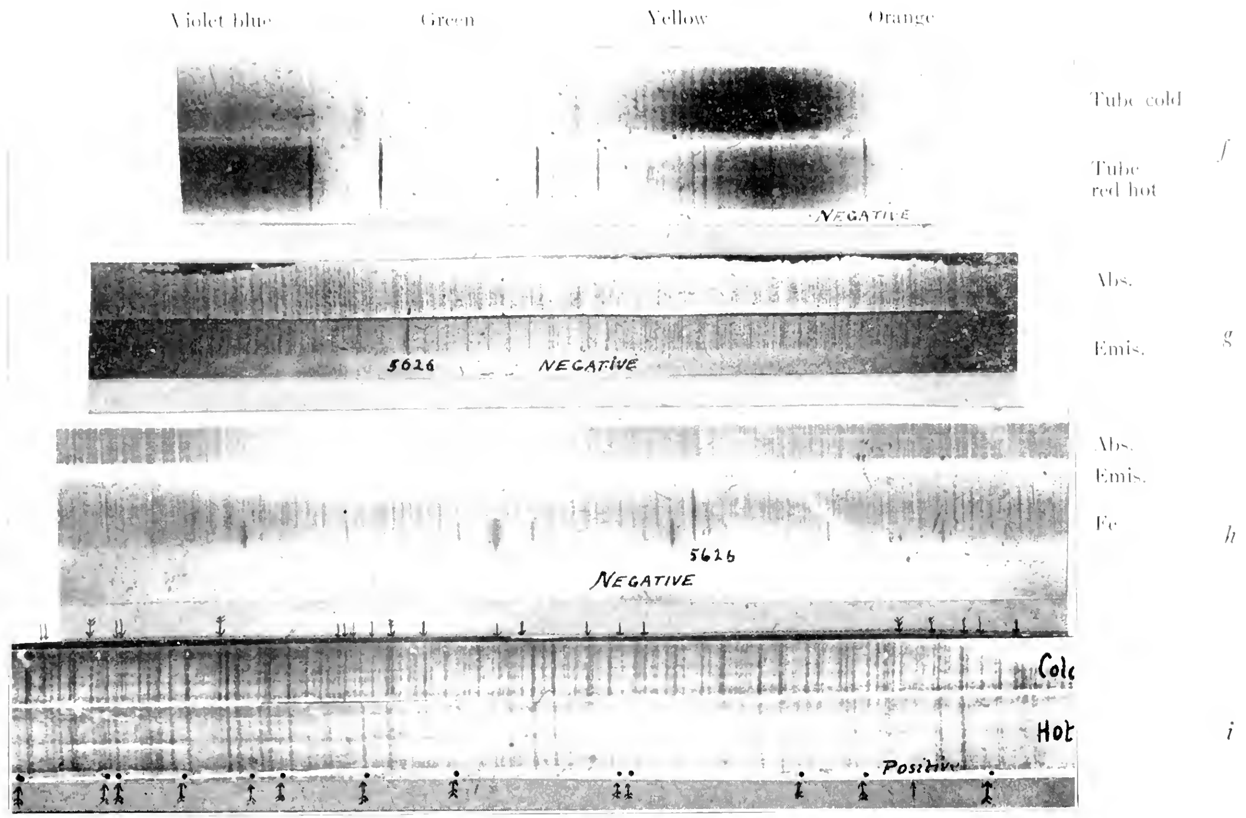

Violet blue
Green
Yellow
Orange
Tube cold
Tube red hot
f
NEGATIVE
Abs.
Emis.
g
5626
NEGATIVE
Abs.
Emis.
Fe
h
5626
NEGATIVE
Cold
Hot
i
Positive

PLATE XV

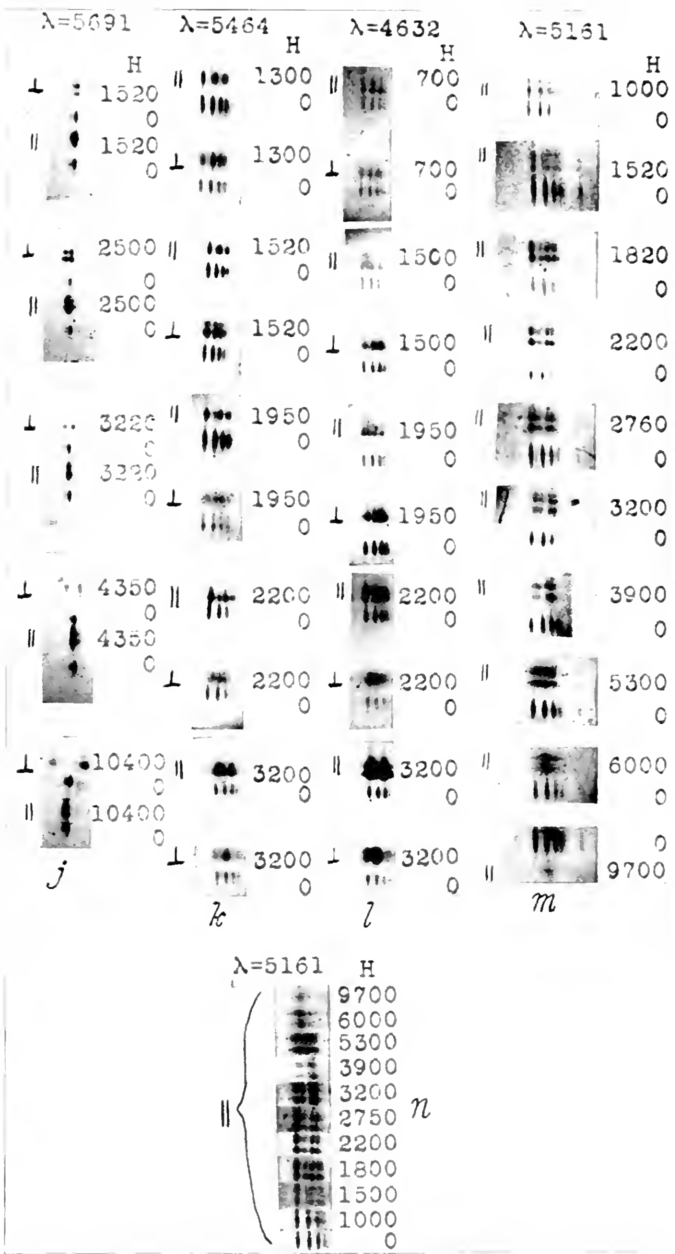
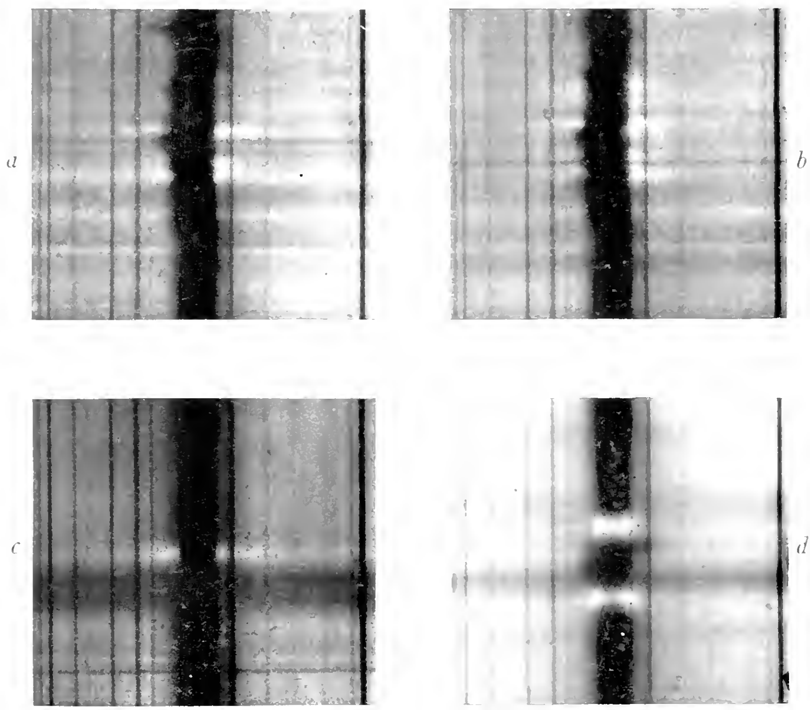


PLATE XVI

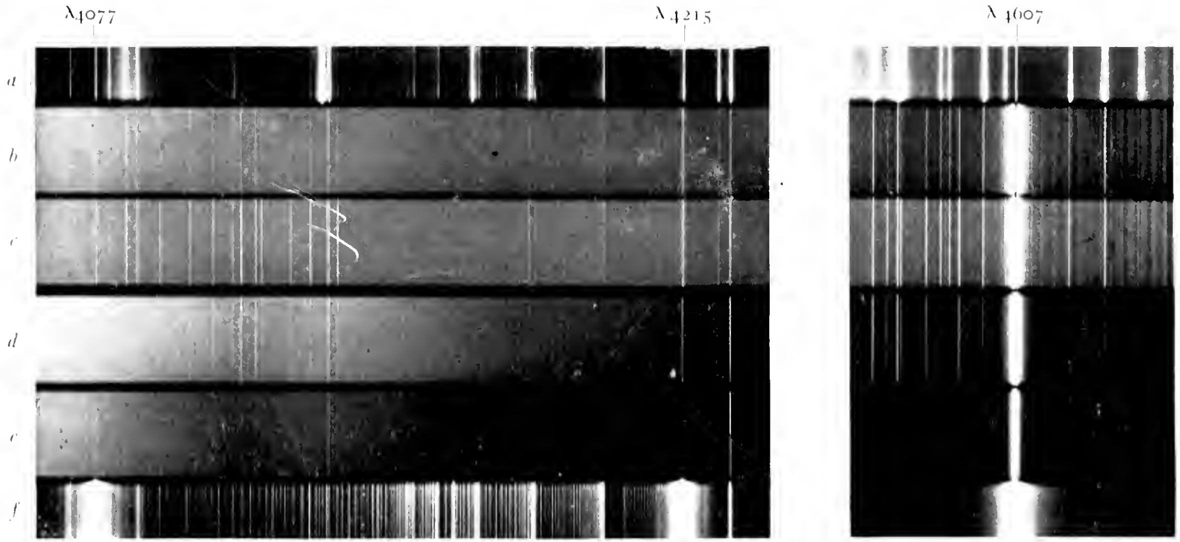


a and *b*. Appearance of the Hα line in an active sun-spot region, showing the bands of "bombs" and the distortions of the line.

c. Spectrum of a "bomb" (close to the dark Hα line) superposed on the spectrum of a facula.

d. Appearance of reversals in an active region.

PLATE XVII



VARIATION WITH TEMPERATURE OF LINES IN FURNACE SPECTRUM OF STRONTIUM

a, Barium arc

b c, Furnace spectra of strontium: *b*, 2350° C., high vapor density; *c*, 2350° C., low vapor density; *d*, 2000° C.; *e*, 1650° C.

f, Strontium arc

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